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GENERALIZED R-WEAKLY COMMUTING MAPPINGS IN NON- ARCHIMEDEAN MENGER SPACE

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ABSTRACT:

We have initiated the concept of generalized R- weakly commuting mappings in non- Archimedean probabilistic metric space for the first time. In fact Sessa initiated weakly commuting mappings in a metric space, Singh and Pant defined the same idea in more general setting of probabilistic metric space. Common fixed point theorems have been obtained by using the concept of generalized R- weakly commuting mappings in non- Archimedean Menger probabilistic metric space in the present paper.

1. INTRODUCTION:

The existence of fixed point theorems for mappings in probabilistic metric space have been obtained by Lee [5], Istratescu [4], Hadzic [3], Singh and Pant [6], [7] Chang [1], and Cho, Sik, Ha and Chang [2] etc. S Sessa [8] has given the concept of weakly commuting mappings and has obtained some fixed point theorems in metric space.

Using the above said concept of Sessa [S 5] was generalized by Singh and Pant [6] by introducing commuting mappings in probabilistic metric space. The above mentioned idea forced us to introduce the definition of generalized R- weakly commuting mappings in non- Archimedean probabilistic metric space. As a consequence of this definition we have obtained some common fixed point theorems in non- Archimedean Menger probabilistic metric space.

NOTE: Through out this paper we consider (X,F,t) a complete non- Archimedean Menger probabilistic metric space of type C_g introduced in [2].

DEFINITION [6]: Two self-mappings f and g on a probabilistic metric space X will be called weakly commuting if $F_{fgp,gfp}(x) \geq F_{fp,gp}(x)\ \forall\ p \in X$ and $x > 0$.

DEFINITION: Two self mappings f and g on a non- Archimedean probabilistic metric space X will be called generalized R- weakly commuting if there exist a real number

$$R > 0 \text{ such that } g(F_{fgp,gfq}(Rx)) \leq g(F_{fp,gq}(x))\ \forall\ p,q \in X \text{ and } x > 0.$$

The following lemma proved by Cho, Sik, Ha and Chang [2].

LEMMA [2]: Let $\{p_n\}$ be a sequence in X such that $\lim_{n\to\infty} F_{p_n,p_{n+1}}(x) = 1\ \forall\ x > 0$. If the sequence $\{p_n\}$ is not a Cauchy sequence in X, then there exist $\varepsilon_0 > 0$, $t_0 > 0$, two sequence $\{m_i\}$ and $\{n_i\}$ of positive integers such that

$$(i)\ m_i > n_i + 1 \text{ and } n_i \to \infty \text{ as } i \to \infty$$

$$(ii)\ g(F_{p_{m_i},p_{n_i}}(t_0)) > g(1-\varepsilon_0) \text{ and } g(F_{p_{m_{i-1}},p_{n_i}}(t_0) \leq g(1-\varepsilon_0)$$

REMARK: If sequence $\{p_n\}$ is not a Cauchy sequence in X and $\lim_{n\to\infty} g(F_{p_n,P_{n+1}}(x) = 0$, then

$$g(1-\varepsilon_0) < g(F_{p_{m_i},p_{n_i}}(t_0)) \leq g(F_{p_{m_i},p_{m_{i-1}}}(t_0)) + g(F_{p_{m_{i-1}},p_{n_i}}(t_0)).$$

Taking $i \to \infty$, $\lim_{i\to\infty} g(F_{p_{m_i},p_{n_i}}(t_0)) = g(1-\varepsilon_0)$ (1)

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Again,

$$g(F_{p_{m_i},p_{n_i}}(t_0)) \le g(F_{p_{m_i},p_{m_{i-1}}}(t_0)) + g(F_{p_{m_{i-1}},p_{n_{i-1}}}(t_0)) + g(F_{p_{n_{i-1}},p_{n_i}}(t_0))$$

and

$$g(F_{p_{m_{i-1}},p_{n_{i-1}}}(t_0)) \le g(F_{p_{m_{i-1}},p_{m_i}}(t_0)) + g(F_{p_{m_i},p_{n_i}}(t_0)) + g(F_{p_{n_i},p_{n_{i-1}}}(t_0))\text{. Taking } i \to \infty,$$

$$\lim_{i\to\infty} g(F_{p_{m_{i-1}},p_{n_{i-1}}}(t_0)) = g(1-\varepsilon_0) \tag{2}$$

Also,

$$g(F_{p_{n_{i-1}},p_{m_{i+1}}}(t_0)) \le g(F_{p_{n_{i-1}},p_{n_i}}(t_0)) + g(F_{p_{n_i},p_{m_i}}(t_0)) + g(F_{p_{m_i},p_{m_{i+1}}}(t_0))$$

and

$$g(F_{p_{m_i},p_{n_i}}(t_0)) \le g(F_{p_{n_i},p_{n_{i-1}}}(t_0)) + g(F_{p_{n_{i-1}},p_{m_{i+1}}}(t_0)) + g(F_{p_{m_{i+1}},p_{m_i}}(t_0)).$$

Taking $i \to \infty$ and from (1), (2) we have

$$\lim_{i\to\infty} g(F_{p_{n_{i-1}},p_{m_{i+1}}}(t_0)) = g(1-\varepsilon_0) \tag{3}$$

At last

$$g(F_{p_{m_i},p_{n_i}}(t_0)) \le g(F_{p_{m_i},p_{n_{i-1}}}(t_0)) + g(F_{p_{n_{i-1}},p_{n_i}}(t_0))$$

and

$$g(F_{p_{m_i},p_{n_{i-1}}}(t_0)) \le g(F_{p_{m_i},p_{n_i}}(t_0)) + g(F_{p_{n_i},p_{n_{i-1}}}(t_0)).$$

As $i \to \infty$ and from (1), (2)

We have

$$\lim_{i\to\infty} g(F_{p_{m_i},p_{n_{i-1}}}(t_0)) = g(1-\varepsilon_0) \tag{4}$$

1.2 LEMMA [2]: If $\varphi:[0,\infty)\to[0,\infty)$ is a function such that φ is upper semi continuous from the right and $\varphi(t) < t$ for all $t > 0$, then

(*a*) For all $t \ge 0, \lim_{n\to\infty} \varphi^n(t) = 0,$ where $\varphi^n(t)$ is the n-th iteration of $\varphi(t)$.

(*b*) If $\{t_n\}$ is a non-decreasing sequence of real numbers and $t_{n+1} \le \varphi(t_n), n = 1, 2, ...$

then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \le \varphi(t)$ for all $t \ge 0$, then $t = 0$.

2 MAIN RESULTS:

2.1 THEOREM: Suppose (X,F,t) be a complete non- Archimedean Menger space and $f, h: X \to X$ be two R- weakly commuting mappings satisfying,

(1) $\forall\ x > 0$, $g(F_{fp,fq}(x)) \le \varphi(\ g(F_{hp,hq}(x)))$, where $\varphi:[0,\infty)\to[0,\infty)$ is a function such that φ is upper semi continuous from the right and $\varphi(t) < t$ for all $t > 0$.

(2) $f(X) \subset h(X)$ and f is continuous.Then f and h have unique common fixed point.

PROOF: Let $p_0 \in X$, choose $p_1 \in X$ such that $f(p_o) = h(p_1)$, because $f(X) \subset h(X)$, so we can construct a sequence $\{p_n\}$ such that $f(p_n) = h(p_{n+1})$, n = 1,2………………

Now,

$$g(F_{fp_n,fp_{n+1}}(x)) \le \varphi(\ g(F_{hp_n,hp_{n+1}}(x))) \Rightarrow\ g(F_{fp_n,fp_{n+1}}(x)) \le \varphi(\ g(F_{fp_{n-1},fp_n}(x)))\text{, so by lemma 1.2}$$

$$\lim_{n\to\infty} g(F_{fp_n,fp_{n+1}}(x)) = 0.$$

$\{fp_n\}$ is a Cauchy sequence. If $\{fp_n\}$ is not a Cauchy sequence then $\exists \in_0 > 0, t_0 > 0$ and set of positive integers $\{m_i\},\{n_i\}$ and then we can apply the above remark for the sequence $\{fp_n\}$.We get $\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) = g(1-\varepsilon_0)$ and

$$\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i}}(t_0)) = g(1-\varepsilon_0)\text{, so}$$

$$g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) = \varphi(g(F_{hp_{m_i},hp_{n_i+1}}(t_0))) < g(F_{hp_{m_i},hp_{n_i+1}}(t_0))$$

$$g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) < g(F_{fp_{m_i-1},fp_{n_i}}(t_0)) \text{ taking } i \to \infty \text{ we get } g(1-\varepsilon_0) < g(1-\varepsilon_0)$$

Which is not possible so $\{fp_n\}$ is a Cauchy sequence.

Since (X,F,t) is complete, $fp_n \to z \in X, hp_n \to z$. Due to continuity of f, $ffp_n \to fz$ and $fhp_n \to fz$. Since f and h are R- weakly commuting so, $g(F_{fhp_n,hfp_n}(Rx)) \leq g(F_{fp_n,hp_n}(x)$, $\forall x > 0$, taking $n \to \infty$ we get,

$$g(F_{fz,hfp_n}(Rx)) \leq g(F_{z,z}(x)) = 0, \forall x > 0 \Rightarrow g(F_{fz,hfp_n}(Rx)) = 0 \Rightarrow hfp_n \to fz$$

z is a common fixed point of f and h, first we prove that $z = fz$ otherwise

$$g(F_{fp_n,ffp_n}(x)) \leq \varphi(\ g(F_{hp_n,hfp_n}(x))),\ \forall x > 0, \text{ taking } n \to \infty \text{ we get}$$

$$g(F_{z,fz}(x)) \leq \varphi(\ g(F_{z,fz}(x))) < (\ g(F_{z,fz}(x))), \text{ which is not possible so } z = fz$$

Again, since $f(X) \subset h(X)$ so $\exists z_1 \in X$ such that $z = fz = hz_1$, then

$$g(F_{ffp_n,fz_1}(x)) \leq \varphi(\ g(F_{hfp_n,hz_1}(x))), \text{ taking } n \to \infty \text{ we get}$$

$$g(F_{fz,fz_1}(x)) \leq \varphi(\ g(F_{fz,fz}(x))) = 0 \text{ so } fz = fz_1 = hz_1 = z$$

Now,

$$g(F_{fz,hz}(Rx)) = \varphi(\ g(F_{fhz_1,hfz_1}(Rx))) \leq g(F_{fz_1,hz_1}(x) = 0 \Rightarrow g(F_{fz,hz}(Rx) = 0 \Rightarrow fz = hz = z$$

Therefore z is a common fixed point of f and h. For uniqueness suppose z_1, z_2 are two common fixed point of f and h. Then

$$g(F_{z_1,z_2}(x)) = g(F_{fz_1,fz_2}(x)) \leq \varphi(g(F_{hz_1,hz_2}(x))) \Rightarrow g(F_{z_1,z_2}(x)) \leq \varphi(g(F_{z_1,z_2}(x)) < g(F_{z_1,z_2}(x)).$$

Which is not possible so $z_1 = z_2$

2.2 THEOREM: Suppose (X,F,t) be a complete non- Archimedean Menger space and $f, h : X \to X$ be two R- weakly commuting mappings satisfying:

(1) $g(F_{fp,fq}(x)) \leq \varphi(\max\{\ g(F_{fp,hp}(x)), g(F_{fq,hq}(x)), g(F_{hp,hq}(x)), g(F_{fp,fq}(x))\ \})$

(2) $f(X) \subset h(X)$ and f is continuous.

Then f and h have unique common fixed point.

PROOF: Since $f(X) \subset h(X)$, so we can construct a sequence $\{p_n\}$ such that $f(p_n) = h(p_{n+1})$, $n = 1,2\ldots$ First we show that $\{fp_n\}$ is a Cauchy sequence,

$$g(F_{fp_n,fp_{n+1}}(x)) \leq \varphi(\ \max\{\ g(F_{fp_n,hp_n}(x)), g(F_{fp_{n+1},hp_{n+1}}(x)), g(F_{hp_n,hp_{n+1}}(x)), g(F_{fp_n,fp_{n+1}}(x))\ \})$$

$$g(F_{fp_n,fp_{n+1}}(x)) \leq \varphi(\ \max\{\ g(F_{fp_n,fp_{n-1}}(x)), g(F_{fp_{n+1},fp_n}(x)), g(F_{fp_{n-1},fp_n}(x)), g(F_{fp_n,fp_{n+1}}(x))\ \})$$

$$g(F_{fp_n,fp_{n+1}}(x)) \leq \varphi(\ \max\{\ g(F_{fp_n,fp_{n-1}}(x)), g(F_{fp_n,fp_{n+1}}(x))\ \}).$$

If $g(F_{fp_n,fp_{n-1}}(x)) \leq g(F_{fp_n,fp_{n+1}}(x))$ then $g(F_{fp_n,fp_{n+1}}(x)) \leq \varphi(g(F_{fp_n,fp_{n+1}}(x)))$, so by lemma 15.1.2 $\lim_{n\to\infty} g(F_{fp_n,fp_{n+1}}(x)) = 0$, again if $g(F_{fp_n,fp_{n-1}}(x)) \geq g(F_{fp_n,fp_{n+1}}(x))$ then $g(F_{fp_n,fp_{n+1}}(x)) \leq \varphi(g(F_{fp_n,fp_{n+1}}(x)))$ so again by lemma 5.1.2 $\lim_{n\to\infty} g(F_{fp_n,fp_{n+1}}(x)) = 0$.

{ fp_n } is a Cauchy sequence. Suppose { fp_n }is not a Cauchy sequence then $\exists \in_0 > 0, t_0 > 0$ and set of positive integers $\{m_i\},\{n_i\}$ and then we can apply the above remark for the sequence { fp_n }. We get, $\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i}}(t_0)) = g(1-\varepsilon_0)$,

$$\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) = g(1-\varepsilon_0),$$

Now,

$$g(F_{p_{n_i+1},P_{m_i}}(t_0)) \le \varphi \,\mathrm{Max}\,\{ g(F_{fp_{n_i+1},hP_{n_i+1}}(t_0)), g(F_{fp_{m_i},hP_{m_i}}(t_0)), g(F_{hp_{n_i+1},hP_{m_i}}(t_0)), g(F_{fp_{n_i+1},fP_{m_i}}(t_0))\}$$
$$g(F_{p_{n_i+1},P_{m_i}}(t_0)) \le \varphi \,\mathrm{Max}\{ g(F_{fp_{n_i+1},fP_{n_i}}(t_0)), g(F_{fp_{m_i},fP_{m_i-1}}(t_0)), g(F_{fp_{n_i},fP_{m_i-1}}(t_0)), g(F_{fp_{n_i+1},fP_{m_i}}(t_0))\}$$

Taking $i \to \infty$ we get, $g(1-\varepsilon_0) \le \varphi\,(\max\{0,0, g(1-\varepsilon_0), g(1-\varepsilon_0)\}) \le \varphi(g(1-\varepsilon_0))$

i.e. $g(1-\varepsilon_0) < g(1-\varepsilon_0)$ which is not possible hence { fp_n }is a Cauchy sequence.

Since *(X,F,t)* is complete, $fp_n \to z \in X, hp_n \to z$. Due to continuity of *f*, $ffp_n \to fz$ and $fhp_n \to fz$. Since *f* and *h* are R-weakly commuting so, as theorem 2.1 $hfp_n \to fz$.

z is a common fixed point of *f* and *h*, first we prove that *z* = *fz* otherwise,

$$g(F_{fp_n,ffp_n}(x)) \le \varphi(\max\{g(F_{fp_n,hp_n}(x)), g(F_{ffp_n,hfp_n}(x)), g(F_{hp_n,hfp_n}(x)), g(F_{fp_n,ffp_n}(x))\}).$$

Taking $n \to \infty$ we get, $g(F_{z,fz}(x) \le \varphi(\max\{g(F_{z,z}(x)), g(F_{fz,fz}(x)), g(F_{z,fz}(x)), g(F_{z,fz}(x))\})$
i.e. $g(F_{z,fz}(x)) \le \varphi(g(F_{z,fz}(x))) < g(F_{z,fz}(x))$, which is not possible so *z* = *fz*.

Again, since $f(X) \subset h(X)$ so $\exists z_1 \in X$ such that $z = fz = hz_1$. Again we show that $z = fz = hz_1 = fz_1$, otherwise

$$g(F_{ffp_n,fz_1}(x)) \le \varphi(\max\{g(F_{ffp_n,hfp_n}(x)), g(F_{fz_1,hz_1}(x)), g(F_{hfp_n,hz_1}(x)), g(F_{ffp_n,fz_1}(x))\}),\ n \to \infty$$
$$g(F_{fz,fz_1}(x)) \le \varphi(\max\{g(F_{fz,fz}(x)), g(F_{fz_1,hz_1}(x)), g(F_{fz,fz_1}(x)), g(F_{fz,fz_1}(x))\}), \text{since } z = fz = hz_1,$$
$$g(F_{z,fz_1}(x)) \le \varphi(\max\{0, g(F_{z,fz_1}(x))\}) \Rightarrow g(F_{z,fz_1}(x)) \le \varphi(g(F_{z,fz_1}(x))) < g(F_{z,fz_1}(x)).$$
Which is not possible so $z = fz = hz_1 = fz_1$.

Again,

$$g(F_{fz,hz}(Rx)) = g(F_{fhz_1,hfz_1}(Rx)) \le g(F_{fz_1,hz_1}(x)) = 0 \Rightarrow g(F_{fz,hz}(Rx) = 0 \Rightarrow fz = hz = z$$

Therefore z is a common fixed point of *f* and *h*. For uniqueness suppose z_1, z_2 are two common fixed point of *f* and *h*, then

$$g(F_{z_1,z_2}(x)) = g(F_{fz_1,fz_2}(x)) \le \varphi(\max\{g(F_{fz_1,hz_1}(x)), g(F_{fz_2,hz_2}(x)), g(F_{hz_1,hz_2}(x)), g(F_{fz_1,fz_2}(x))\})$$
$$g(F_{z_1,z_2}(x)) \le \varphi(\max\{0, g(F_{z_1,z_2}(x))\}) \Rightarrow g(F_{z_1,z_2}(x)) \le \varphi(g(F_{z_1,z_2}(x))) < g(F_{z_1,z_2}(x))$$
Which is not possible so $z_1 = z_2$

2.3 THEOREM: Suppose *(X,F,t)* be a complete non- Archimedean Menger space and $f,\phi,h : X \to X$ are three mappings satisfying

(1) The pairs (f,ϕ) and (f,h) are generalized R –weakly commuting.
(2) $f(X) \subset \phi(X), f(X) \subset h(X)$ and *f* is continuous.
(3) $g(F_{fp,fq}(x)) \le \varphi(\max\{ g(F_{fp,hq}(x)), g(F_{fp,\phi q}(x)), g(F_{hp,fp}(x)), g(F_{\phi p,fp}(x))\}), \forall x > 0$
(4) If $\exists p,q \in X$ such that $\phi p = hq = t$ then $\phi q = hp = t$
Then *f*, ϕ and *h* have unique common fixed point.

PROOF: Since $f(X) \subset \phi(X), f(X) \subset h(X)$ so we can construct a sequence $\{p_n\}$ by using (4) as $fp_{n-1} = \phi p_n = hp_n, n = 1,2,\ldots\ldots\ldots\ldots$. First we show that $\{ fp_n \}$ is a Cauchy sequence.

For $x > 0$,

$$g(F_{fp_n,fp_{n+1}}(x)) \le \varphi(\max\{g(F_{fp_n,hp_{n+1}}(x)), g(F_{fp_n,\phi p_{n+1}}(x)), g(F_{hp_n,fp_n}(x)), g(F_{\phi p_n,fp_n}(x))\})$$
$$g(F_{fp_n,fp_{n+1}}(x)) \le \varphi(\max\{0, g(F_{fp_n,fp_{n-1}}(x))\}) \Rightarrow g(F_{fp_n,fp_{n+1}}(x)) \le \varphi(g(F_{fp_n,fp_{n-1}}(x)))$$

so by lemma 5.1.2 $\lim_{n\to\infty} g(F_{fp_n,fp_{n+1}}(x)) = 0$ for all $x > 0$.

$\{ fp_n \}$ is a Cauchy sequence. Suppose $\{ fp_n \}$ is not a Cauchy sequence then $\exists \in_0 > 0, t_0 > 0$ and set of positive integers $\{m_i\},\{n_i\}$ and then we can apply the above remark for the sequence $\{ fp_n \}$.

We get, $\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i}}(t_0)) = g(1-\varepsilon_0)$, $\lim_{i\to\infty} g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) = g(1-\varepsilon_0)$.

Again,

$$g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) \le \varphi(\max\{g(F_{fp_{m_i},hp_{n_i+1}}(t_0)), g(F_{fp_{m_i},\phi p_{n_i+1}}(t_0)), g(F_{hp_{m_i},fp_{m_i}}(t_0)), g(F_{\phi p_{m_i},fp_{m_i}}(t_0))\})$$

$g(F_{fp_{m_i},fp_{n_i+1}}(t_0)) \le \varphi(\max\{g(F_{fp_{m_i},fp_{n_i}}(t_0)),0\})$, taking $i \to \infty$ we get,

$$g(1-\varepsilon_0) \le \varphi(g(1-\varepsilon_0)) < g(1-\varepsilon_0),$$

which is not possible hence $\{ fp_n \}$ is a Cauchy sequence.

Since (X,F,t) is complete, $fp_n \to z \in X, hp_n \to z, \phi p_n \to z$. Due to continuity of f, $ffp_n \to fz$, $fhp_n \to fz$ and $f\phi p_n \to fz$. Since The pairs (f,ϕ) and (f,h) are generalized R –weakly commuting so as above theorem 5.2.1 $hfp_n \to fz$ and $\phi fp_n \to fz$.

z is a common fixed point of f, ϕ and h, first we prove that $z = fz$ otherwise,

$$g(F_{fp_n,ffp_n}(x)) \le \varphi(\max\{g(F_{fp_n,hfp_n}(x)), g(F_{fp_n,\phi fp_n}(x)), g(F_{hp_n,fp_n}(x)), g(F_{\phi p_n,fp_n}(x))\}), \forall x > 0$$

Taking $n \to \infty$ we get, $g(F_{z,fz}(x)) \le \varphi(\max\{g(F_{z,fz}(x)), g(F_{z,z}(x))\}), \forall x > 0$

i.e. $g(F_{z,fz}(x)) \le \varphi(g(F_{z,fz}(x))) < g(F_{z,fz}(x)), \forall x > 0$, which is not possible so $z = fz$.

Since $f(X) \subset \phi(X), f(X) \subset h(X)$ so $\exists z_1, z_2 \in X$ such that $z = fz = hz_1$ and $z = \phi z_2 = fz$ i.e. by the given condition (4) $z = \phi z_2 = hz_1 = \phi z_1 = hz_2 = fz$. Again we show that $z = fz = hz_1 = \phi z_2 = fz_1 = fz_2$, for this

$$g(F_{ffp_n,fz_1}(x)) \le \varphi(\max\{g(F_{ffp_n,hz_1}(x)), g(F_{ffp_n,\phi z_1}(x)), g(F_{hfp_n,ffp_n}(x)), g(F_{\phi fp_n,ffp_n}(x))\}), \forall x > 0$$

Taking $n \to \infty$, we have

$$g(F_{fz,fz_1}(x)) \le \varphi(\max\{g(F_{fz,z}(x)), g(F_{fz,fz}(x))\}) \Rightarrow g(F_{fz,fz_1}(x)) \le \varphi(0)) \Rightarrow g(F_{fz,fz_1}(x)) = 0$$

i.e. $z = fz = fz_1$ similarly we can prove $z = fz = fz_2$ i.e. $z = fz = hz_1 = \phi z_2 = fz_1 = fz_2$

Since The pairs (f,ϕ) and (f,h) are generalized R –weakly commuting so as above theorem $z = fz = hz$ and $z = fz = \phi z$, i.e. z is a common fixed point of f, ϕ and h. For uniqueness suppose z_1, z_2 are two common fixed point of f, ϕ and h then

$$g(F_{z_1,z_2}(x)) = g(F_{fz_1,fz_2}(x)) \le \varphi(\max\{g(F_{fz_1,hz_2}(x)), g(F_{fz_1,\phi z_2}(x)), g(F_{hz_1,fz_1}(x)), g(F_{\phi z_1,fz_1}(x))\})$$
$$g(F_{z_1,z_2}(x)) \le \varphi(\max\{g(F_{z_1,z_2}(x)),0\}) \Rightarrow g(F_{z_1,z_2}(x)) = 0 \Rightarrow z_1 = z_2$$

2.4 THEOREM: Suppose (X,F,t) be a complete Menger probabilistic metric space. Suppose $f,\phi_i : X \to X$ are $n+1$ mappings ($i = 1, 2 \ldots n$), satisfying,

(a) $F_{fp,fq}(x) \le \varphi(\max\{F_{fp,\phi_i q}(x), F_{\phi_i p,fp}(x)\}$ $\forall\ p,q \in X$ and $x > 0$, $(i = 1,2........n)$

(b) $f(X) \subset \phi_i(X), i = 1,2,..................n$ and f is continuous.

(c) The pairs $(f,\phi_i), i = 1,2,..................n$ are generalized R - weakly commuting mappings.

(d) If $\exists u_1, u_2, ...u_n \in X$ such that $\phi u_1 = \phi u_2 .... = \phi u_n = t$ then $\phi(\sigma(u_1)) = \phi(\sigma(u_2)), ... = \phi(\sigma(u_n)) = t$ where $\sigma : \{x_1, x_2 ....x_n\} \to \{x_1, x_2 ....x_n\}$ are any mapping. Then there exist a unique fixed point of mappings f,ϕ_i, ($i = 1, 2 \ldots\ldots\ldots n$).

Proof is similar to as theorem 2.3.

REFERENCERS:

1. Chang S. S. *Basic theory and application of probabilistic metric spaces* (1), Appl. Math. And Mech. (Eng. Edition) 9 (1988), No.2, 123-133.
2. Cho Y. J., Ha K. S. and Chang S. S. *Common fixed point theorems for compatible mappings of type (A) in non-Archimedean Menger PM-spaces*, Math. Japon. 46 (1) (1997), 169-179.
3. Hadžić O. *A generalization of the contraction principle in probabilistic metric spaces*, Review Research Prirod. Mat. Fak. Novi Sad 10 (1980), 13-21.
4. Istrătescu V. I. *On generalized complete probabilistic metric spaces*, Sem. Probab. Metric Space, Univ. Of Timisoara, No. 25 (1974).
5. Lee B. S. *Fixed point theorems in probabilistic metric spaces*, Math. Japon. 45(1) (1997), 89- 96.
6. Singh S. L. and Pant B. D. *Coincidence theorems*, Math. Japon. 31 (1986), 783-789.
7. Singh S. L. and Pant B. D. *Common* fixed *points of weakly commuting mappings on non –Archimedean Menger spaces*, The Vikram Math. J. 6 (1985), 27-31.
8. Sessa S. *On a weak commutativity condition of mappings in fixed point contractions*, Publ. Inst. Math. (N. S.) (Beograd) 32 (1982), 149-153. MR 85f:54107