A NOTE ON INTUITIONISTIC FUZZY $\pi$-GENERALIZED SEMI IRRESOLUTE MAPPINGS

1S. Maragathavalli & 2K. Ramesh*

1Department of Mathematics, Sree Saraswathi thyagaraja College, Pollachi, Tamilnadu, India
E-mail: smvalli@rediffmail.com

2Department of Mathematics, SVS College of Engineering, Coimbatore, Tamilnadu, India
E-mail: rameshfuzzy@gmail.com

(Received on: 11-02-12; Accepted on: 26-03-12)

ABSTRACT

In this paper a new class of mapping called intuitionistic fuzzy $\pi$-generalized semi irresolute mapping in intuitionistic fuzzy topological space is introduced and some of its properties are studied.

Keywords and Phrases: Intuitionistic fuzzy topology, intuitionistic fuzzy $\pi$-generalized semi closed set, intuitionistic fuzzy $\pi$-generalized semi open set, intuitionistic fuzzy $\pi$-generalized semi irresolute mapping, intuitionistic fuzzy $\pi_{1/2}$ space and intuitionistic fuzzy $\pi_{1/2}$ space.

AMS Classification Code: 54A40.

1. INTRODUCTION

After the introduction of Fuzzy set (FS) by Zadeh [12] in 1965 and fuzzy topology by Chang [3] in 1967, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy set (IFS) was introduced by Atanassov in 1983 as a generalization of fuzzy sets. In 1997, Coker [4] introduced the concept of intuitionistic fuzzy topological space. In this paper, we introduce the notion of intuitionistic fuzzy $\pi$-generalized semi irresolute mapping in intuitionistic fuzzy topological space and studied some of their properties. We provide some characterizations of intuitionistic fuzzy $\pi$-generalized semi irresolute mapping and established the relationships with other classes of early defined forms of intuitionistic fuzzy mappings.

1. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) $A$ in $X$ is an object having the form $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set $A$ respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let $A$ and $B$ be IFSs of the forms $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ and $B = \{ (x, \mu_B(x), \nu_B(x)) / x \in X \}$. Then

(1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(3) $A^c = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$

(4) $A \cap B = \{ (x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)) / x \in X \}$

(5) $A \cup B = \{ (x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)) / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$ instead of $A = \{ (x, \mu_A(x), \nu_A(x)) / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ (x, (\mu_A, \mu_B), (\nu_A, \nu_B)) / x \in X \}$ instead of $A = \{ (x, (\mu_A, \mu_B), (\nu_A, \nu_B)) / x \in X \}$.

*Corresponding author: 2K. Ramesh*, E-mail: rameshfuzzy@gmail.com
**Definition 2.3:** [4] An intuitionistic fuzzy topology (IFT in short) on a non empty $X$ is a family $\tau$ of IFSs in $X$ satisfying the following axioms:

(a) $\emptyset, X \in \tau$,
(b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(c) $\bigcup_{i \in I} G_i \in \tau$ for any arbitrary family $\{G_i / i \in I\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in $\tau$ is known as an intuitionistic fuzzy open set (IFOS for short) in $X$. The complement $A^c$ of an IFOS $A$ in an IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS for short) in $X$.

**Definition 2.4:** [4] Let $(X, \tau)$ be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in $X$. Then, the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \bigcup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \bigcap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$
Definition 2.12: [7] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be (a) intuitionistic fuzzy semi continuous (IFS continuous in short) if \( f^{-1}(B) \in \text{IFSO}(X) \) for every \( B \in \sigma \) (b) intuitionistic fuzzy \( \alpha \)-continuous (IF\( \alpha \) continuous in short) if \( f^{-1}(B) \in \text{IFIo}(X) \) for every \( B \in \sigma \) (c) intuitionistic fuzzy pre continuous (IFP continuous in short) if \( f^{-1}(B) \in \text{IFPO}(X) \) for every \( B \in \sigma \) (d) intuitionistic fuzzy completely continuous if \( f^{-1}(B) \in \text{IFRO}(X) \) for every \( B \in \sigma \).

Definition 2.13: [6] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \gamma \) continuous (IF\( \gamma \) continuous in short) if \( f^{-1}(B) \) is an IF\( \gamma \)OS in \((X, \tau)\) for every \( B \in \sigma \).

Definition 2.14: [12] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if \( f^{-1}(B) \in \text{IFGCS}(X) \) for every IFCS \( B \) in \( Y \).

Result 2.15: [12] Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

Definition 2.16: [10] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if \( f^{-1}(B) \) is an IFGCS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

Definition 2.17: [9] A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \pi \)-generalized continuous (IF\( \pi \)GS continuous in short) if \( f^{-1}(B) \) is an IF\( \pi \)GCS in \((X, \tau)\) for every IFCS \( B \) of \((Y, \sigma)\).

Definition 2.18: [12] An IFTS \((X, \tau)\) is called an intuitionistic fuzzy \( T_{1/2} \) (IFT\( T_{1/2} \) in short) space if every IFGCS in \( X \) is an IFCS in \( X \).

Definition 2.19: [11] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy irresolute (IF irresolute in short) if \( f^{-1}(B) \in \text{IFRO}(X) \) for every IFCS \( B \) in \( Y \).

Definition 2.20: [11] Let \( f \) be a mapping from an IFTS \((X, \tau)\) into an IFTS \((Y, \sigma)\). Then \( f \) is said to be an intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if \( f^{-1}(B) \in \text{IFGCS}(X) \) for every IFGCS \( B \) in \( Y \).

Result 2.21: [8] Every IFGCS is an IF\( \pi \)GCS but not conversely.

Definition 2.22: [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \( \pi_\alpha T_{1/2} \) (IF\( \pi_\alpha T_{1/2} \) in short) space if every IF\( \pi \)GCS in \( X \) is an IFCS in \( X \).

Definition 2.23: [8] An IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy \( \pi_\beta T_{1/2} \) (IF\( \pi_\beta T_{1/2} \) in short) space if every IF\( \pi \)GCS in \( X \) is an IFCS in \( X \).

3. INTUITIONISTIC FUZZY \( \pi \)-GENERALIZED SEMI IRRESOLUTE MAPPINGS

In this section, we have introduced intuitionistic fuzzy \( \pi \)-generalized semi irresolute mappings and studied some of their properties.

Definition 3.1: A mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) is called an intuitionistic fuzzy \( \pi \)-generalized semi irresolute (IF\( \pi \)GS irresolute) mapping if \( f^{-1}(A) \) is an IF\( \pi \)GCS in \((X, \tau)\) for every IF\( \pi \)GCS \( A \) of \((Y, \sigma)\).

Theorem 3.2: If \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \pi \)GS irresolute mapping, then \( f \) is an IF\( \pi \)GS continuous mapping but not conversely.

Proof: Let \( A \) be any IFCS in \( Y \). Since every IFCS is an IF\( \pi \)GCS, \( A \) is an IF\( \pi \)GCS in \( Y \). Since \( f \) is an IF\( \pi \)GS irresolute mapping, \( f^{-1}(A) \) is an IF\( \pi \)GSCS in \( X \). Hence \( f \) is an IF\( \pi \)GS continuous mapping.

Example 3.3: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \langle x, (0.1_a, 0.2_b), (0.3_a, 0.3_b) \rangle \), \( G_2 = \langle x, (0.1_a, 0.2_b), (0.3_a, 0.3_b) \rangle \), \( G_3 = \langle x, (0.1_a, 0.2_b), (0.3_a, 0.3_b) \rangle \). Then \( \tau = \{ 0, G_1, G_2, G_3 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IF\( \pi \)GS continuous mapping. Let \( B = \langle y, (0.1_u, 0.1_v) \rangle \),
Theorem 3.4: If \( f : (X, \tau) \to (Y, \sigma) \) is an IF\( \pi \)GS irresolute mapping, then \( f \) is an IFGS continuous mapping but not conversely.

Proof: Let \( A \) be an IFCS in \( Z \). Then \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f^{-1}(A) \) is an IF\( \pi \)GCS in \( X \). Therefore \( f \) is not an IF\( \pi \)GS irresolute mapping.

Example 3.5: Let \( X = \{ a, b \} \), \( Y = \{ u, v \} \) and \( G_1 = \{ x, (0.2a, 0.4b), (0.3a, 0.4b) \}, G_2 = \{ x, (0.1a, 0.3b), (0.3a, 0.4b) \}, G_3 = \{ x, (0.2a, 0.4b), (0.3a, 0.4b) \}, G_4 = \{ x, (0.1a, 0.3b), (0.2a, 0.5b) \}, G_5 = \{ x, (0.1a, 0.3b), (0.3a, 0.4b) \}, G_6 = \{ x, (0.2a, 0.4b), (0.2a, 0.5b) \} \). Then \( \tau = \{ 0, G_1, G_2, G_3, G_4, G_5, 1 \} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f : (X, \tau) \to (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). Then \( f \) is an IFGS continuous mapping. Let \( B = \{ y, (0a, 0.3b), (0.5a, 0.4b) \} \) is an IF\( \pi \)GCS in \( Y \). But \( f^{-1}(B) = \{ x, (0a, 0.3b), (0.3a, 0.4b) \} \) is not an IF\( \pi \)GCS in \( X \). Therefore \( f \) is not an IF\( \pi \)GS irresolute mapping.

Theorem 3.6: If \( f : (X, \tau) \to (Y, \sigma) \) is an IF\( \pi \)GS irresolute mapping, then \( f \) is an IF continuous mapping if \( X \) is an IF\( \pi \)T_{1/2} space.

Proof: Let \( A \) be an IFCS in \( Z \). Then \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f^{-1}(A) \) is an IFGCS in \( X \). Therefore \( f \) is not an IF\( \pi \)GS irresolute mapping. Conversely.

Theorem 3.7: If \( f : (X, \tau) \to (Y, \sigma) \) is an IF\( \pi \)GS irresolute mapping, then \( f \) is an IF continuous mapping.

Proof: Let \( A \) be an IF\( \pi \)GCS in \( Z \). Then \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f^{-1}(A) \) is an IF\( \pi \)GCS in \( X \). Therefore \( f \) is not an IF\( \pi \)GS irresolute mapping.

Theorem 3.8: If \( f : (X, \tau) \to (Y, \sigma) \) is an IF\( \pi \)GS irresolute mapping, then \( f \) is an IFGCS in \( X \). Therefore \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f \) is an IF\( \pi \)GCS in \( X \).

Proof: Let \( A \) be an IFCS in \( Z \). Then \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f^{-1}(A) \) is an IFGCS in \( X \). Therefore \( f \) is not an IF\( \pi \)GS irresolute mapping.

Theorem 3.9: If \( f : (X, \tau) \to (Y, \sigma) \) is an IF\( \pi \)GS irresolute mapping, then \( f \) is an IFGCS in \( X \). Therefore \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f \) is an IF\( \pi \)GCS in \( X \).

Proof: (i) \( \Rightarrow \) (ii): Obviously true.

(ii) \( \Rightarrow \) (iii): Let \( B \) be any IFS in \( Y \). Clearly \( B \subseteq \text{cl}(B) \). Since \( \text{cl}(B) \) is an IFCS in \( Y \), \( \text{cl}(B) \) is an IFGCS in \( Y \). Therefore \( f^{-1}(\text{cl}(B)) \) is an IF\( \pi \)GCS in \( X \). Since \( X \) is an IF\( \pi \)T_{1/2} space, \( f^{-1}(\text{cl}(B)) \) is an IF\( \pi \)GCS in \( X \). Hence \( \text{cl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B)) \) for each IFS \( B \) of \( Y \).

(iii) \( \Rightarrow \) (i): Let \( B \) be an IF\( \pi \)GCS in \( Y \). Since \( Y \) is an IF\( \pi \)T_{1/2} space, \( B \) is an IFCS in \( Y \) and \( \text{cl}(B) = B \).
Hence $f^{-1}(B) = f^{-1}(\text{cl}(B)) \supseteq \text{cl}(f^{-1}(B))$, by hypothesis. But clearly $f^{-1}(B) \subseteq \text{cl}(f^{-1}(B))$. Therefore, $\text{cl}(f^{-1}(B)) = f^{-1}(B)$.

This implies $f^{-1}(B)$ is an IFCS in X and hence it is an IFGTCS in X. Thus $f$ is an IFGTG irresolute mapping.

**Theorem 3.11:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping and $(Y, \sigma)$ is an IF $\pi_{T_{1/2}}$ space. Then the following statements are equivalent:

(i) $f$ is an IFGTG irresolute mapping

(ii) $f$ is an IFGTG continuous mapping

**Proof:** (i) $\Rightarrow$ (ii): Follows from the theorem 3.2.

(ii) $\Rightarrow$ (i): Let $f$ be an IFGTG continuous mapping. Let $A$ be an IFGTGCS in $(Y, \sigma)$. Since $(Y, \sigma)$ is an IF $\pi_{T_{1/2}}$ space, $A$ is an IFCS in $(Y, \sigma)$ and by hypothesis $f^{-1}(A)$ is an IFGTGCS in $(X, \tau)$. Therefore $f$ is an IFGTG irresolute mapping.

**Theorem 3.12:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into IFTS Y. Then the following conditions are equivalent

(i) $f$ is an IFGTG irresolute mapping

(ii) $f^{-1}(B)$ is an IFGTGOS in X for every IFGTGOS $B$ in Y.

**Proof:** (i) $\Rightarrow$ (ii): Let $B$ be an IFGTGOS in Y, then $B^c$ is an IFGTGCS in Y. Since $f$ is an IFGTG irresolute mapping, $f^{-1}(B^c)$ is an IFGTGCS in X. But $f^{-1}(B^c) = (f^{-1}(B))^c$, implies $f^{-1}(B)$ is an IFGTGOS in X.

(ii) $\Rightarrow$ (i): Let $B$ be an IFGTGCS in Y. By our assumption $f^{-1}(B^c)$ is an IFGTGOS in X for every IFGTGOS $B^c$ in Y. But $f^{-1}(B^c) = (f^{-1}(B))^c$, which implies $f^{-1}(B)$ is an IFGTGCS in X. Hence $f$ is an IFGTG irresolute mapping.

**Theorem 3.13:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFGTG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF$\alpha$ continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFGTG continuous mapping.

**Proof:** Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IF$\alpha$CS in Y. Since $g$ is IF$\alpha$ continuous, every IF$\alpha$CS is an IFGTGCS, $g^{-1}(A)$ is an IFGTGCS in Y. But $f$ is an IFGTG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFGTGSCS in X. Hence $g \circ f$ is an IFGTG continuous mapping.

**Theorem 3.14:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFGTG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF$\alpha$ continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFGTG continuous mapping.

**Proof:** Let A be an IFCS in Z. By assumption, $g^{-1}(A)$ is an IF$\alpha$CS in Y. Since every IF$\alpha$CS is an IFGTGCS, $g^{-1}(A)$ is an IFGTGCS in Y. But $f$ is an IFGTG irresolute mapping, implies $f^{-1}(g^{-1}(A))$ is an IFGTGSCS in X. Hence $g \circ f$ is an IFGTG continuous mapping.

**Theorem 3.15:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFGTG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IF$\delta$ continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFGTG continuous mapping.

**Proof:** Let A be an IFCS in Z. By assumption, $g^{-1}(A)$ is an IFGCS in Y. Since every IFGCS is an IFGTGCS, $g^{-1}(A)$ is an IFGTGCS in Y. But $f$ is an IFGTG irresolute mapping. Therefore $f^{-1}(g^{-1}(A))$ is an IFGTGSCS in X. Hence $g \circ f$ is an IFGTG continuous mapping.

**Theorem 3.16:** Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFGTG irresolute mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is an IFG continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFGTG continuous mapping.

**Proof:** Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFGCS in Y. Since $g$ is an IFG continuous. Since every IFGCS is an IFGTGCS, $g^{-1}(A)$ is an IFGTGCS in Y. But $f$ is an IFGTG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFGTGSCS in X. Hence $g \circ f$ is an IFGTG continuous mapping.

**Definition 3.17:** Let $A$ be an IFS in an IFTS $(X, \tau)$. Then $\pi$-generalized Semi closure of A ($\pi_{gscl}(A)$ in short) and $\pi$-generalized semi interior of A ($\pi_{gsint}(A)$ in short) are defined by
Proposition 3.18: If $A$ is an IFS in $X$, then $A \subseteq \pi_{gscl}(A) \subseteq \text{cl}(A)$.

Proof: The result follows from the definition.

Theorem 3.19: If $A$ is an IF$\pi$GSCS in $X$ then $\pi_{gscl}(A) = A$.

Proof: Since $A$ is an IF$\pi$GSCS, $\pi_{gscl}(A)$ is the smallest IF$\pi$GSCS which contains $A$, which is nothing but $A$. Hence $\pi_{gscl}(A) = A$.

Theorem 3.20: If $A$ is an IF$\pi$GSOS in $X$ then $\pi_{gsint}(A) = A$.

Proof: Similar to above theorem.

Proposition 3.21: Let $(X, \tau)$ be any IFTS. Let $A$ and $B$ be any two intuitionistic fuzzy sets in $(X, \tau)$. Then the intuitionistic fuzzy $\pi$-generalized Semi closure operator satisfies the following properties.

(i) $A \subseteq \pi_{gscl}(A)$
(ii) $\pi_{gsint}(A) \subseteq A$
(iii) $A \subseteq B \Rightarrow \pi_{gscl}(A) \subseteq \pi_{gscl}(B)$
(iv) $A \subseteq B \Rightarrow \pi_{gsint}(A) \subseteq \pi_{gsint}(B)$

Theorem 3.22: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IF$\pi$GS irresolute mapping, then $f(\pi_{gscl}(A)) \subseteq \text{cl}(A)$ for every IFS $A$ of $X$.

Proof: Let $A$ be an IFCS of $X$. Then $\text{cl}(f(A))$ is an IFCS of $Y$. Since every IFCS is an IF$\pi$GSCS, $\text{cl}(f(A))$ is an IF$\pi$GSCS in $Y$. Since $f$ is IF$\pi$GS irresolute, $f^{-1}(\text{cl}(f(A)))$ is IF$\pi$GSCS in $X$. Clearly $A \subseteq f^{-1}(\text{cl}(f(A)))$.

Therefore $\pi_{gscl}(A) \subseteq \pi_{gscl}(f^{-1}(\text{cl}(f(A)))) = f^{-1}(\text{cl}(f(A)))$. Hence $f(\pi_{gscl}(A)) \subseteq \text{cl}(A)$ for every IFS $A$ of $X$.

Theorem 3.23: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is IF$\pi$GS irresolute, then $\pi_{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS $B$ of $Y$.

Proof: Let $B$ be an IFS of $Y$. Then $\text{cl}(B)$ is an IFCS of $Y$. Since every IFCS is an IF$\pi$GSCS, $\text{cl}(B)$ is an IF$\pi$GSCS in $Y$. By hypothesis, $f^{-1}(\text{cl}(B))$ is IF$\pi$GSCS in $X$. Clearly $B \subseteq \text{cl}(B)$ implies $f^{-1}(B) \subseteq f^{-1}(\text{cl}(B))$. Therefore, $\pi_{gscl}(f^{-1}(B)) \subseteq \pi_{gscl}(f^{-1}(\text{cl}(B))) = f^{-1}(\text{cl}(B))$. Hence $\pi_{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS $B$ of $Y$.

Theorem 3.24: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS $X$ into IFTS $Y$. Then the following conditions are equivalent

(i) $f$ is an IF$\pi$GS irresolute mapping
(ii) $f^{-1}(B)$ is an IF$\pi$GSOS in $X$, for each IF$\pi$GSOS in $Y$
(iii) $f^{-1}(\pi_{gsint}(B)) \subseteq \pi_{gsint}(f^{-1}(B))$
(iv) $\pi_{gscl}(f^{-1}(B)) \subseteq f^{-1}(\text{cl}(B))$ for every IFS $B$ of $Y$.

Proof:

(i) $\Rightarrow$ (ii): is obviously true.

(ii) $\Rightarrow$ (iii): Let $B$ be an IF$\pi$GSCS in $Y$ and $\pi_{gsint}(B) \subseteq B$. Then $f^{-1}(\pi_{gsint}(B)) \subseteq f^{-1}(B)$. Since $\pi_{gsint}(B)$ is an IF$\pi$GSOS in $Y$, $f^{-1}(\pi_{gsint}(B))$ is an IF$\pi$GSOS in $X$, by hypothesis. Hence $f^{-1}(\pi_{gsint}(B)) \subseteq \pi_{gsint}(f^{-1}(B))$.

(iii) $\Rightarrow$ (iv): is obvious by taking complement in (iii).

(iv) $\Rightarrow$ (i): Let $B$ be an IF$\pi$GSCS in $Y$ and $\pi_{gscl}(B) = B$. Hence $f^{-1}(B) = f^{-1}(\pi_{gscl}(B)) \subseteq \pi_{gscl}(f^{-1}(B))$. Therefore, $\pi_{gscl}(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IF$\pi$GSCS in $X$. Thus $f$ is an IF$\pi$GS irresolute mapping.
REFERENCES


********************