On $\mathcal{M}_X\alpha\delta$-closed sets in $\mathcal{M}$-Structures

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ABSTRACT

We introduce a new set called $\mathcal{M}_X\alpha\delta$-closed which are defined on a family of sets satisfying some minimal conditions. Further we studied the properties of $\mathcal{M}_X\alpha\delta$-closed sets.

Keywords: $\mathcal{M}_X\alpha\delta$-closed set.

1. INTRODUCTION

In 1950, H. Maki, J. Umehara and T. Noiri [3] introduced the notions of minimal structure and minimal space. Also they introduced the notion of $m_X$-open set and $m_X$-closed set and characterize those sets using $m_X$-cl and $m_X$-int operators respectively. Further they introduced $m$-continuous functions [11] and studied some of its basic properties. They achieved many important results compatible by the general topology case. Some other results about minimal spaces can be found in [4–11]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [4], [3] and [7].

In this paper we introduce $\mathcal{M}_X\alpha\delta$-closed set. Further, we obtain some characterizations and some properties.

2. PRELIMINARIES

In this section, we introduce the $\mathcal{M}$-structure and define some important subsets associated to the $\mathcal{M}$-structure and the relation between them.

Definition 2.1: [3] Let $X$ be a nonempty set and let $m_X \subseteq P(X)$, where $P(X)$ denote the power set of $X$. Where $m_X$ is an $\mathcal{M}$-structure (or a minimal structure) on $X$, if $\emptyset$ and $X$ belong to $m_X$.

The members of the minimal structure $m_X$ are called $m_X$-open sets, and the pair $(X, m_X)$ is called an $m$-space. The complement of $m_X$-open set is said to be $m_X$-closed.

Definition 2.2: [3] Let $X$ be a nonempty set and $m_X$ an $\mathcal{M}$-structure on $X$. For a subset $A$ of $X$, $m_X$-closure of $A$ and $m_X$-interior of $A$ are defined as follows:

$$m_X\text{-cl}(A) = \bigcap \{F : A \subseteq F, X - F \in m_X\}$$

$$m_X\text{-int}(A) = \bigcup \{U : U \subseteq A, U \in m_X\}$$

Lemma 2.3: [3] Let $X$ be a nonempty set and $m_X$ an $\mathcal{M}$-structure on $X$. For subsets $A$ and $B$ of $X$, the following properties hold:

(a) $m_X\text{-cl}(X - A) = X - m_X\text{-int}(A)$ and $m_X\text{-int}(X - A) = X - m_X\text{-cl}(A)$.

(b) If $X - A \in m_X$, then $m_X\text{-cl}(A) = A$ and if $A \in m_X$ then $m_X\text{-int}(A) = A$.

(c) $m_X\text{-cl}(\emptyset) = \emptyset$ and $m_X\text{-int}(X) = X$.

(d) $A \subseteq B$ then $m_X\text{-cl}(A) \subseteq m_X\text{-cl}(B)$ and $m_X\text{-int}(A) \subseteq m_X\text{-int}(B)$.

(e) $A \subseteq m_X\text{-cl}(A)$ and $m_X\text{-int}(A) \subseteq A$.

(f) $m_X\text{-cl}(m_X\text{-cl}(A)) = m_X\text{-cl}(A)$ and $m_X\text{-int}(m_X\text{-int}(A)) = m_X\text{-int}(A)$.

(g) $m_X\text{-int}(A \cap B) = (m_X\text{-int}(A)) \cap (m_X\text{-int}(B))$ and $(m_X\text{-int}(A)) \cup (m_X\text{-int}(B)) \subseteq m_X\text{-int}(A \cup B)$.

(h) $m_X\text{-cl}(A \cup B) = (m_X\text{-cl}(A)) \cup (m_X\text{-cl}(B))$ and $m_X\text{-int}(A \cup B) \subseteq (m_X\text{-cl}(A)) \cap (m_X\text{-cl}(B))$. 

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Lemma 2.4: [7] Let $(X, m_X)$ be an $m$-space and $A$ a subset of $X$. Then $x \in m_X-cl(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \subseteq m_X$ containing $x$.

Definition 2.5: [10] A minimal structure $m_X$ on a nonempty set $X$ is said to have the property $\mathcal{B}$ if the union of any family of subsets belonging to $m_X$ belongs to $m_X$.

Remark 2.6: A minimal structure $m_X$ with the property $\mathcal{B}$ coincides with a generalized topology on the sense of Lugojan.

Lemma 2.7: [5] Let $X$ be a nonempty set and $m_X$ an $\mathcal{M}$-structure on $X$ satisfying the property $\mathcal{B}$. For a subset $A$ of $X$, the following property hold:

(a) $A \in m_X$ iff $m_X-int(A) = A$
(b) $A \in m_X$ iff $m_X-cl(A) = A$
(c) $m_X-int(A) \in m_X$ and $m_X-cl(A) \in m_X$.

3. $\mathcal{M}_\alpha\delta$-CLOSED SETS

Definition 3.2: A subset $A$ of an $m$-space $(X, m_X)$ is called $\mathcal{M}_\alpha\delta$-closed set if $m_Xα\delta-cl(\{x\}) \subseteq m_Xα\delta-cl(A)$ whenever $A \subseteq U$ and $U$ is $m_X$-open in $(X, m_X)$.

Example 3.3: Let $X = \{a, b, c\}$. Define $\mathcal{M}$-structure on $X$ as follows: $m_X = \{\emptyset, X, \{a\}\}$. Then $m_Xα\delta-open = \{\emptyset, X, \{a, b\}, \{a, c\}\}$, $m_Xα\delta-open = \{\emptyset, X, \{a, b\}, \{a, c\}\}$ and $m_Xα\delta-open = \{\emptyset, X, \{a\}\}$.

Example 3.4: Let $X = \{a, b, c\}$. Define $\mathcal{M}$-structure on $X$ as follows: $m_X = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $m_Xα-open = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$, $m_Xα\delta-open = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $m_Xα\delta-open = \{\emptyset, X, \{a\}\}$.

Definition 3.5: The intersection of all $m_Xα\delta-open$ subsets of $(X, m_X)$ containing $A$ is called the $m_Xα\delta$-kernel of $A$ (briefly, $m_Xα\delta ker(A)$), i.e., $m_Xα\delta ker(A) = \bigcap \{G \in m_Xα\delta O(X) : A \subseteq G\}$.

Theorem 3.6: Let $A$ be a subset of $(X, m_X)$, then $A$ is $m_Xα\delta$-closed if and only if $m_Xα\delta ker(A)$.

Proof: Suppose that $A$ is $m_Xα\delta$-closed and let $D = \{S : S \subseteq X, A \subseteq S : S$ is an $m_Xα\delta open\}$.

Then $m_Xα\delta ker(A) = \bigcap_{S \in D}S$. Observe that $S \in D$ implies that $A \subseteq S$ follows $m_Xα\delta ker(A) \subseteq S$ for all $S \in D$.

Conversely, if $m_Xα\delta ker(A) \subseteq m_Xα\delta ker(A)$, take $S \in α\delta O(X, m_X)$ such that $A \subseteq S$ then by hypothesis, $m_Xα\delta ker(A) \subseteq m_Xα\delta ker(A) \subseteq S$. This shows that $A$ is $m_Xα\delta$-closed.

Theorem 3.7: For subsets $A$ and $B$ of $(X, m_X)$, the following properties hold:

(a) If $A$ is $m_Xα\delta$-closed, then $A$ is $m_Xα\delta$-closed.
(b) If $m_X$ has the property $\mathcal{B}$ and $A$ is $m_Xα\delta$-closed and $m_Xα\delta-open$ then $A$ is $m_Xα\delta$-closed.
(c) If $A$ is $m_Xα\delta$-closed and $A \subseteq B \subseteq cl_{\delta}(A)$ then $B$ is $m_Xα\delta$-closed.

Proof: (a) Let $A$ be an $m_Xα\delta$-closed set in $(X, m_X)$. Let $A \subseteq U$, where $U$ is $m_Xα\delta$-open in $(X, m_X)$. Since $A$ is $m_Xα\delta$-closed, $m_Xα\delta(A) = A$, $m_Xα\delta cl_{\delta}(A) \subseteq U$. Therefore, $A$ is $m_Xα\delta$-closed.

(b) Since $A$ is $m_Xα\delta$-open and $m_Xα\delta$-closed, we have $m_Xα\delta cl_{\delta}(A) \subseteq A$. Therefore, $A$ is $m_Xα\delta$-closed.

(c) Let $U$ be an $m_Xα\delta$-open set of $(X, m_X)$ such that $B \subseteq U$, then $A \subseteq U$. Since $A$ is $m_Xα\delta$-closed, $m_Xα\delta cl_{\delta}(A) \subseteq U$. 

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823
Now \( m_X cl_\delta(B) \subseteq m_X cl_\delta(m_X cl_\delta(A)) \subseteq U \). Therefore, \( B \) is also an \( \mathcal{M}_\alpha \delta \)-closed set of \((X, m_X)\).

**Theorem 3.8:** Union of two \( \mathcal{M}_\alpha \delta \)-closed sets is \( \mathcal{M}_\alpha \delta \)-closed.

**Proof:** Let \( A \) and \( B \) be two \( \mathcal{M}_\alpha \delta \)-closed sets in \((X, m_X)\). Let \( A \cup B \subseteq U \). Since \( A \) and \( B \) are \( \mathcal{M}_\alpha \delta \)-closed sets, \( m_X cl_\delta(A) \subseteq U \) and \( m_X cl_\delta(B) \subseteq U \). This implies that \( m_X cl_\delta(A \cup B) \subseteq m_X cl_\delta(A) \cup m_X cl_\delta(B) \subseteq U \) and so \( m_X cl_\delta(A \cup B) \subseteq U \). Therefore \( A \cup B \) is \( \mathcal{M}_\alpha \delta \)-closed.

**Theorem 3.9:** Let \( m_X \) be an \( \mathcal{M} \) -structure on \( X \) satisfying the property \( \mathcal{B} \) and \( A \subseteq X \). Then \( A \) is an \( \mathcal{M}_\alpha \delta \)-closed set if and only if there does not exist a nonempty \( m_X \alpha \gamma \)-closed set \( F \) such that \( F \neq \emptyset \) and \( F \subseteq m_X cl_\delta(A) \setminus A \).

**Proof:** Suppose that \( A \) is an \( \mathcal{M}_\alpha \delta \)-closed set and let \( F \subseteq X \) be an \( m_X \alpha \gamma \)-closed set such that \( F \subseteq m_X cl_\delta(A) \setminus A \). It follows that, \( A \subseteq X - F \) and \( X - F \) is an \( m_X \alpha \gamma \)-open set. Since \( A \) is an \( \mathcal{M}_\alpha \delta \)-closed set, we have that \( m_X cl_\delta(A) \subseteq X - F \) and \( F \subseteq X - m_X cl_\delta(A) \). Follows that, \( F \subseteq (X - m_X cl_\delta(A)) \cap (X - m_X cl_\delta(A)) = \emptyset \), implying that \( F = \emptyset \).

Conversely, if \( A \subseteq U \) and \( U \) is an \( m_X \alpha \gamma \)-open set, then \( m_X cl_\delta(A) \cap (U - U) \subseteq m_X cl_\delta(A) \cap (X - A) = m_X cl_\delta(A) \setminus A \). Since \( m_X cl_\delta(A) \setminus A \) does not contain subsets \( m_X \alpha \gamma \)-closed sets different from the empty set, we obtain that \( m_X cl_\delta(A) \subseteq U \) and this implies that \( m_X cl_\delta(A) \subseteq U \), in consequence \( A \) is \( m_X \alpha \gamma \)-closed.

**Theorem 3.10:** Let \((X, m_X)\) be an \( m \) -space and \( A \subseteq X \), then \( A \) is \( \mathcal{M}_\alpha \delta \)-open if and only if \( F \subseteq m_X int_\delta(A) \) where \( F \) is \( m_X \alpha \gamma \)-closed and \( F \subseteq A \).

**Proof:** Let \( A \) be an \( \mathcal{M}_\alpha \delta \)-open, \( F \) be \( m_X \alpha \gamma \)-closed set such that \( F \subseteq A \). Then \( X - A \subseteq X - F \), but \( X - F \) is \( m_X \alpha \gamma \)-closed and \( X - A \) is \( \mathcal{M}_\alpha \delta \)-closed implies that \( m_X cl_\delta(X - A) \subseteq X - F \). Follows that \( X - m_X int_\delta(A) \subseteq X - F \). In consequence \( F \subseteq m_X int_\delta(A) \).

Conversely, if \( F \) is \( m_X \alpha \gamma \)-closed, \( F \subseteq A \) and \( F \subseteq m_X int_\delta(A) \). Let \( X - A \subseteq A \) where \( U \) is \( m_X \alpha \gamma \)-open, then \( X - U \subseteq A \) and \( X - U \) is \( m_X \alpha \gamma \)-closed. By hypothesis, \( X - A \subseteq m_X int_\delta(A) \). Follows \( X - m_X cl_\delta(X - A) \subseteq U \). Therefore, \( X - A \) is \( \mathcal{M}_\alpha \delta \)-closed and hence \( A \) is \( \mathcal{M}_\alpha \delta \)-open.

**REFERENCES**


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