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ON CERTAIN GENERATING FUNCTIONS BY GROUP-THEORETIC METHOD FOR GENERALIZED HYPERGEOMETRIC POLYNOMIALS

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ABSTRACT

An attempt is made to derive recurrence relations of ascending, descending type and certain generating functions for the generalized hypergeometric polynomials $u_n(\beta;\gamma;x)$ by Weisner's group-theoretic method which are useful for deriving many other properties of $u_n(\beta;\gamma;x)$. Furthermore, some particular cases of generalized hypergeometric polynomials $u_n(\beta;\gamma;x)$, namely Laguerre, Mexiner, Gottlieb, Krawtchouk and Mexiner-pollaczek polynomials are also pointed out, which are of great important in engineering, science and technology and constitute good models for many systems in various fields.

Mathematics Subject Classification: *33C45, 33C50, 33C80.*

Keywords: *Special Functions, Hypergeometric polynomials, integral representations.*

1. INTRODUCTION:

Recently, V. S. Bhagavan[3] studied some classical properties of $u_n(\beta;\gamma;x)$ such as the addition, multiplication theorems, finite difference formula and various integral representations. It is interesting to note that the polynomial set $u_n(\beta;\gamma;x)$ is a product of x^n and hypergeometric function which enable to derive varies types of generating functions .Because of the important role which hypergeometric polynomials/functions play in problems of physics and applied mathematics , the theory of generating functions has been developed into various directions and found wide applications in different branches of analysis namely infinite series, general theories of linear differential equations, Statistics (various type of distributions), operations research and functions of a complex variables. The hypergeometric functions have also retained its significance in engineering, science and technology. In this paper, an attempt is made to derive some simple generating functions, recurrence relations of ascending and descending type of the generalized hypergeometric polynomials $u_n(\beta;\gamma;x)$ by the series manipulation method which are useful for obtaining many other properties of $u_n(\beta;\gamma;x)$ .The principle interest in our results lies in the fact that a number of orthogonal polynomials, that is, the Laguerre , Meixner, Gottlieb, Krawchouk and Mexiner-Pollaczek polynomial, are derived as the special cases of our results. Some generating relations of these polynomials are well known but some of them are believed to be new in the theory of special functions.

2. DEFINATION:

S. D. Bajpai and M. S. Arora [2] studied the semi-orthogonality property and an integral involving Fox's H-function of $u_n(\beta;\gamma;x)$ defined as

$$u_n(\beta;\gamma;x) \quad = x^n \; {}_2F_1[-n,\beta;\gamma;\frac{1}{x}]\,, \tag{2.1}$$

where n is a non- negative integer , x is any non-zero complex variable and β,γ are independent of n.

Remark: If β,γ are dependent of n then many properties which are valid for β,γ independent of n fail to be valid for β,γ dependent upon n.

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In the paper, after obtaining ascending and descending recurrence relations, the main aim is to derive generating functions for $u_n(\beta;\gamma;x)$ by series manipulation method which are of great important for deriving other properties of $u_n(\beta;\gamma;x)$. The hypergeometric polynomial $u_n(\beta;\gamma;x)$ satisfies the differential equation

$$\{x(1-x)\ D^2 - [(n+\beta-1)-(\gamma+2n-2)x] - n(\gamma+n-1)\}\ u_n(x) \quad = 0,$$
$$where\ u_n(x) = u_n(\beta;\gamma;x)\quad and\ D \equiv \frac{d}{dx}. \tag{2.2}$$

APPLICATIONS:

1. $$\lim_{\beta\to\infty}\left\{\beta^{-n}u_n\left(\beta;1+\alpha;\frac{\beta}{x}\right)\right\} = \frac{n!}{(1+\alpha)_n}x^{-n}L_n^{\alpha}(x), \tag{2.3}$$
   where $L_n^{\alpha}(x)$ is the Laguerre polynomial . [9]

2. $$u_n\left(-y;\gamma;(1-\rho^{-1})^{-1}\right) = (1-\rho^{-1})^{-n}M_n(Y;\gamma,\rho),\ provided\ \gamma>0, 0<\rho<1, y=0,1,2... \tag{2.4}$$
   where $M_n(y\ ;\ \gamma,\rho)$ is the Mexiner polynomial.[10]

3. $$u_n(-y;1;(1-e^{\lambda})^{-\lambda}) = (e^{-\lambda}-1)^{-n}\phi_n(y,\lambda), \tag{2.5}$$
   where $\phi_n(y,\lambda)$ is the Gottlieb polynomial.[9]

4. $$u_n(-y;-N;P) = P^nK_n(y;P,N), \tag{2.6}$$
   where $K_n(y;P,N)$ is the Krawtchouk polynomial.[10]

5. $$u_n(\lambda+i\,y;\ 2\lambda;(1-e^{-2i\phi})^{-1}) = \frac{n!}{(2\lambda)_n}(2i)^{-n}\cos ec^n\phi\ \ P_n^{\lambda}(y\ ;\phi), \tag{2.7}$$
   where $P_n^{\lambda}(y;\phi)$ is the Miexner –pollaczek polynomial.[10]

3. GENERATIG RELATIONS:

Here, we shall deduce only simple generating relations which are useful for obtaining many properties of $u_n(\beta;\gamma;x)$

Consider the series

$$\sum_{n=0}^{\infty}\frac{(\alpha)_n u_n(\beta;\gamma;x)t^n}{n!} = \sum_{n=0}^{\infty}\sum_{k=o}^{n}\frac{(-)^k(\alpha)_n(\beta)_n x^{n-k}t^n}{(n-k)!k!(\gamma)_k}$$
$$= \sum_{n=0}^{\infty}\frac{(\alpha)_k(\beta)_k(-t)^n}{k!(\gamma)_k}\left[\sum_{n=0}^{\infty}\frac{(\alpha+k)_n(xt)^n}{n!}\right]$$
$$\text{(i)}\qquad = (1-xt)^{-\alpha}\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{k!(\gamma)_k}\left[\frac{-t}{1-xt}\right]^k.$$

Therefore,

$$\sum_{n=0}^{\infty}\frac{(\alpha)_n u_n(\beta;\gamma;x)t^n}{n!} = (1-xt)^{-\alpha}\ {}_2F_1\left[\alpha,\beta;\gamma;\frac{-t}{1-xt}\right]. \tag{3.1}$$

Corollary: In particular, if $\alpha=\gamma$ then, we get

(ii) Similarly

$$\sum_{n=0}^{\infty}\frac{u_n(\beta;\gamma;x)t^n}{n!} = \sum_{n=0}^{\infty}\frac{(xt)^n}{n!}\sum_{k=0}^{\infty}\frac{(\beta)_k(-t)^k}{k!(\gamma)_k}$$

Which yields

$$\sum_{n=0}^{\infty}\frac{u_n(\beta;\gamma;x)\,t^n}{n!}=\exp\left(xt\right){}_1F_1\left[\beta;\gamma;-t\right].$$

$$\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^n}{n!}=\left(1-xt\right)^{-\gamma}\ {}_1F_0\left[\beta;-;\frac{-t}{1-xt}\right]$$
$$=\left(1-xt\right)^{\beta-\gamma}\left(1-xt+t\right)^{-\beta}. \tag{3.2}$$

3. RECURRENCE RELATIONS:

(i) Let $G=\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^n}{n!}=(1-xt)^{\beta-\gamma}(1-xt+t)^{-\beta}$.

Then $\frac{\partial G}{\partial x}=-(\beta-\gamma)\,t\,(1-xt)^{-1}\,G+\beta\,t(1-xt+t)^{-1}\,G$ (3.3)

and $\frac{\partial G}{\partial x}=-(\beta-\gamma)\,x\,(1-xt)^{-1}\,G-\beta(1-x)\,t(1-xt+t)^{-1}\,G.$ (3.4)

This implies that $x\frac{\partial G}{\partial x}-t\frac{\partial G}{\partial t}=\beta\,t(1-xt)^{-1}\,G,$ (3.5)

or, $x\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n'(\beta;\gamma;x)\,t^n}{n!}+x\left(1-x\right)\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n'(\beta;\gamma;x)\,t^{n+1}}{n!}-.$

$$\sum_{n=1}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^n}{(n-1)!}-(1-x)\sum_{n=1}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^{n+1}}{(n-1)!}=\beta\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^{n+1}}{n!}.$$

Now, comparing the coefficient of t^n, we finally have

$$(\gamma+n-1)(1-x)\,u_n'(\beta;\gamma;x)+n\,x(1-x)\ \ u_{n-1}'(\beta;\gamma;x)-n\,(\gamma+n-1)$$
$$u_n(\beta;\gamma;x)-n[(n-1)(1-x)+\beta]u_n'(\beta;\gamma;x)=0 \tag{3.6}$$

(ii) Further, from (2.8) and (2.9), we have

$$(1-x)\frac{\partial G}{\partial x}+t\ \frac{\partial G}{\partial t}=-(\beta-\gamma)\,t\,(1-xt)^{-1}\,G, \tag{3.7}$$

or, $(1-x)\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n'(\beta;\gamma;x)\,t^n}{n!}-x(1-x)\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n'(\beta;\gamma;x)\,t^{n+1}}{n!}+.$

$$\sum_{n=1}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^n}{(n-1)!}-\ x\sum_{n=1}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^{n+1}}{(n-1)!}=-(\beta-\gamma)\sum_{n=0}^{\infty}\frac{(\gamma)_n\,u_n(\beta;\gamma;x)\,t^{n+1}}{n!}.$$

which implies that

$$(\gamma+n-1)(1-x)\,u_n'(\beta;\gamma;x)-n\,x(1-x)\ \ u_{n-1}'(\beta;\gamma;x)+n\,(\gamma+n-1)$$

$$u_n(\beta;\gamma;x) - n[(n-1)x - \beta + \gamma]\ u_{n-1}(\beta;\gamma;x) = 0. \tag{3.8}$$

Eliminating $u'_n(\beta;\gamma;x)$ and $u'_{n-1}(\beta;\gamma;x)$ respectively from (3.6) and (3.8), we have the following recurrence relations:

$$D\ u_n(\beta;\gamma;x) = n\ u_{n-1}(\beta;\gamma;x). \tag{3.9}$$

and

$$D\ u_n(\beta;\gamma;x) = \frac{1}{x(1-x)}\{(\gamma+n)\ \ u_{n+1}(\beta;\gamma;x) + [(n+\beta)-(\gamma+2n)x]\,u_n(\beta;\gamma;x)\,. \tag{3.10}$$

These two independent differential recurrence relations determine the linear ordinary differential equation

$$\{x(1-x)D^2 - [(n+\beta-1)-(\gamma+2n-2)x]D - n(\gamma+n-1)\}\ u_n(\beta;\gamma;x) = 0, \tag{3.11}$$

where $D \equiv \frac{d}{dx}$. We now seek linearly independent lowering and raising differential operators B and C each of the form

$$A_1(x,y)\frac{\partial}{\partial x} + A_2(x,y)\frac{\partial}{\partial x} + A_3(x,y)$$

such that

$$B[y^n\ u_n(\beta;\gamma;x)] = a_n y^{n-1} u_{n-1}(\beta;\gamma;x)],\quad n \geq 1, \tag{3.12}$$

and

$$C[y^n\ u_n(\beta;\gamma;x)] = b_n y^{n+1} u_{n+1}(\beta;\gamma;x)],\quad n \geq 1, \tag{3.13}$$

where a_n *and* b_n are expressions in 'n' which are independent of x and y, but may contains β *and* γ.

Using (3.12), (3.13) and recurrence relations (3.9) and (3.10), we get the following linear partial differential operators (lowering and raising) respectively:

$$B = -y^{-1}\frac{\partial}{\partial x}, \tag{3.14}$$

and

$$C = xy(1-x)\frac{\partial}{\partial x} + (2x-1)y^2\frac{\partial}{\partial y} + (\gamma x - \beta)\,y. \tag{3.15}$$

4. BILATERAL GENERATING FUNCTIONS:

In this section, we have derived a theorem and its two corollaries on bilateral generating relations for the ypergeometric polynomial set $u_n(\beta;\gamma;x)$ through Weisner's group- theoretic technique. It is followed by its applications to various classical polynomials.

From (3.15), we have derived the transformed group generated by C is given by

$$\exp(wC)\,f(x,y) = (1-wxy)^{\beta-\gamma}\{1+wy(1-x)\}^{-\beta}\ f\left(x+wxy(1-x), \frac{y}{(1-wxy)\{1+wy(1-x)\}}\right). \tag{4.1}$$

Let us suppose that

$G(x,t)=\sum_{k=0}^{\infty} \mu_k \, U_k(\beta;\gamma;x)t^k$ .Replacing t by twy , we have $G(x,twy)=\sum_{k=o}^{\infty}\mu_k U_k(\beta;\gamma;x)t^k \mathrm{w}^k \mathrm{y}^k$.

(4.2)

Operating both sides of (4.2) by $\exp(wC)$, we get

$$\exp(wC)\,G(x,twy)=\exp(wC)\sum_{k=o}^{\infty}\mu_k U_k(\beta;\gamma;x)t^k \mathrm{w}^k \mathrm{y}^k. \tag{4.3}$$

By using (3.15) and (4.1), we obtain $\exp(wC)G(x,twy)=(1-wxy)^{\beta-\gamma}\{1+wy(1-x)\}^{-\beta}$

$$G\left(x+wxy(1-x),\ \frac{twy}{(1-wxy)\{1+wy(1-x)\}}\right). \tag{4.4}$$

On the other hand, the right hand reduces to $\exp(wC)\sum_{k=o}^{\infty}\mu_k U_k(\beta;\gamma;x)t^k \mathrm{w}^k \mathrm{y}^k$

$$=\sum_{n=0}^{\infty}\frac{(wC)^n}{n!}\left(\sum_{k=o}^{\infty}\mu_k U_k(\beta;\gamma;x)t^k \mathrm{w}^k \mathrm{y}^k\right)$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{\mu_k w^{n+k}}{n\,!}C[y^k U_k(\beta;\gamma;x)]t^k$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{\mu_k(\gamma+k)_{n-k}}{(n-k)\,!}U_k(\beta;\gamma;x)(wy)^n t^k. \tag{4.5}$$

Now equating (4.4) and (4.5) we obtain

$$(1-wxy)^{\beta-\gamma}\{1+wy(1-x)\}^{-\beta}\,G\left(x+wxy(1-x),\ \frac{twy}{(1-wxy)\{1+wy(1-x)\}}\right).$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{\mu_k(\gamma+k)_{n-k}}{(n-k)\,!}U_k(\beta;\gamma;x)(wy)^n t^k. \tag{4.6}$$

Putting y=1 in (4.6), we get

$$(1-wx)^{\beta-\gamma}\{1+wy(1-x)\}^{-\beta}\,G\left(x+wx(1-x),\ \frac{tw}{(1-wx)\{1+w(1-x)\}}\right).$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{\mu_k(\gamma+k)_{n-k}}{(n-k)\,!}U_n(\beta;\gamma;x)\,w^n t^k. \tag{4.7}$$

Thus, we arrive at the following theorem:

Theorem: If there exists a linear generating relation of the form $G(x,t)=\sum_{k=0}^{\infty} \mu_k \, U_k(\beta;\gamma;x)t^k$, where μ_k is independent of x and t , then the following linear generating relation will exit

$$(1-wx)^{\beta-\gamma}\{1+w(1-x)\}^{-\beta}\,G\left(x+wx(1-x),\ \frac{tw}{(1-wx)\{1+w(1-x)\}}\right).=\sum_{n=0}^{\infty}\sigma_n(t)U_n(\beta;\gamma;x)\,w^n, \tag{4.8}$$

where $\sigma_n(t)=\sum_{n=0}^{\infty}\frac{\mu_k(\gamma+k)_{n-k}t^k}{(n-k)!}$. (4.9)

Corollary 1:

Consider the generating function (3.2) ,that is,

$$\sum_{n=0}^{\infty} \frac{u_n(\beta;\gamma;x)t^n}{n!} = \exp(xt)\,{}_1F_1[\beta;\gamma;-t]. \tag{4.10}$$

Let us suppose that $\mu_k = \frac{1}{k!}$ *and* $G(x,t) = \exp(xt)\,{}_1F_1[\beta;\gamma;-t]$. By the above theorem, we have the following generating relation $\sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{(\gamma+k)_{n-k}}{k!\,(n-k)!} u_n(\beta;\gamma;x)\, w^n t^k$

$$= (1-wx)^{\beta-\gamma}\{1+wy(1-x)\}^{-\beta} \exp\left(\frac{xtw}{1-wx}\right) {}_1F_1\left[\beta;\gamma;\frac{-tw}{(1-wx)\{1+wy(1-x)\}}\right]. \tag{4.11}$$

Left hand side can be deduced to

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{(\gamma+k)_{n-k}}{k!\,(n-k)!} u_n(\beta;\gamma;x)\, w^n t^k = \sum_{n=0}^{\infty} \frac{(\gamma)_n\,{}_1F_1[-n;\gamma;-t]}{n!} u_n(\beta;\gamma;x)\, w^n. \tag{4.12}$$

Then $\sum_{n=0}^{\infty} \frac{(\gamma)_n\,{}_1F_1[-n;\gamma;-t]}{n!} u_n(\beta;\gamma;x)\, w^n$

$$= (1-wx)^{\beta-\gamma}\{1+w(1-x)\}^{-\beta} \exp\left(\frac{xtw}{1-wx}\right) {}_1F_1\left[\beta;\gamma;\frac{-tw}{(1-wx)\{1+w(1-x)\}}\right]. \tag{4.13}$$

Now , we replacing –t by z and w by t, we get

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n\,{}_1F_1[-n;\gamma;-z]}{n!} u_n(\beta;\gamma;x)\, t^n$$

$$= (1-xt)^{\beta-\gamma}\{1+t(1-x)\}^{-\beta} \exp\left(\frac{-xzt}{1-xt}\right) {}_1F_1\left[\beta;\gamma;\frac{zt}{(1-xt)\{1+t(1-x)\}}\right]. \tag{4.14}$$

which is a bilateral generating relation for $u_n(\beta;\gamma;x)$.

Corollary 2:

Consider the another generating function (3.1) for $u_n(\beta;\gamma;x)$

i.e., $$\sum_{k=0}^{\infty} \frac{(\alpha)_k\, u_k(\beta;\gamma;x)t^k}{k!} = (1-xt)^{-\alpha}\,{}_2F_1\left[\alpha,\beta;\gamma;\frac{-t}{1-xt}\right]. \tag{4.15}$$

Let us suppose that $\mu_k = \frac{(\alpha)_k}{k!}$ *and* $G(x,t) = (1-xt)^{-\alpha}\,{}_2F_1\left[\alpha,\beta;\gamma;\frac{-t}{1-xt}\right]$.

In a similar way, by applying previous theorem, we get the following generating relation

$$\sum_{n=0}^{\infty} \frac{(\gamma)_n\,{}_2F_1[-n,\alpha;\gamma;z]u_n(\beta;\gamma;x)t^n}{n!} = (1-xt)^{\alpha+\beta-\gamma}\{1+t(1-x)\}^{-\beta}\{1-xt(1-z)\}^{-\alpha}$$

$$\cdot {}_2F_1\left[\alpha,\beta;\gamma;\frac{-zt}{\{1+t(1-x)\}\{1-xt(1-z)\}}\right], \tag{4.16}$$

which is other bilateral generating relation for $u_n(\beta;\gamma;x)$.

5. APPLICATIONS

The following generating relations have been derived from (4.14) and (4.16) by using the conditions of (4.1)

1. $$\sum_{n=0}^{\infty}\frac{n!\,x^{-n}}{(1+\alpha)_n}L_n^{(\alpha)}(x)L_n^{(\alpha)}(z)t^n=(x-t)^{-1-\alpha}x^{1+\alpha}\exp\left(\frac{-t(x+z)}{x-t}\right){}_0F_1\left[-;1+\alpha;\frac{x^2zt}{(x-t)^2}\right].$$

2. $$\sum_{n=0}^{\infty}{}_2F_1\left[-n,\alpha;1+\lambda;z\right]x^{-n}L_n^{(\lambda)}(x)t^n$$
$$=(x-t)^{\alpha-1-\lambda}x^{1+\lambda}\exp\left(\frac{-xt}{x-t}\right)(x-t+tz)^{-\alpha}{}_1F_1\left[\alpha;1+\lambda;\frac{x^2zt}{(x-t)(x-t+tz)}\right].$$

3. $$\sum_{n=0}^{\infty}\frac{(\gamma)_n}{n!}(1-\rho^{-1})^{-n}{}_1F_1[-n;\gamma;z]M_n(y;\gamma,\rho)t^n$$
$$=(1-\rho^{-1})^{\gamma}(1-\rho^{-1}-t)^{-y-\gamma}(1-\rho^{-1}-t\rho^{-1})^{y}.\exp\left(\frac{-zt}{1-\rho^{-1}-t}\right){}_1F_1\left[-y;\gamma;\frac{zt(1-\rho^{-1})^2}{(1-\rho^{-1}-t)(1-\rho^{-1}-\rho^{-1}t)}\right],$$
provided $\gamma>0,\ 0<\rho<1,\ y=0,1,2,\ldots$

4. $$\sum_{n=0}^{\infty}\frac{(\gamma)_n}{n!}(1-\rho^{-1})^{-n}{}_2F_1[-n,\alpha;\gamma;z]M_n(y;\gamma,\rho)t^n$$
$$=(1-\rho^{-1})^{\gamma}(1-\rho^{-1}-t)^{\alpha-y-\gamma}(1-\rho^{-1}-t\rho^{-1})^{y}$$
$$.\{1-\rho^{-1}-t(1-z)\}^{-\alpha}\,{}_2F_1\left[\alpha,-y;\gamma;\frac{zt(1-\rho^{-1})^2}{\{(1-\rho^{-1}-t\rho^{-1})(1-\rho^{-1}-t(1-z)\}}\right],$$
provided $\gamma>0,\ 0<\rho<1,\ y=0,1,2,3,\ldots$

5. $$\sum_{n=0}^{\infty}(e^{-\lambda}-1)^{-n}{}_1F_1[-n,\alpha;\gamma;z]\phi_n(y;\lambda)t^n$$
$$=(1-e^{\lambda})(1-e^{\lambda}-t)^{-y-1}(1-e^{\lambda}-te^{\lambda})^{y}\exp\left(\frac{-zt}{(1-e^{\lambda}-t)}\right).{}_1F_1\left[-y;1;\frac{zt(1-e^{\lambda})^2}{(1-e^{\lambda}-t)(1-e^{\lambda}-te^{\lambda})}\right].$$

6. $$\sum_{n=0}^{\infty}(e^{-\lambda}-1)^{-n}{}_2F_1[-n,\alpha;1;z]\phi_n(y;\lambda)t^n$$
$$=(1-e^{\lambda})(1-e^{\lambda}-t)^{\alpha-y-1}(1-e^{\lambda}-te^{\lambda})^{y}\left\{1-e^{\lambda}-t(1-z)\right\}^{-\alpha}$$
$$.{}_2F_1\left[\alpha,-y;1;\frac{zt(1-e^{\lambda})^2}{(1-e^{\lambda}-te^{\lambda})\{1-e^{\lambda}-t(1-z)\}}\right].$$

7. $$\sum_{n=0}^{N}\frac{(-N)_n}{n!}{}_1F_1[-n;-N;z]K_n(y;P,N)(Pt)^n$$
$$=(1-Pt)^{N-y}(1+t-Pt)^{\lambda}\exp\left(\frac{-zPt}{1-Pt}\right){}_1F_1\left[-y;-N;\frac{zy}{(1-Pt)(1+t-Pt)}\right],$$
provided $0<P<1,\ y=0,1,2,\ldots N.$

8. $$\sum_{n=0}^{N}\frac{(-N)_n}{n!}{}_2F_1[-n;\alpha;-N;z]K_n(y;P,N)(Pt)^n$$
$$=(1-Pt)^{\alpha-y+N}(1+t-Pt)^{y}(1-Pt+zPt)^{-\alpha}\,{}_2F_1\left[\alpha,-y;-N;\frac{zy}{(1+t-Pt)(1-Pt+zPt)}\right],$$
provided $0<P<1,\ y=0,1,2,\ldots N.$

9. $$\sum_{n=0}^{\infty}(2i)^{-n}\operatorname{cosec}^n\phi\ {}_1F_1\left[-n;2\lambda;z\right]P_n^{\lambda}(y;\phi)\,t^n$$

$$=(1-e^{-2i\phi})^{2\lambda}(1-e^{-2i\phi}-t)^{-\lambda+iy}\left\{(1+t)(1-e^{-2i\phi})-t\right\}^{-\lambda-iy}$$

$$.\exp\left\{\frac{-zt}{(1-e^{-2i\phi}-t)}\right\}\ {}_1F_1\left[\lambda;+iy;2\lambda;\frac{zt(1-e^{-2i\phi})^2}{(1-e^{-2i\phi}-t)\{(1+t)(1-e^{-2i\phi})-t\}}\right].$$

10. $$\sum_{n=0}^{\infty}(2i)^{-n}\operatorname{cosec}^n\phi\ {}_2F_1\left[-n,\alpha;2\lambda;z\right]P_n^{\lambda}(y;\phi)\,t^n$$

$$=(1-e^{-2i\phi})^{2\lambda}(1-e^{-2i\phi}-t)^{\alpha-\lambda+iy}\left\{(1+t)(1-e^{-2i\phi})-t\right\}^{-\lambda-iy}(1-e^{-2i\phi}-t+zt)^{-\alpha}$$

$$.\ {}_2F_1\left[\alpha,\lambda+iy;2\lambda;\frac{zt(1-e^{-2i\phi})^2}{\{(1+t)(1-e^{-2i\phi})-t\}\{1-e^{-2i\phi}-t+zt\}}\right].$$

These are all the bilateral (bilinear) generating relations for the Laguerre. Polynomials whereas the results for the Meixner, Gottlieb, Krawtchouk and Meixner – Pollaczek polynomials are believed to be new.

Remark: In a similar way, one can be deduced other types of generating functions from the lowering differential operator "B", which are of great importance in the theory of special functions of mathematical physics.

REFERENCES:

1. Bride Mc, E.B. *Obtaining Generating Functions.* Springer –Verlag, New York, Heidelberg and Berlin (1971).
2. Bajpai, S.D.and Arora M.S.*Semi-orthogonality of a class of the Gauss's hypergeometric polynomials*. Anna.Math.Blaisc Pascal 1(1), 75-83, (1994).
3. Khanna, I.K. and Srinivasa Bhagavan, V. *Generating Functions of Gegenbaur Polynomials* $C_n^{\,v}(x)$ *from a view point of Lie Algebra. Proc. Math. Soc.* 12, 93-98 (1996).
4. Khanna, I.K. and Srinivasa Bhagavan, V. *Weisener's method to obtain generating relations for the generalized polynomial set .J. Phys . A : Math. Gen*.32, 989-998 (1999).
5. Khanna, I.K. and Srinivasa Bhagavan, V and Singh M. N: *Generating relations of the hypergeometric functions by Lie group-theoretic method*, J. Mathematical Physics , Analysis and Geometry, 3, 287-303(2000).
6. Khanna , I.K. and Srinivasa Bhagavan, V: *Lie group-theoretic origins of certain generating functions of the generalized hypergeometric polynomials*, Integral Transforms and Special Functions, Vol.II , No. 2 , 177-188(2001) .
7. Mathai, A.M. *A Handbook of Generalized Special Functions for Statistics and Physical Sciences*, Clarendon Press, Oxford (1993).
8. Miller, W. Jr. *Lie Theory and Special Functions*. Academic Press, New York and London (1968).
9. Rainville, E.D. *Special Functions*. Macmillan, New York; Reprinted by Chelsea Publ. Co., Bronx, New York,(1971).
10. Srivastava, H.M. and Manocha H.L. *A Treatise on Generating Functions*. Ellis Harwood Ltd (1984).11. Wesiner, L. *Group-theoretic origin of certain generating functions.* Pacific J. Math. 5, 1033-1039 (1955).

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