



# A NOTE ON INTUITIONISTIC FUZZY $\mathbb{T}$ -GENERALIZED SEMI IRRESOLUTE MAPPINGS

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## ABSTRACT

*In this paper a new class of mapping called intuitionistic fuzzy  $\mathbb{T}$ -generalized semi irresolute mapping in intuitionistic fuzzy topological space is introduced and some of its properties are studied.*

**Keywords and Phrases:** Intuitionistic fuzzy topology, intuitionistic fuzzy  $\mathbb{T}$ -generalized semi closed set, intuitionistic fuzzy  $\mathbb{T}$ -generalized semi open set, intuitionistic fuzzy  $\mathbb{T}$ -generalized semi irresolute mapping, intuitionistic fuzzy  $\mathbb{T}_{1/2}$  space and intuitionistic fuzzy  $\mathbb{T}_{1/2}$  space.

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## 1. INTRODUCTION

After the introduction of Fuzzy set (FS) by Zadeh [12] in 1965 and fuzzy topology by Chang [3] in 1967, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1983 as a generalization of fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy  $\mathbb{T}$ -generalized semi irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy  $\mathbb{T}$ -generalized semi irresolute mapping and established the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

## 1. PRELIMINARIES

**Definition 2.1:** [1] Let  $X$  be a non empty fixed set. An intuitionistic fuzzy set (IFS in short)  $A$  in  $X$  is an object having the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  where the functions  $\mu_A(x): X \rightarrow [0, 1]$  and  $\nu_A(x): X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$  respectively and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ .

**Definition 2.2:** [1] Let  $A$  and  $B$  be IFSs of the forms

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$ . Then

- (1)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$
- (2)  $A = B$  if and only if  $A \subseteq B$  and  $B \subseteq A$
- (3)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (4)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (5)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation  $A = \langle x, \mu_A, \nu_A \rangle$  instead of  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ . Also for the sake of simplicity, we shall use the notation  $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$  instead of  $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$ . The intuitionistic fuzzy sets  $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$  and  $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$  are respectively the empty set and the whole set of  $X$ .

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**Definition 2.3:** [4] An intuitionistic fuzzy topology (IFT in short) on a non empty  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms:

- (a)  $0_-, 1_- \in \tau$ ,
- (b)  $G_1 \cap G_2 \in \tau$ , for any  $G_1, G_2 \in \tau$ ,
- (c)  $\cup G_i \in \tau$  for any arbitrary family  $\{G_i / i \in J\} \subseteq \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS for short) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS for short) in  $X$ .

**Definition 2.4:** [4] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then, the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

**Definition 2.5:** [7] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be

- (i) An intuitionistic fuzzy closed (IF closed in short) mapping if  $f(A)$  is an IFCS  $A$  in  $Y$  for each IFCS in  $X$
- (ii) An intuitionistic fuzzy  $\alpha$ -closed (IF $\alpha$  closed in short) mapping if  $f(A)$  is an IF $\alpha$ CS in  $Y$  for every IFCS  $A$  in  $X$
- (iii) An intuitionistic fuzzy semiclosed (IFS closed in short) mapping if  $f(A)$  is an IFSCS in  $Y$  for every IFCS  $A$  in  $X$
- (iv) An intuitionistic fuzzy preclosed (IFP closed in short) mapping if  $f(A)$  is an IFPCS in  $Y$  for every IFCS  $A$  in  $X$ .

**Definition 2.6:** [7] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be

- (i) An intuitionistic fuzzy generalized closed (IFG closed in short) mapping if  $f(A)$  is an IFGCS in  $Y$  for every IFCS  $A$  in  $X$
- (ii) An intuitionistic fuzzy pre-regular closed (IFPR closed in short) mapping if  $f(A)$  is an IFRCS in  $Y$  for every IFRCS  $A$  in  $X$ .

**Definition 2.7:**[7] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an intuitionistic fuzzy almost closed (IFA closed in short) mapping if  $f(A)$  is an IFCS in  $Y$  for every IFRCS  $A$  in  $X$ .

**Definition 2.8:**[8] A subset of  $A$  of a space  $(X, \tau)$  is called:

- (i) regular open if  $A = \text{int}(\text{cl}(A))$
- (ii)  $\mathbb{T}$  open if  $A$  is the union of regular open sets.

**Definition 2.9:** An IFS  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$  in an IFTS  $(X, \tau)$  is called an

- (a) intuitionistic fuzzy semi closed set [7] (IFSCS) if  $\text{int}(\text{cl}(A)) \subseteq A$
- (b) intuitionistic fuzzy  $\alpha$ -closed set [7] (IF $\alpha$ CS) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (c) intuitionistic fuzzy pre-closed set [7] (IFPCS) if  $\text{cl}(\text{int}(A)) \subseteq A$
- (d) intuitionistic fuzzy regular closed set [7] (IFRCS) if  $\text{cl}(\text{int}(A)) = A$
- (e) intuitionistic fuzzy generalized closed set [9] (IFGCS) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS
- (f) intuitionistic fuzzy generalized semi closed set [8] (IFGSCS) if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS
- (g) intuitionistic fuzzy  $\alpha$  generalized closed set [8] (IF $\alpha$ GCS) if  $\alpha\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS.
- (h) intuitionistic fuzzy  $\mathbb{T}$ -generalized semi closed set [8] (IF $\mathbb{T}$ GSCS) if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IF $\mathbb{T}$ OS.

An IFS  $A$  is called intuitionistic fuzzy semi open set, intuitionistic fuzzy  $\alpha$ -open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set, intuitionistic fuzzy  $\alpha$  generalized open set and intuitionistic fuzzy  $\mathbb{T}$ -generalized semi open set (IFSOS, IF $\alpha$ OS, IFPOS, IFROS, IFGOS, IFGSOS, IF $\alpha$ GOS and IF $\mathbb{T}$ GSOS) if the complement of  $A^c$  is an IFSCS, IF $\alpha$ CS, IFPCS, IFRCS, IFGCS, IFGSCS, IF $\alpha$ GCS and IF $\mathbb{T}$ GSCS respectively.

**Result 2.10:** [8] Every IFCS, IFSCS, IFGCS, IFRCS, IF $\alpha$ CS, IFGSCS is an IF $\mathbb{T}$ GSCS but the converses may not be true in general. (Every IFOS, IFSOS, IFGOS, IFROS, IF $\alpha$ OS, IFGSOS is an IF $\mathbb{T}$ GSOS but the converses may not be true in general).

**Definition 2.11:** [5] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be intuitionistic fuzzy continuous (IF continuous in short) if  $f^{-1}(B) \in \text{IFO}(X)$  for every  $B \in \sigma$ .

**Definition 2.12:** [7] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be  
(a) intuitionistic fuzzy semi continuous (IFS continuous in short) if  $f^{-1}(B) \in \text{IFSO}(X)$  for every  $B \in \sigma$   
(b) intuitionistic fuzzy  $\alpha$ - continuous ( $\text{IF}\alpha$  continuous in short) if  $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$  for every  $B \in \sigma$   
(c) intuitionistic fuzzy pre continuous (IFP continuous in short) if  $f^{-1}(B) \in \text{IFPO}(X)$  for every  $B \in \sigma$   
(d) intuitionistic fuzzy completely continuous if  $f^{-1}(B) \in \text{IFRO}(X)$  for every  $B \in \sigma$ .

**Definition 2.13:** [6] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\gamma$  continuous ( $\text{IF}\gamma$  continuous in short) if  $f^{-1}(B)$  is an  $\text{IF}\gamma\text{OS}$  in  $(X, \tau)$  for every  $B \in \sigma$ .

**Definition 2.14:** [12] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if  $f^{-1}(B) \in \text{IFGCS}(X)$  for every IFCS  $B$  in  $Y$ .

**Result 2.15:** [12] Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

**Definition 2.16:** [10] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if  $f^{-1}(B)$  is an IFGSCS in  $(X, \tau)$  for every IFCS  $B$  of  $(Y, \sigma)$ .

**Definition 2.17:** [9] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\mathbb{T}$ - generalized continuous ( $\text{IF}\mathbb{T}\text{GS}$  continuous in short) if  $f^{-1}(B)$  is an  $\text{IF}\mathbb{T}\text{GSCS}$  in  $(X, \tau)$  for every IFCS  $B$  of  $(Y, \sigma)$ .

**Definition 2.18:** [12] An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $T_{1/2}$  ( $\text{IFT}_{1/2}$  in short) space if every IFGCS in  $X$  is an IFCS in  $X$ .

**Definition 2.19:** [11] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if  $f^{-1}(B) \in \text{IFCS}(X)$  for every IFCS  $B$  in  $Y$ .

**Definition 2.20:** [11] Let  $f$  be a mapping from an IFTS  $(X, \tau)$  into an IFTS  $(Y, \sigma)$ . Then  $f$  is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if  $f^{-1}(B) \in \text{IFGCS}(X)$  for every IFGCS  $B$  in  $Y$ .

**Result 2.21:** [8] Every IFGSCS is an  $\text{IF}\mathbb{T}\text{GSCS}$  but not conversely.

**Definition 2.22:**[8] An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\mathbb{T}T_{1/2}$  ( $\text{IF}\mathbb{T}T_{1/2}$  in short) space if every  $\text{IF}\mathbb{T}\text{GSCS}$  in  $X$  is an IFCS in  $X$ .

**Definition 2.23:**[8] An IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\mathbb{T}T_{1/2}$  ( $\text{IF}\mathbb{T}T_{1/2}$  in short) space if every  $\text{IF}\mathbb{T}\text{GSCS}$  in  $X$  is an IFGCS in  $X$ .

### 3. INTUITIONISTIC FUZZY $\mathbb{T}$ - GENERALIZED SEMI IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy  $\mathbb{T}$ - generalized semi irresolute mappings and studied some of their properties.

**Definition 3.1:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\mathbb{T}$ - generalized semi irresolute ( $\text{IF}\mathbb{T}\text{GS}$  irresolute) mapping if  $f^{-1}(A)$  is an  $\text{IF}\mathbb{T}\text{GSCS}$  in  $(X, \tau)$  for every  $\text{IF}\mathbb{T}\text{GSCS}$   $A$  of  $(Y, \sigma)$ .

**Theorem 3.2:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $\text{IF}\mathbb{T}\text{GS}$  irresolute mapping, then  $f$  is an  $\text{IF}\mathbb{T}\text{GS}$  continuous mapping but not conversely.

**Proof:** Let  $A$  be any IFCS in  $Y$ . Since every IFCS is an  $\text{IF}\mathbb{T}\text{GSCS}$ ,  $A$  is an  $\text{IF}\mathbb{T}\text{GSCS}$  in  $Y$ . Since  $f$  is an  $\text{IF}\mathbb{T}\text{GS}$  irresolute mapping,  $f^{-1}(A)$  is an  $\text{IF}\mathbb{T}\text{GSCS}$  in  $X$ . Hence  $f$  is an  $\text{IF}\mathbb{T}\text{GS}$  continuous mapping.

**Example 3.3:** Let  $X = \{ a, b \}$ ,  $Y = \{ u, v \}$  and  $G_1 = \langle x, (0.1_a, 0.2_b), (0.3_a, 0.3_b) \rangle$ ,  $G_2 = \langle x, (0.1_a, 0.1_b), (0.2_a, 0.3_b) \rangle$ ,  $G_3 = \langle x, (0.1_a, 0.2_b), (0.2_a, 0.3_b) \rangle$ ,  $G_4 = \langle x, (0.1_a, 0.1_b), (0.3_a, 0.3_b) \rangle$ ,  $G_5 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.3_b) \rangle$ ,  $G_6 = \langle y, (0.4_u, 0.2_v), (0.6_u, 0.7_v) \rangle$ . Then  $\tau = \{ 0, G_1, G_2, G_3, G_4, G_5, 1 \}$  and  $\sigma = \{ 0, G_6, 1 \}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then  $f$  is an  $\text{IF}\mathbb{T}\text{GS}$  continuous mapping. Let  $B = \langle y, (0.1_u, 0.1_v) \rangle$ ,

$(0.3_u, 0.3_v) \rangle$  is an IF $\mathbb{T}$ GSCS in Y. But  $f^{-1}(B) = \langle x, (0.1_a, 0.1_b), (0.3_a, 0.3_b) \rangle$  is not an IF $\mathbb{T}$ GSCS in X. Therefore f is not an IF $\mathbb{T}$ GS irresolute mapping.

**Theorem 3.4:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an IF $\mathbb{T}$ GS irresolute mapping, then f is an IFGS continuous mapping but not conversely.

**Proof:** Let A be an IFCS in Y. Since every IFCS is an IF $\mathbb{T}$ GSCS, A is an IF $\mathbb{T}$ GSCS in Y. By hypothesis,  $f^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in X. This implies  $f^{-1}(A)$  is an IFGSCS in X. Hence f is an IFGS continuous mapping.

**Example 3.5:** Let  $X = \{ a, b \}$ ,  $Y = \{ u, v \}$  and  $G_1 = \langle x, (0.2_a, 0.4_b), (0.5_a, 0.4_b) \rangle$ ,  $G_2 = \langle x, (0.1_a, 0.3_b), (0.3_a, 0.4_b) \rangle$ ,  $G_3 = \langle x, (0.1_a, 0.3_b), (0.5_a, 0.4_b) \rangle$ ,  $G_4 = \langle x, (0.2_a, 0.4_b), (0.3_a, 0.4_b) \rangle$ ,  $G_5 = \langle x, (0.4_a, 0.4_b), (0.3_a, 0.4_b) \rangle$ ,  $G_6 = \langle y, (0.4_u, 0.2_v), (0.5_u, 0.5_v) \rangle$ . Then  $\tau = \{ 0_-, G_1, G_2, G_3, G_4, G_5, 1_- \}$  and  $\sigma = \{ 0_-, G_6, 1_- \}$  are IFTs on X and Y respectively. Define a mapping  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . Then f is an IFGS continuous mapping. Let  $B = \langle y, (0_u, 0.3_v), (0.5_u, 0.4_v) \rangle$  is an IF $\mathbb{T}$ GSCS in Y. But  $f^{-1}(B) = \langle x, (0_a, 0.3_b), (0.5_a, 0.4_b) \rangle$  is not an IF $\mathbb{T}$ GSCS in X. Therefore f is not an IF $\mathbb{T}$ GS irresolute mapping.

**Theorem 3.6:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is an IF $\mathbb{T}$ GS irresolute mapping, then f is an IF continuous mapping if X is an IF $\mathbb{T}$  $T_{1/2}$  space.

**Proof:** Let A be an IFCS in Y. Then A is an IF $\mathbb{T}$ GSCS in Y. Since f is an IF $\mathbb{T}$ GS irresolute mapping,  $f^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in X. Since X is an IF $\mathbb{T}$  $T_{1/2}$  space,  $f^{-1}(A)$  is an IFCS in X. Hence f is an IF continuous mapping.

**Theorem 3.7:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be any two IF $\mathbb{T}$ GS irresolute mappings. Then  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is an IF $\mathbb{T}$ GS irresolute mapping.

**Proof:** Let A be an IF $\mathbb{T}$ GSCS in Z. Then by hypothesis,  $g^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in Y. Since f is an IF $\mathbb{T}$ GSCS irresolute mapping,  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{T}$ GSCS in X. That is  $(g \circ f)^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in X. Hence  $g \circ f$  is an IF $\mathbb{T}$ GS irresolute mapping.

**Theorem 3.8:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{T}$ GS irresolute mapping and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be an IF $\mathbb{T}$ GS continuous mapping. Then  $g \circ f : (X, \tau) \rightarrow (Z, \eta)$  is an IF $\mathbb{T}$ GS continuous mapping.

**Proof:** Let A be an IFCS in Z. Then by hypothesis,  $g^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in Y. Since f is an IF $\mathbb{T}$ GS irresolute mapping,  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{T}$ GSCS in X. That is  $(g \circ f)^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in X. Hence  $g \circ f$  is an IF $\mathbb{T}$ GS continuous mapping.

**Theorem 3.9:** If  $f : (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{T}$ GS irresolute mapping, then f is an IFG irresolute mapping if X is an IF $\mathbb{T}$  $T_{1/2}$  space.

**Proof:** Let A be an IFGCS in Y. Then A is an IF $\mathbb{T}$ GSCS in Y. Therefore  $f^{-1}(A)$  is an IF $\mathbb{T}$ GSCS in X, by hypothesis. Since X is an IF $\mathbb{T}$  $T_{1/2}$  space,  $f^{-1}(A)$  is an IFGCS in X. Hence f is an IFG irresolute mapping.

**Theorem 3.10:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF $\mathbb{T}$  $T_{1/2}$  spaces:

- (i) f is an IF $\mathbb{T}$ GS irresolute mapping
- (ii)  $f^{-1}(B)$  is an IF $\mathbb{T}$ GSOS in X for each IF $\mathbb{T}$ GSOS B in Y
- (iii)  $\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$  for each IFS B of Y.

**Proof:** (i)  $\Rightarrow$  (ii): Obviously true.

(ii)  $\Rightarrow$  (iii): Let B be any IFS in Y. Clearly  $B \subseteq \text{cl}(B)$ . Then  $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B))$ . Since  $\text{cl}(B)$  is an IFCS in Y,  $\text{cl}(B)$  is an IF $\mathbb{T}$ GSCS in Y. Therefore  $f^{-1}(\text{cl}(B))$  is an IF $\mathbb{T}$ GSCS in X, by hypothesis. Since X is an IF $\mathbb{T}$  $T_{1/2}$  space,  $f^{-1}(\text{cl}(B))$  is an IFCS in X. Hence  $\text{cl}(f^{-1}(B)) \subseteq \text{cl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$ . That is  $\text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ .

(iii)  $\Rightarrow$  (i): Let B be an IF $\mathbb{T}$ GSCS in Y. Since Y is an IF $\mathbb{T}$  $T_{1/2}$  space, B is an IFCS in Y and  $\text{cl}(B) = B$ .

Hence  $f^{-1}(B) = f^{-1}(\text{cl}(B)) \supseteq \text{cl}(f^{-1}(B))$ , by hypothesis. But clearly  $f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$ . Therefore,  $\text{cl}(f^{-1}(B)) = f^{-1}(B)$ .

This implies  $f^{-1}(B)$  is an IFCS in  $X$  and hence it is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Thus  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping.

**Theorem 3.11:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF continuous mapping and  $(Y, \sigma)$  is an IF $\mathbb{I}\mathbb{F}$  $T_{1/2}$  space. Then the following statements are equivalent:

- (i)  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping
- (ii)  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping

**Proof:** (i)  $\Rightarrow$  (ii): Follows from the theorem 3.2.

(ii)  $\Rightarrow$  (i): Let  $f$  be an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping. Let  $A$  be an IF $\mathbb{I}\mathbb{F}$ GSCS in  $(Y, \sigma)$ . Since  $(Y, \sigma)$  is an IF $\mathbb{I}\mathbb{F}$  $T_{1/2}$ space,  $A$  is an IFCS in  $(Y, \sigma)$  and by hypothesis  $f^{-1}(A)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $(X, \tau)$ . Therefore  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping.

**Theorem 3.12:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from an IFTS  $X$  into IFTS  $Y$ . Then the following conditions are equivalent

- (i)  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping
- (ii)  $f^{-1}(B)$  is an IF $\mathbb{I}\mathbb{F}$ GSOS in  $X$  for every IF $\mathbb{I}\mathbb{F}$ GSOS  $B$  in  $Y$ .

**Proof:** (i)  $\Rightarrow$  (ii): Let  $B$  be an IF $\mathbb{I}\mathbb{F}$ GSOS in  $Y$ , then  $B^c$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . Since  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping,  $f^{-1}(B^c)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . But  $f^{-1}(B^c) = (f^{-1}(B))^c$ , implies  $f^{-1}(B)$  is an IF $\mathbb{I}\mathbb{F}$ GSOS in  $X$ .

(ii)  $\Rightarrow$  (i): Let  $B$  be an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . By our assumption  $f^{-1}(B^c)$  is an IF $\mathbb{I}\mathbb{F}$ GSOS in  $X$  for every IF $\mathbb{I}\mathbb{F}$ GSOS  $B^c$  in  $Y$ . But  $f^{-1}(B^c) = (f^{-1}(B))^c$ , which implies  $f^{-1}(B)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Hence  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping.

**Theorem 3.13:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  is an IF $\alpha$  continuous mapping, then  $g \circ f: (X, \tau) \rightarrow (Z, \delta)$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Proof:** Let  $A$  be an IFCS in  $Z$ . Then  $g^{-1}(A)$  is an IF $\alpha$ CS in  $Y$ , Since  $g$  is IF $\alpha$  continuous. Since every IF $\alpha$ CS is an IF $\mathbb{I}\mathbb{F}$ GSCS,  $g^{-1}(A)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . But  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping. Therefore  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Hence  $g \circ f$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Theorem 3.14:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  is an IF $\alpha$ G continuous mapping, then  $g \circ f: (X, \tau) \rightarrow (Z, \delta)$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Proof:** Let  $A$  be an IFCS in  $Z$ . By assumption,  $g^{-1}(A)$  is an IF $\alpha$ GCS in  $Y$ . Since every IF $\alpha$ GCS is an IF $\mathbb{I}\mathbb{F}$ GSCS,  $g^{-1}(A)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . But  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping, implies  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Hence  $g \circ f$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Theorem 3.15:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  is an IFG continuous mapping, then  $g \circ f: (X, \tau) \rightarrow (Z, \delta)$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Proof:** Let  $A$  be an IFCS in  $Z$ . By assumption,  $g^{-1}(A)$  is an IFGCS in  $Y$ . Since every IFGCS is an IF $\mathbb{I}\mathbb{F}$ GSCS,  $g^{-1}(A)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . But  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping. Therefore  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Hence  $g \circ f$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Theorem 3.16:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  is an IFGS continuous mapping, then  $g \circ f: (X, \tau) \rightarrow (Z, \delta)$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Proof:** Let  $A$  be an IFCS in  $Z$ . Then  $g^{-1}(A)$  is an IFGSCS in  $Y$ , Since  $g$  is an IFGS continuous. Since every IFGSCS is an IF $\mathbb{I}\mathbb{F}$ GSCS,  $g^{-1}(A)$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $Y$ . Since  $f$  is an IF $\mathbb{I}\mathbb{F}$ GS irresolute mapping,  $f^{-1}(g^{-1}(A))$  is an IF $\mathbb{I}\mathbb{F}$ GSCS in  $X$ . Hence  $g \circ f$  is an IF $\mathbb{I}\mathbb{F}$ GS continuous mapping.

**Definition 3.17:** Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then  $\mathbb{I}\mathbb{F}$ -generalized Semi closure of  $A$  ( $\mathbb{I}\mathbb{F}\text{gscl}(A)$  in short) and  $\mathbb{I}\mathbb{F}$ - generalized semi interior of  $A$  ( $\mathbb{I}\mathbb{F}\text{gsint}(A)$  in short) are defined by

$$\pi_{\text{gsint}}(A) = \cup \{ G / G \text{ is an IF}\mathcal{IT}\text{GSOS in } X \text{ and } G \subseteq A \}$$

$$\pi_{\text{gscl}}(A) = \cap \{ K / K \text{ is an IF}\mathcal{IT}\text{GSCS in } X \text{ and } A \subseteq K \}.$$

**Proposition 3.18:** If A is an IFS in X, then  $A \subseteq \pi_{\text{gscl}}(A) \subseteq \text{cl}(A)$ .

**Proof:** The result follows from the definition.

**Theorem 3.19:** If A is an IF $\mathcal{IT}$ GSCS in X then  $\pi_{\text{gscl}}(A) = A$ .

**Proof:** Since A is an IF $\mathcal{IT}$ GSCS,  $\pi_{\text{gscl}}(A)$  is the smallest IF $\mathcal{IT}$ GSCS which contains A, which is nothing but A. Hence  $\pi_{\text{gscl}}(A) = A$ .

**Theorem 3.20:** If A is an IF $\mathcal{IT}$ GSOS in X then  $\pi_{\text{gsint}}(A) = A$ .

**Proof:** Similar to above theorem.

**Proposition 3.21:** Let  $(X, \tau)$  be any IFTS. Let A and B be any two intuitionistic fuzzy sets in  $(X, \tau)$ . Then the intuitionistic fuzzy  $\mathcal{IT}$ -generalized Semi closure operator satisfies the following properties.

- (i)  $A \subseteq \pi_{\text{gscl}}(A)$
- (ii)  $\pi_{\text{gsint}}(A) \subseteq A$
- (iii)  $A \subseteq B \Rightarrow \pi_{\text{gscl}}(A) \subseteq \pi_{\text{gscl}}(B)$
- (iv)  $A \subseteq B \Rightarrow \pi_{\text{gsint}}(A) \subseteq \pi_{\text{gsint}}(B)$

**Theorem 3.22:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an IF $\mathcal{IT}$ GS irresolute mapping, then  $f(\pi_{\text{gscl}}(A)) \subseteq \text{cl}(A)$  for every IFS A of X.

**Proof:** Let A be an IFCS of X. Then  $\text{cl}(f(A))$  is an IFCS of Y. Since every IFCS is an IF $\mathcal{IT}$ GSCS,  $\text{cl}(f(A))$  is an IF $\mathcal{IT}$ GSCS

in Y. Since f is IF $\mathcal{IT}$ GS irresolute,  $f^{-1}(\text{cl}(f(A)))$  is IF $\mathcal{IT}$ GSCS in X. Clearly  $A \subseteq f^{-1}(\text{cl}(f(A)))$ .

Therefore  $\pi_{\text{gscl}}(A) \subseteq \pi_{\text{gscl}}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$ . Hence  $f(\pi_{\text{gscl}}(A)) \subseteq \text{cl}(f(A))$  for every IFS A of X.

**Theorem 3.23:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is IF $\mathcal{IT}$ GS irresolute, then  $\pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$  for every IFS B of Y.

**Proof:** Let B be an IFS of Y. Then  $\text{cl}(B)$  is an IFCS of Y. since every IFCS is an IF $\mathcal{IT}$ GSCS,  $\text{cl}(B)$  is an IF $\mathcal{IT}$ GSCS in Y. By hypothesis,  $f^{-1}(\text{cl}(B))$  is IF $\mathcal{IT}$ GSCS in X. Clearly  $B \subseteq \text{cl}(B)$  implies  $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B))$ . Therefore,  $\pi_{\text{gscl}}(f^{-1}(B)) \subseteq \pi_{\text{gscl}}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$ . Hence  $\pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$  for every IFS B of Y.

**Theorem 3.24:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a mapping from an IFTS X into IFTS Y. Then the following conditions are equivalent

- (i) f is an IF $\mathcal{IT}$ GS irresolute mapping
- (ii)  $f^{-1}(B)$  is an IF $\mathcal{IT}$ GSOS in X, for each IF $\mathcal{IT}$ GSOS in Y
- (iii)  $f^{-1}(\pi_{\text{gsint}}(B)) \subseteq \pi_{\text{gsint}}(f^{-1}(B))$
- (iv)  $\pi_{\text{gscl}}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$  for every IFS B of Y.

**Proof:**

(i)  $\Rightarrow$  (ii): is obviously true.

(ii)  $\Rightarrow$  (iii): Let B be an IF $\mathcal{IT}$ GSCS in Y and  $\pi_{\text{gsint}}(B) \subseteq B$ . Then  $f^{-1}(\pi_{\text{gsint}}(B)) \subseteq f^{-1}(B)$ . Since  $\pi_{\text{gsint}}(B)$  is an IF $\mathcal{IT}$ GSOS in Y,  $f^{-1}(\pi_{\text{gsint}}(B))$  is an IF $\mathcal{IT}$ GSOS in X, by hypothesis. Hence  $f^{-1}(\pi_{\text{gsint}}(B)) \subseteq \pi_{\text{gsint}}(f^{-1}(B))$ .

(iii)  $\Rightarrow$  (iv): is obvious by taking complement in (iii).

(iv)  $\Rightarrow$  (i): Let B be an IF $\mathcal{IT}$ GSCS in Y and  $\pi_{\text{gscl}}(B) = B$ . Hence  $f^{-1}(B) = f^{-1}(\pi_{\text{gscl}}(B)) \supseteq \pi_{\text{gscl}}(f^{-1}(B))$ . Therefore,  $\pi_{\text{gscl}}(f^{-1}(B)) = f^{-1}(B)$ . This implies  $f^{-1}(B)$  is an IF $\mathcal{IT}$ GSCS in X. Thus f is an IF $\mathcal{IT}$ GS irresolute mapping.



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