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ON SEPARATION INDEX OF FUZZY SETS

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ABSTRACT

The objective of this paper is to provide information regarding the fact that the way in which fuzzy variables separation index was introduced cannot give us a logical result if the complementation of fuzzy sets is defined on the basis of reference function. Here efforts have been made to show how it is not possible to define separation index in the manner.

***Keywords**: Complement of a fuzzy set, ultra metric measure, separation index of fuzzy sets*

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1. INTRODUCTION:

Data clustering is a process of dividing data elements into classes or clusters so that items in the same class are as similar as possible and items in the different classes are as dissimilar as possible. Depending on the nature of the data and purpose for which clustering is used, different measure of similarity may be used to place items into classes where similarity measures controls how the clusters are formed. In fuzzy clustering data elements can belong to more than one cluster, and associated with each element a set of membership levels.

Separation is an indicator of isolation of clusters from one another .In other words, it can be said that separation measure provides us with the knowledge of the isolation between two fuzzy clusters which is obtained by computing the distance between the fuzzy clusters. Different authors have defined separation index of two fuzzy subsets in different ways.

Polit ([1], p-85) has defined a separation index between two fuzzy subsets A and B as

$$d^*(A,B) = 1 - \frac{\sum_{n=1}^{N}(\mu_{A_n} \cap \mu_{B_n})}{\sum_{n=1}^{N}(\mu_{A_n} \cup \mu_{B_n})}$$

where $\mu_{A_n} \cap \mu_{B_n} = \min(\mu_{A_n}, \mu_{B_n})$ and $\mu_{A_n} \cup \mu_{B_n} = \max(\mu_{A_n}, \mu_{B_n})$

Here μ_{A_n} and μ_{B_n} denote the membership functions of N available data of the two fuzzy subsets A and B. Then it was proved that $d^*(A,B)$ is a ultra metric measure of P(X) of the fuzzy parts of X data. Further, proceeding in this way it was concluded that a possible way of measuring compatibility of two fuzzy sets A and B which was denoted by G(A,B) and was defined in the following manner:

$$G(A,B) = \frac{\sum_{n=1}^{N}(\mu_{A_n} \cap \mu_{B_n})}{\sum_{n=1}^{N}(\mu_{A_n} \cup \mu_{B_n})}$$

One important thing to be remembered here is that this type of compatibility was found by replacing A^c by B in the fuzzy entropy formula proposed by Kosko [2] which is

$$H_A = \frac{M(A \cap A^c)}{M(A \cup A^c)} \text{ and } M(A) = \sum_{n=1}^{N} \mu_{A_n}$$

In this work, we shall show that the aforesaid way of finding compatibility between fuzzy sets can never logically give us the desired result.It is because of the fact that the definition was derived from the formula of finding entropy of fuzzy sets which is defined on the basis of the assumption that fuzzy sets violates excluded middle laws .It is to be noticed here that this phenomenon is caused by the fuzzy properties of fuzzy sets but here we can prove that these assumptions no longer holds if complementation is seen from our standpoints. Excluded middle laws consist of two laws: The first, known as the law of excluded middle, deals with the union of a set A and its complement A^c .

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The second law, known as the law of contradiction, represents the intersection of the set A and its complement A^c. The following equations describe these two laws symbolically:

$$A \cap A^c = \text{the null set } \varphi \quad \text{and } A \cup A^c = \text{the universal set } \Omega$$

On the contrary, in case of fuzzy sets, from the inception it was accepted that $A \cap A^c \neq$ the null set φ and $A \cup A^c \neq$ the universal set Ω

Here the complement A^c of a fuzzy set A is defined with the help of membership function

$$\mu_{A^c}(\mathbf{x}) = 1 - \mu_A(\mathbf{x}), \forall x \in \Omega$$

It is to be worth mentioning here that in order to establish our claim, we have to take the help of the definition of complementation of fuzzy sets on the basis of reference function as defined by Baruah [3].This definition claim to satisfy all the properties of classical sets.The way of defining complementation in the process described seems logical and hence this is taken as a tool for reaching our point.

2. RELATED WORKS:

Qing-Shi Gao, Xiao-Yu Gao and Yue Hu [10] went a step further and found that there is some mistakes Zadeh's fuzzy sets and found that it is incorrect to define the set complement as one minus the membership function of the given fuzzy set. Becuseuse it can be proved that set complement may not exist in Zadeh's fuzzy set. And it also leads to logical confusion, and is seriously mistaken to believe that logics of fuzzy sets necessarily go against classical and normal thinking, logic, and concepts. Hence they went a step forward and found some shortcomings in the Zadeh's fuzzy set theory and consequently they wanted to remove the shortcomings which seemed to them debarred fuzzy sets to satisfy all the properties of classical sets. They introduced a new fuzzy set theory,called *C*-fuzzy set theory which satisfies all the formulas of the classical set theory. The *C*-fuzzy set theory proposed by them was shown to overcome all of the errors and shortcomings, and more reasonably reflects fuzzy phenomenon in the natural world. It satisfies all relations, formulas, and operations of the classical set theory.

From above it is clear that the author of the aforesaid paper was not satisfied with Zadehian theory and hence found an alternate definition. Now let us have a look at the definition which we are going to use.

3. BARUAH'S DEFINITION OF COMPLEMENT OF FUZZY SETS:

According to Baruah ([3] & [4]), a fuzzy set should be defined with the help of two functions of which, one is named as fuzzy membership function and the other is named as fuzzy reference function from which the membership values are to be counted. The membership value is defined to be the difference between the membership function and reference function. Again, the membership value is taken to be different from membership function in the sense that in the Zadehian case these two were taken to be equivalent.

A possible justification behind such a claim is as follows: if a glass is half filled with water then the empty portion is to be counted from the portion which is already filled and hence there arises the necessity of reference function.

Accordingly, for a normal fuzzy number N, if the fuzzy membership function and reference function are denoted by $\mu_2(x)$ and $\mu_1(x)$ respectively such that both lying between 0 and 1 and $\mu_2(x) \geq \mu_1(x)$ then for a fuzzy number denoted by $\{x, \mu_1(x), \mu_2(x), x \epsilon \Omega\}$, we would call the difference $\{\mu_2(x) - \mu_1(x)\}$ as the fuzzy membership value.

As a consequence of the above discussion, for a normal fuzzy number $N = [\alpha, \beta, \gamma]$

defined with a membership function $\mu_N(x)$ and if $\psi_1(x)$, is a continuous nondecreasing function in the interval $[\alpha, \beta]$ and $\psi_2(x)$ is a continuous non increasing function in the interval $[\beta, \gamma]$, then we must have the following:

$$\mu_N(x) = \psi_1(x), \text{ if } \alpha \leq x \leq \beta$$

$$= \psi_2(x), \text{ if } \beta \leq x \leq \gamma$$

$$= 0, \text{ otherwise.}$$

while $\psi_1(\alpha) = \psi_2(\gamma) = 0, \psi_1(\beta) = \psi_2(\beta) = 1$, The membership function of the complement N^c will have to be written as

$$\mu_N c(x) = 1\ ,\ \ -\infty < x < \infty$$

But it is to be pointed out here that $\mu_N c(x)$ is to be counted from $\psi_1(x)$, if $\alpha \le x \le \beta$, from $\psi_2(x)$, if $\beta \le x \le \gamma$ and from zero otherwise in order to keep a difference between the fuzzy membership function and fuzzy membership value. Accordingly, it is defined that the fuzzy membership function of the normal fuzzy number N to be equal to 1 for entire real line and the membership value would have to be counted from the membership value of N.

Thus from the view of extended definition of complementation of fuzzy sets using a reference function, we would get the following two results which were considered most debatable from Zadeh's initial conception.

$A \cap A^c =$ the null set φ and $A \cup A^c =$the universal set Ω

Thus it found that the assumption which was from the very beginning that given a fuzzy set neither its intersection with complement is the null set, nor its union with the complement is the universal set no longer holds. Hence the definition of entropy on the basis of these assumptions cannot produce the desired result. Thus the framework which was taken as the basis for finding the formulae for separation index and compatibility between two fuzzy sets becomes unacceptable since it becomes illogical from the view of complementation of fuzzy sets on the basis of reference function Anything without a proper logical foundation cannot be considered for further use .So from our standpoints the proposed definition is based on a very weak base and hence have nothing to do for further studies.
The above lines are given to yield a deeper understanding of the principle involved and the principle is discussed and this is similar to those obtained for crisp sets. Hence is our claim.

3. CONCLUSIONS:

In this article, one of the definitions of fuzzy variable separation index has been discussed and commented on .The main objective of the paper is to show that these definitions which were derived on the assumptions that fuzzy sets violates excluded middle laws are not at all dependable since these are not logical. The reasons for such claims are also discussed and the different views of what it should include have been proposed. The complementation of fuzzy sets on the basis of reference function which was derived with the help of superimposition of sets played a crucial role in establishing our claim. The main contribution of this paper is to provide information that the separation index and compatibility between fuzzy sets should have to be defined in such a manner that would produce a result leaving no contradiction of any kind.

REFERENCES:

[1] Monique Polit: Artificial intelligence Research and Development, p-85, IOS Press, 2006

[2] Bart Kosko: Fuzzy Entropy and Conditioning, Information Sciences 40, 165-174 (1986)

[3] Hemanta. K. Baruah, Fuzzy Membership with respect to a Reference Function, Journal of the Assam Science Society, Vol.40,No.3,1999,65-73

[4] Hemanta. K. Baruah , Theory of Fuzzy sets Beliefs and Realities , IJEIC, Vol. 2, Issue 2,1-22, 2011

[5] Mamoni Dhar: Fuzzy sets Towards forming Boolean algebra, International Journal Of Energy, Information and communications, Korea,Vol.2 Issue4, 137-142, November, 2011

[6] Mamoni Dhar, Rituparno Chutia, Supahi Mahanta, A note on existing Definition of Fuzzy Entropy, International Journal Of Energy, Information and communications, Korea, Vol. 3 Issue 1, 17-21, February.2012

[7] Mamoni Dhar, On Hwang and Yang's definition of Entropy of Fuzzy sets, International Journal of Latest Trends in Computing, United Kingdom, 496-497 December 2011

[8] Lotfi Zadeh, Fuzzy Sets, Information and Control, Vol8, No.3, 965, 338-353

[9] Andrzej Piegat, A New definition of the Fuzzy sets, Int. J. Appl. Math. Comput. Sci., 2005, Vol15, No.1, 125-140

[10] Qing. Shi Gao, Xiao-Yu Gao and Yue Hu, A new fuzzy set theory satisfying all classical set formulas, journal of computer Science and Technology, 24(4), 798-804, July 2009.

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