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SOME RESULTS ON ELEGANT GRAPHS

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ABSTRACT

In 1981, Chang, Hsu and Rogers [1] defined an elegant labeling f of a graph G with q edges as an injective function from the vertices of G to the set {0, 1, 2, . . . , q} such that when each edge xy is assigned the label (f(x) + f(y)) (mod (q+1)), the resulting edge labels are distinct and non – zero. In this paper, certain families of graphs are shown to be elegant.

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1. INTRODUCTION

In this paper, by a graph we mean an undirected graph without loops or multiple edges. For notations and terminology, we follow Bondy and Murthy [2].

Throughout this paper, we denote the cycle on n vertices by C_n and the path on n vertices by P_n. Also, f stands for a 1 – 1 function from V (G) to a subset of the set of non – negative integers and for any edge e = xy ∈ E (G), f *(xy) = f(x) + f(y). We call f * the induced edge labeling of G (induced by f).

Chang, Hsu and Rogers [1] defined an elegant labeling f of a graph G with q edges as an injective function from the vertices of G to the set {0, 1, 2. . . q} such that when each edge xy is assigned the label (f(x) + f(y)) (mod (q+1)), the resulting edge labels are distinct and non – zero. In this paper, certain families of graphs are shown to be elegant.

Balakrishnan, Selvam and Yegnanarayanan [3] have shown that the bistar $B_{n,n}$ is elegant if and only if n is even. For example, an elegant labeling of $B_{2,2}$ is shown in Figure 1.1.

Fig. 1.1

Theorem 1.1: The total possibilities of the edge labeling in an elegant graph is $\frac{q^2}{2}$ when q is even and $\frac{q^2+1}{2}$ when q is odd.

Proof: case (i) when q is even.

Sub case (i): Let the edge label be k.

The possible edge labels are $\{(i, q-i+k+1) : k + 1 \le i \le \frac{q}{2} + \lceil \frac{k-1}{2} \rceil\} \cup \{(i, k-i) : 0 \le i \le \lceil \frac{k}{2} \rceil -1\}$ and its total is $\frac{q}{2}$.

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Hence, the total possible edge labels is $q\,(\frac{q}{2}) = \frac{q^2}{2}$

Case (ii) when q is odd.

Sub case (i): Let the edge label be k and let k be odd with $1 \le k \le q$.

The possible edge labels are $\{(i, q-i+k+1) : k+1 \le i \le \frac{q+1}{2} + \frac{k-1}{2}\} \cup \{(i, k-i) : 0 \le i \le \frac{k-1}{2}\}$ and its total is $\frac{q+1}{2}$.

Hence, the total possible odd edge label is $(\frac{q+1}{2})(\frac{q+1}{2})$ (1)

Sub case (ii): Let the edge label be k and let k be even with $2 \le k \le q - 1$.

The possible edge labels are $\{(i, q-i+k+1) : k+1 \le i \le \frac{q+1}{2} + \frac{k-2}{2}\} \cup \{(i, k-i) : 0 \le i \le \frac{k}{2} - 1\}$ and its total is $\frac{q-1}{2}$.

Hence, the total possible even edge labels is $(\frac{q-1}{2})(\frac{q-1}{2})$ (2)

Therefore, the total possible edge label is,

$$(\frac{q+1}{2})(\frac{q+1}{2}) + (\frac{q-1}{2})(\frac{q-1}{2}) = \frac{q^2+1}{2}, \text{ when q is odd}$$

Hence, the total possibilities of the edge labeling in an elegant graph is $\frac{q^2}{2}$ when q is even and $\frac{q^2+1}{2}$ when q is odd.

2. DEFINITIONS:

Definition 2.1: Consider the graph $C_n \times P_m$. Let C_n^i, $1 \le i \le m$ denote the m cycles in the graph $C_n \times P_m$, corresponding to each vertex of v_i of P_m. Add a new vertex v and join it to all the vertices of $C_n^1, C_n^2, C_n^3, \ldots, C_n^m$. The resulting graph be called as $C_{n,m}$.

Definition 2.2: [4] Let $C_{n,m}^{\dagger}$ denotes the graph obtained from $C_n \times P_m$ by taking two new distinct vertices, say u and v and joining u to all the vertices of C_n^1 and v to all the vertices of C_n^m.

Definition 2.3: The total graph T (G) of G has the vertex set $V(G) \cup E(G)$ in which two vertices are adjacent whenever they are either adjacent or incident in G. The vertex set of $T(P_n)$ is $\{u_i, v_j : 1 \le i \le n,\ 1 \le j \le n-1\}$ and the edge set of $T(P_n)$ is $\{u_i u_{i+1},\ v_j v_{j+1}, u_i v_i, u_{i+1} v_i : 1 \le i \le n-1,\ 1 \le j \le n-2\}$.

For example, T (P_3) is shown in Figure 2.1.

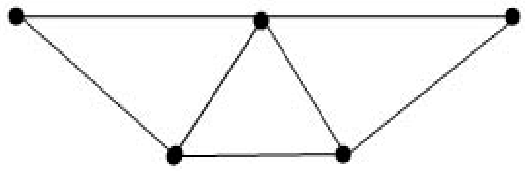

Fig. 2.1

Definition 2.4: The graph P_n^2 is a graph with vertex set $V(P_n^2) = \{u_i : 1 \le i \le n\}$ and $E(P_n^2) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_i u_{i+2} : 1 \le i \le n-2\}$.

Definition 2.5: The graph $K_2 + mK_1$ is the join of the graph K_2 and m disjoint copies of K_1. Some authors call this graph a Book with triangular pages.

For example, $K_2 + 4K_1$ is shown in Figure 2.2.

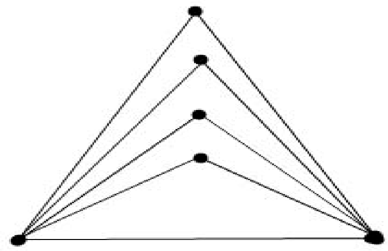

Fig. 2.2

Definition 2.6: The graph $(P_2 \cup mK_1) + N_2$ is a graph with vertex set $\{z_1, z_2, x_1, x_2. . . x_m\} \cup \{y_1, y_2\}$ and the edge set $\{z_1z_2, y_1z_1, y_1z_2, y_2z_1, y_2z_2\} \cup \{y_1 x_i, y_2 x_i / 1 \le i \le m\}$.

For example, $(P_2 \cup 2K_1) + N_2$ is shown in Figure 2.3. z_1

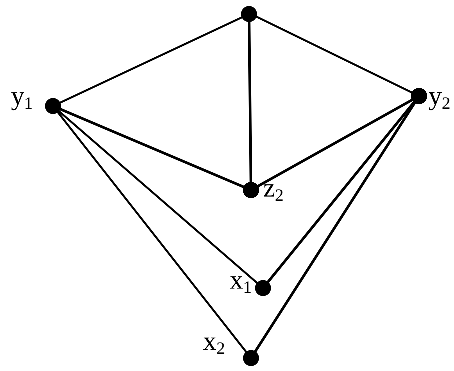

Fig. 2.3

Definition 2.7: For integers m, n ≥ 0, we consider the graph Jelly Fish J(m,n) with vertex set V(J(m, n)) = {u, v, x, y} $\cup \{x_1, x_2, . . . , x_m\} \cup \{y_1, y_2, . . . , y_n\}$ and the edge set E(J(m, n)) = {(u, x), (u, y), (u, v), (v, x), (v, y)} $\cup \{(x_i, x) / 1 \le i \le m\} \cup \{(y_j, y) / 1 \le j \le n\}$.

For example, J (3, 4) is shown in Figure 2.4.

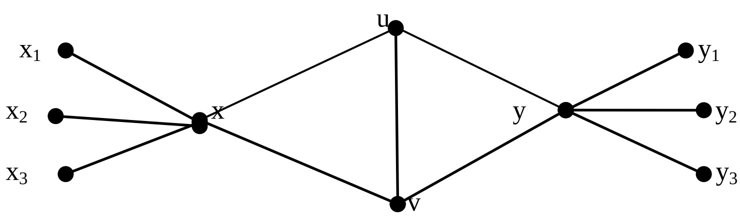

Fig. 2.4

Definition 2.8: $\langle C_3, K_{1,m}\rangle$ (m ≥ 1) be the graph obtained by attaching $K_{1,m}$ to one vertex of the cycle C_3.

For example, $\langle C_3, K_{1,4}\rangle$ is shown in Figure 2.5.

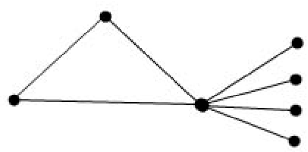

Fig. 2.5

Definition 2.9: $(K_4 – \{e\})_t$ is the one edge union of $K_4 – \{e\}$.

For example, $(K_4 – \{e\})_3$ is shown in Figure 2.6.

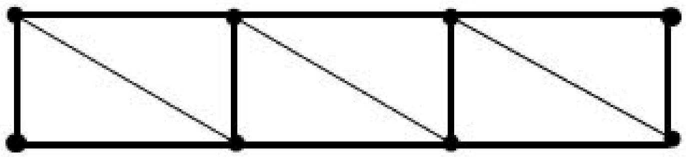

Fig. 2.6

Definition 2.10: Let T be any tree. Denote the tree obtained from T by considering 2 copies of T by adding an edge between them by T (2) and in general, the graph obtained from T_{n-1} and T by adding an edge between them is denoted by T(n). Note that T(1) is nothing but T.

For example, T and T (2) are shown in Figure 2.7.

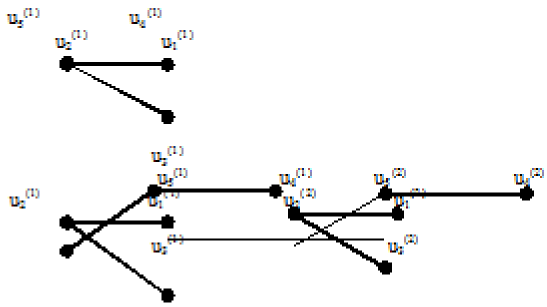

Fig. 2.7

Definition 2.11: Let G be a graph with a fixed vertex v_o and let $v_{10}, v_{20}, \ldots, v_{mo}$ be the vertices in m copies of G respectively corresponding to the vertex v_o. The graph $[P_m, G]$ is a graph obtained from m copies of G by joining v_{i0} and $v_{(i+1)0}$ by an edge for each i, $1 \leq i \leq m-1$.

For example, $[P_2, C_3]$ is shown in Figure 2.8

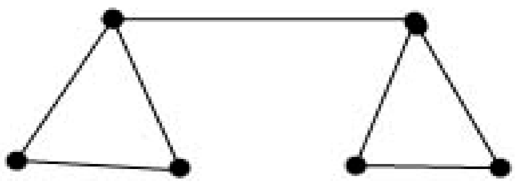

Fig. 2.8

3. MAIN RESULTS:

Theorem 3.1: $[P_{2m-1}, C_3]$ is an elegant graph for $m \geq 1$.

Proof: Let u_1^j, u_2^j, u_3^j be the vertices of j^{th} copy of C_3.

Define a function $f: V \to \{0, 1, 2, \ldots, q = 8m – 5\}$ as follows :

$f(u_1^j) = 4(j – 1), \quad 1 \leq j \leq 2m – 1$

$f(u_2^j) = 4j – 2, 1 \leq j \leq 2m – 1$

$f(u_3^j) = 4j – 1, 1 \leq j \leq 2m – 1$

The induced edge labels are given as,

$$f(u_1^j u_2^j) = \begin{cases} 8j - 6, & 1 \le j \le m \\ 8(j - m) - 2, & m + 1 \le j \le 2m - 1 \end{cases}$$

$$f(u_2^j u_3^j) = \begin{cases} 8j - 3, & 1 \le j \le m - 1 \\ 8(j - m) + 1, & m \le j \le 2m - 1 \end{cases}$$

$$f(u_1^j u_3^j) = \begin{cases} 8j - 5, & 1 \le j \le m \\ 8(j - m) - 1, & m + 1 \le j \le 2m - 1 \end{cases}$$

$$f(u_1^j u_1^{j+1}) = \begin{cases} 8j, & 1 \le j \le m - 1 \\ 8(j - m) + 4, & m + 1 \le j \le 2m - 1 \end{cases}$$

Hence, $[P_{2m-1}, C_3]$ is an elegant graph for $m \ge 1$.

For example, an elegant labeling of $[P_3, C_3]$ is shown in Figure 3.1.

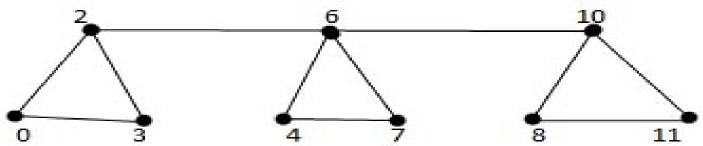

Fig. 3.1

Theorem 3.2: Comb $P_n \odot K_1$ is an elegant graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and $v_1, v_2, \ldots, v_n$ be the corresponding pendant vertices.

Define an one to one function $f : V \to \{0, 1, 2, \ldots, q = 2n - 1\}$ as follows:

$$f(u_i) = 2i - 1, \; 1 \le i \le n$$

$$f(v_i) = 2(i - 1), \; 1 \le i \le n$$

The induced edge labels are given as,

$$f(u_i u_{i+1}) = \begin{cases} 4i, & 1 \le i \le \frac{n-1}{2} \\ 4i - 2n, & \frac{n-1}{2} + 1 \le i \le n - 1 \end{cases}$$

$$f(u_i v_i) = \begin{cases} 4i - 3, & 1 \le i \le \frac{n+1}{2} \\ 4i - 2n - 3, & \frac{n+1}{2} + 1 \le i \le n \end{cases}$$

It is easy to check that $f(E) = (1, 2, 3, \ldots, q\}$. Hence, comb $P_n \odot K_1$ is an elegant graph.

For example, an elegant labeling of $P_5 \odot K_1$ is shown in Figure 3.2.

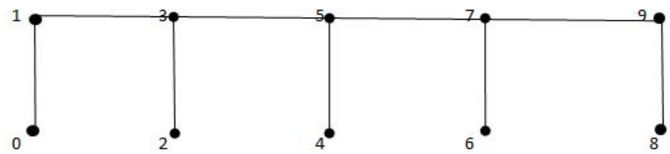

Fig. 3.2

Theorem 3.3: The graph $K_2 + mK_1$ is an elegant graph for all m.

Proof : Let u, v be the vertices of K_2 and $u_1, u_2, \ldots, u_m$ be the remaining vertices of the graph $K_2 + mK_1$ with edges $\{(u\, u_i), (v\, u_i) : 1 \le i \le m\}$.

Define an one to one function $f : V \to \{0, 1, 2, 3, \ldots, q = 2m + 1\}$ by

$f(u) = 0$, $f(v) = 2m + 1$,

$f(u_i) = 2i$, $1 \le i \le m$.

The induced edge labels are given as,

$f(uu_i) = 2i$, $1 \le i \le m$

$f(uv) = 2m + 1$

$f(vu_i) = 2i - 1$, $1 \le i \le m$

Hence, the graph $K_2 + mK_1$ is an elegant graph for all m.

For example, an elegant labeling of $K_2 + 4K_1$ is shown in Figure 3.3.

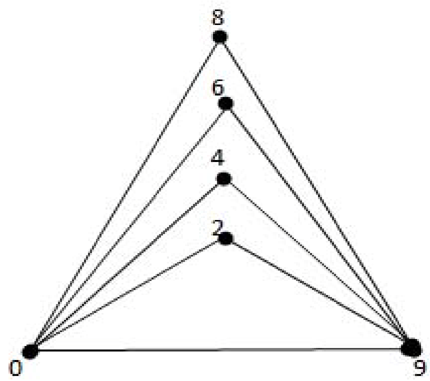

Fig. 3.3

Lemma 3.4: $C_3 \times P_n$ is an elegant graph.

Proof: Let $V(C_3 \times P_n) = \{u_{ij} / 1 \le i \le 3 \;\; \& \;\; 1 \le j \le n\}$ and $E(C_3 \times P_n) = \{(u_{1j}\; u_{2j}), \{(u_{2j}\; u_{3j}), \{(u_{3j}\; u_{1j}), : 1 \le j \le n\} \cup \{(u_{i,j}\; u_{i, j+1}) : 1 \le j \le n - 1\}$.

Define an one to one function $f : V \to \{0,1,2,\ldots,q = 6n - 3\}$ as follows:

$f(u_{ij}) = i - 1$, $1 \le i \le 3$ for $j = 1$

$f(u_{21}) = 4$, $f(u_{22}) = 5$, $f(u_{23}) = 6$ and

Let $a = i + j$ where the summation is taken modulo 3 with residues 1,2,3.

$f(u_{aj}) = f(u_{(a+1)\,(j-1)}) + i$, $1 \le i \le 3$ for $3 \le j \le n$

Clearly, the edge labels $1, 2, 3, \ldots, q = 6n - 3$.

For example, an elegant labeling of C_3 x P_4 is shown in Figure 3.4.

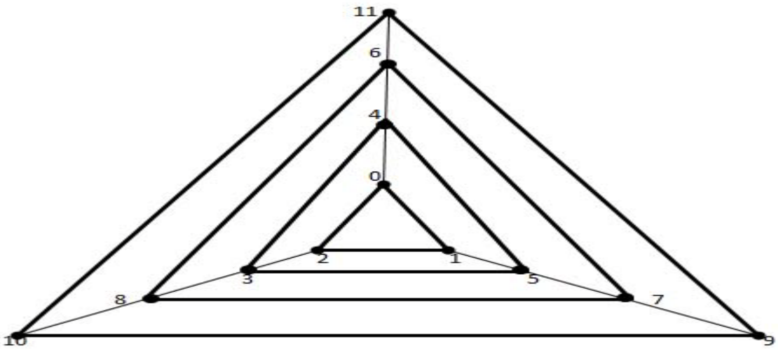

Fig. 3.4

Theorem 3.5: $C_{3,n}$ is an elegant graph for any n.

Proof: C_3 x P_n is an elegant graph by lemma 3.4. Let $V(C_{3,n}) = \{v, u_{ij} : 1 \le i \le 3,\ 1 \le j \le n\}$ and $E(C_{3,n}) = E(C_3 \text{ x } P_n) \cup \{v\, u_{ij} : 1 \le i \le 3,\ 1 \le j \le n\}$

Define $f(u_{ij})$ as in lemma 3.4 and

$$f(v) = 6n - 2$$

The edge labels of u_{ij} v is $6n - 2 + f(u_{ij})$, $1 \le i \le 3$ and $1 \le j \le n$.

Clearly, the edge labels of C_3 x P_n are distinct and non – zero.

For example, an elegant labeling of $C_{3,4}$ is shown in Figure 3.5.

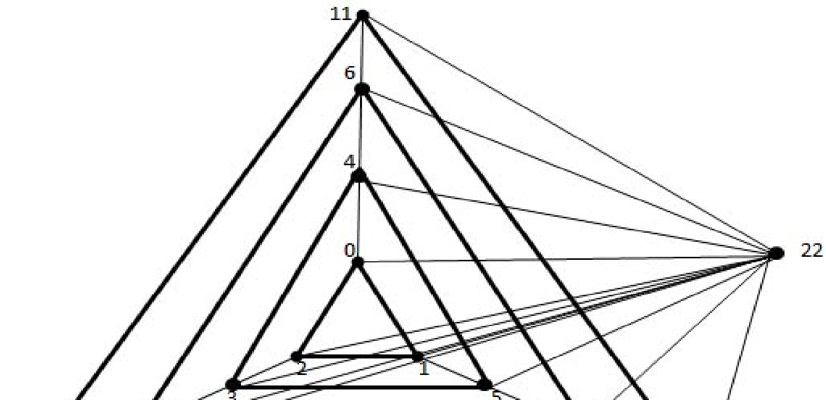

Fig. 3.5

Theorem 3.6: $C_{3,n}{}^{\dagger}$ is an elegant graph for any m.

Proof: C_3 x P_n is an elegant graph by lemma 3.4. Let $V(C_{3,n}{}^{\dagger}) = V(C_3 \text{ x } P_n) \cup \{u, v\}$ and $E(C_{3,n}{}^{\dagger}) = E(C_3 \text{ x } P_n) \cup \{(u\, u_{i1}), (v\, u_{in}) : 1 \le i \le 3,\ 1 \le j \le n\}$

Define $f(u_{ij})$ as in lemma 3.4 and $f(u) = 6n - 2$, $f(v) = 3n + 4$

The labels of the edges u_{11} u, u_{12}u, u_{13} u, u_{n1}v, u_{n2}v, u_{n3} v as $6n - 2, 6n - 1, \ldots 6n + 3$.

Hence, the edge labels of $C_{3,n}{}^{\dagger}$ distinct and non – zero.

For example, an elegant labeling of $C_{3,4}^{\dagger}$ is shown in Figure 3.6.

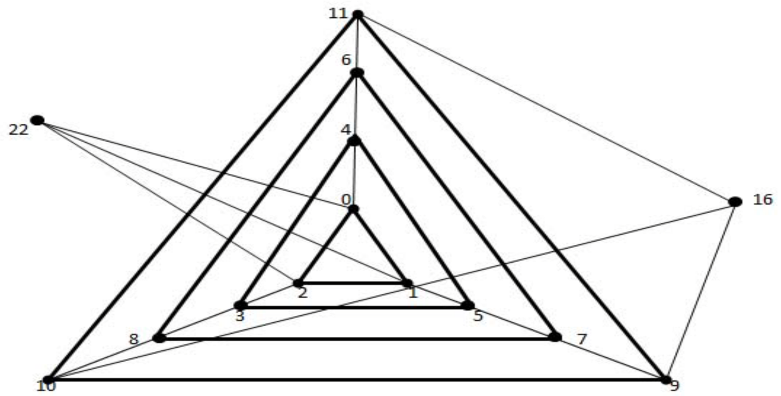

Fig. 3.6

Theorem 3.7: The total graph $T(P_n)$ is an elegant for any positive integer n.

Proof: Let $P_n = u_1, u_2, \ldots, u_n$ and let $V(T(P_n)) = V(P_n) \cup \{v_i : 1 \le i \le n-1\}$ and $E(T(P_n)) = E(P_n) \cup \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{(u_i v_i), (v_i u_{i+1}) : 1 \le i \le n-1\}$. The total number of edges is $3n - 4$.

Define an one to one function f: $V \to \{0, 1, 2, 3 \ldots q = 4n - 5\}$ by

$$f(u_i) = \begin{cases} i, & 1 \le i \le 2 \\ 2i - 3, & 3 \le i \le n \end{cases}$$

$$f(v_j) = \begin{cases} 0, & j = 1 \\ 2j, & 2 \le j \le n - 1 \end{cases}$$

The labels of the edges are given as:

$$f(u_i u_{i+1}) = \begin{cases} 2i + 1, & 1 \le i \le 2 \\ 4(i-1), & 3 \le i \le n - 1 \end{cases}$$

$$f(v_j v_{j+1}) = \begin{cases} 4, & i = 1 \\ 4i + 2, & 2 \le i \le n - 2 \end{cases}$$

$$f(u_i v_i) = \begin{cases} 5i - 4, & 1 \le i \le 2 \\ 4i - 3, & 3 \le i \le n - 1 \end{cases}$$

$$f(u_{i+1} v_i) = \begin{cases} 5i - 3, & 1 \le i \le 2 \\ 4i - 1, & 3 \le i \le n - 1 \end{cases}$$

Hence, the total graph $T(P_n)$ is an elegant for any positive integer n.

For example, an elegant labeling of $T(P_5)$ is shown in Figure 3.7.

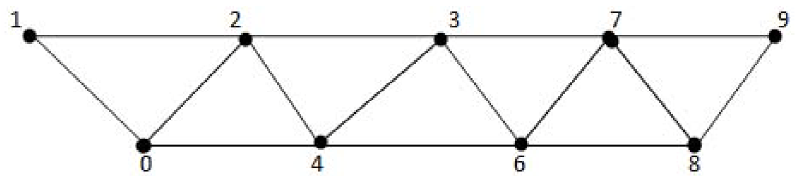

Fig. 3.7

Theorem 3.8: The graph P_n^2 is an elegant graph.

Proof: Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n .

Define an one to one function $f : V \rightarrow \{0, 1, 2, 3, \ldots, q\}$ by

$f(u_i) = i - 1, 1 \le i \le n$

The labels of the edges are given as :

$f(u_i\, u_{i+1}) = 2i - 1, 1 \le i \le n - 1$

$f(u_i\, u_{i+2}) = 2i, 1 \le i \le n - 2$

Hence, the graph P_n^2 is an almost elegant graph.

For example, the elegant labeling of P_5^2 is given in the Figure 3.8.

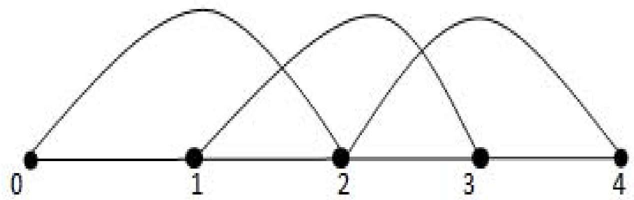

Fig. 3.8

Theorem 3.9: $(P_2 \cup mK_1) + N_2$ is an elegant graph.

Proof: Let z_1 and z_2 and y_1 and y_2 and x_j, $1 \le j \le m$ be the vertices of $(P_2 \cup mK_1) + N_2$.

Define an one to one function $f: V \rightarrow \{0, 1, 2, 3, \ldots, q = 2m + 5\}$ by

$f(z_i) = 3(i - 1), 1 \le i \le 2$

$f(y_i) = i, 1 \le i \le 2$

$f(x_j) = 2j + 3, 1 \le j \le m$

The labels of the edges are given as:

$f(y_1 z_1) = 1,$

$f(z_1 y_2) = 2,$

$f(z_1 z_2) = 3,$

$f(y_1 z_2) = 4,$

$f(y_2 z_2) = 5,$

$f(y_1 x_j) = 2j + 4, 1 \le j \le m$

$f(y_2 x_j) = 2j + 5, 1 \le j \le m$

Hence, $(P_2 \cup mK_1) + N_2$ is an elegant graph.

For example, an elegant labeling of $(P_2 \cup 2K_1) + N_2$ is shown in Figure 3.9.

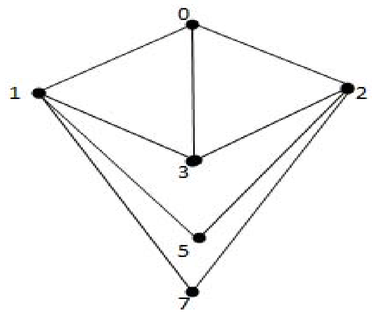

Fig. 3.9

Theorem 3.10: Jelly fish J (m, n) is an elegant graph for any positive integers m,n.

Proof: Let u, v, x, y, x_i, $1 \le i \le m$ and y_j, $1 \le j \le n$ be the vertices of Jelly fish. Let $V(J(m, n)) = \{u, v, x, y\} \cup \{x_i : 1 \le i \le m\} \cup \{y_j : 1 \le j \le n\}$ and $E(J(m, n)) = \{(u, x), (u, y), (u, v), (v, x), (v, y)\} \cup \{(x_i, x) : 1 \le i \le m\} \cup \{(y_j, y) : 1 \le j \le n\}$.

Define an one to one function $f : V \rightarrow \{0, 1, 2, 3, \ldots, q = m + n + 5\}$ by

$f(u) = 0$,

$f(v) = 3$,

$f(x) = 1$,

$f(y) = 2$,

$f(y_j) = 3 + j, \ 1 \le j \le n$

$f(x_i) = n + 4 + i, \ 1 \le i \le m$

The labels of the edges are given as follows:

$f(ux) = 1$,

$f(uy) = 2$,

$f(uv) = 3$,

$f(xv) = 4$,

$f(yv) = 5$,

$f(xx_i) = n + 5 + i, \ 1 \le i \le m$

$f(yy_j) = 5 + j, \ 1 \le j \le n$

Clearly, the edge values are distinct and non – zero. Hence, Jelly fish J(m,n) is an elegant graph for any positive integers m,n.

For example, an elegant labeling of J (3, 4) is shown in Figure 3.10.

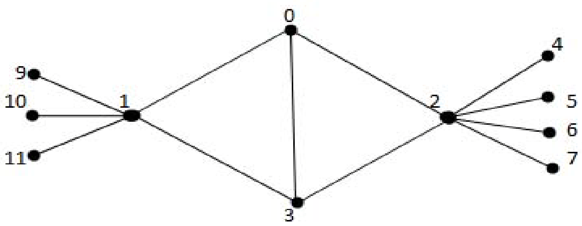

Fig. 3.10

Proposition 3.11: $C_3 \hat{o} K_{1,m}$ $(m \geq 1)$ is an elegant graph.

Proof: Let $V(C_3 \hat{o} K_{1,m})=\{u_1, u_2, u_3, v_1, v_2, v_3, . . . v_m\}$and $E(C_3 \hat{o} K_{1,m}) = \{(u_1 u_2), (u_2 u_3), (u_3 u_1)\} \cup \{u_2 v_i :1 \leq i \leq m\}$.

Let u_2 be the common vertex (centre vertex) of $K_{1,m}$.

Define an one to one function $f: V \to \{0, 1, 2, 3, . . . , q = m + 3\}$ by

$f(u_i) = i – 1, 1 \leq i \leq 3$

$f(v_j) = 2 + j, 1 \leq j \leq m$

The labels of the edges are given as :

$f(u_1u_2) = 1, f(u_2u_3) = 3, f(u_3u_1) = 2,$

$f(u_2v_j) = 3 + j, 1 \leq j \leq m$

Clearly, the edge labels are distinct and non – zero. Hence, $C_3 \hat{o} K_{1,m}$ $(m \geq 1)$ is an elegant graph.

For example, an elegant labeling of $C_3 \hat{o} K_{1,6}$ is shown in Figure 3.11.

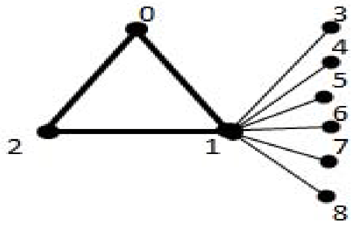

Fig. 3.11

Theorem 3.12: $(K_4 – \{e\})_t$ is an elegant graph for $t \geq 1$.

Proof: Let $V((K_4 – \{e\})_t) = \{ u_i, v_i: 1 \leq i \leq n \}$ and $E((K_4 – \{e\})_t) = \{(u_i u_{i+1}), (v_i v_{i+1}), (u_i v_{i+1}) : 1 \leq i \leq n – 1\} \cup \{u_i v_i : 1 \leq i \leq n\}$.

Define an one to one function $f: V \to \{0, 1, 2, 3, . . . , q\}$ by

$f(u_i) = 2i – 2, 1 \leq i \leq n$

$f(v_i) = 2i – 1, 1 \leq i \leq n$

The labels of the edges are given as :

$f(u_i u_{i+1}) = 4i – 2, 1 \leq i \leq n – 1,$

$f(v_i v_{i+1}) = 4i, 1 \leq i \leq n – 1,$

$f(u_i v_i) = 4i – 3, 1 \leq i \leq n,$

$f(u_i v_{i+1}) = 4i – 1, 1 \leq i \leq n – 1.$

Clearly, the edge labels are distinct and non – zero. Hence, $(K_4 – \{e\})_t$ is a near felicitous graph for $t \geq 1$.
For example, an elegant labeling of $(K_4 – \{e\})_4$ is shown in Figure 3.12.

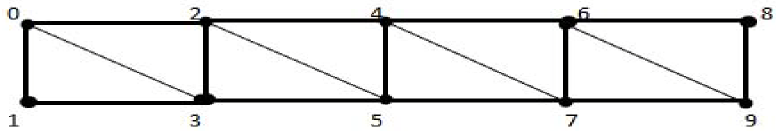

Fig. 3.12

Theorem 3.13: n – armed crown $C_3 \odot K_m$, $m \geq 1$ is an elegant graph.

Proof: Let $V(C_3 \odot K_m) = \{u_1, u_2, u_3, :1 \leq j \leq m\}$and $E(C_3 \odot K_m) = \{(u_1\, u_2), (u_2\, u_3), (u_3\, u_1)\} \cup \{(u_1\, v_j), (u_2\, v_j), (u_3\, v_j):1 \leq j \leq m\}$.

Define an one to one function $f: V \rightarrow \{0, 1, 2, \ldots, q = 3m + 3\}$ by

$$f(u_i) = i - 1,\ 1 \leq i \leq 3$$

For $1 \leq i \leq 2$,

$$f(u_{(i+2)\,j}) = i + 2j,\ \ 1 \leq j \leq m$$

For $i = 3$,

$$f(u_{(i+2)\,j}) = f(u_{1j}) + j,\ 1 \leq j \leq m$$

The label of the edges are given as :

$$f(u_3\, v_j) = 2j + 1,\ 1 \leq j \leq m$$

$$f(u_1\, v_j) = 2(j + 1),\ 1 \leq j \leq m$$

$$f(u_2\, v_j) = 2m + 4 + (j - 1),\ \ 1 \leq j \leq m$$

Clearly, the edge labels are distinct and non – zero. Hence, n – armed crown $C_3 \odot K_m$, $m \geq 1$ is an elegant graph.

For example, an elegant labeling of $C_3 \odot K_3$ is shown in Figure 3.13.

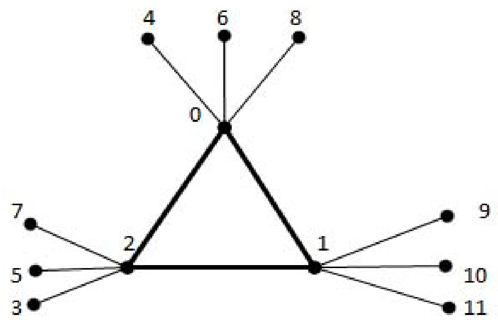

Fig. 3.13

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