ON RARELY gp-CONTINUOUS MULTIFUNCTIONS

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(Received on: 24-02-12; Accepted on: 18-03-12)

ABSTRACT

Popa [13] introduced the notion of rare continuity. The authors [5] introduced and investigated a new class of functions called rarely gp-continuous functions. This paper is devoted to the study of upper (and lower) rarely gp-continuous multifunctions.

Keywords and Phrases: Rare set, gp-open, rarely gp-continuous multifunctions.

2000 Mathematics Subject Classification: 54C60, 54C08; Secondary 54D05.

1. INTRODUCTION

In 1979, Popa [13] introduced the notion of rare continuity as a generalization of weak continuity [8] which has been further investigated by Long and Herrington [10] and Jafari [6] and [7]. Levine [9] introduced the concept of generalized closed sets of a topological space. Authors [1] introduce the concept of gp-continuous functions. The authors [5] introduced and investigated rarely gp-continuous in topological spaces. In this paper we study some characterization of rarely gp-continuous multifunctions.

2. PRELIMINARIES

Throughout this paper, X and Y are topological spaces. Recall that a rare set is a set R such that \( \text{Int}(R) = \phi \). Noiri et al[11]. Introduced the notion of gp-closed sets: A set A in X is called gp-closed if \( \text{Cl}(A) \subseteq G \) whenever A \( \subseteq G \) and G is open in X. The complement of a gp-closed set is called gp-open [11]. The family of all gp-open (resp. open) sets will be denoted by GPO(X) (resp. O(X)). We set GPO(X, x) = \{U/ x \in U \in GPO(X)\}, GO(X, x) = \{U/ x \in U \in GO(X)\} and O(X, x) = \{U/ x \in U \in O(X)\}.

Definition 1: A function \( f: X \rightarrow Y \) is called:

i) weakly continuous [7] (resp. weakly-g-continuous [4] and weakly-gp-continuous[5] ) if for each \( x \in X \) and each open set \( G \) containing \( f(x) \), there exists \( U \in O(X, x) \) (resp. \( U \in GO(X, x) \) and \( U \in GPO(X, x) \) ) such that \( f(U) \subseteq C(f(G)) \),

ii) rarely continuous [13] (resp. rarely-g-continuous [2] and rarely-gp-continuous[5] ) if for each \( x \in X \) and each \( G \subseteq O(Y, f(x)) \), there exists a rare set \( R_G \) with \( G \cap C(R_G) = \phi \) and \( U \in O(X, x) \) (resp. \( U \in GO(X, x) \) and \( U \in GPO(X, x) \) ) such that \( f(U) \subseteq G \cup R_G \),

iii) gp-continuous [1] if the inverse image of every closed set in Y is gp-closed in X.

3. UPPER (LOWER) RARELY gp-CONTINUOUS MULTIFUNCTIONS

We provide the following definitions which will be used in the sequel. Let \( F: X \rightarrow Y \) be a multifunction. The upper and lower inverses of a set \( V \subseteq Y \) are denoted by \( F^+(V) \) and \( F^-(V) \) respectively, that is,

\[
F^+(V) = \{x \in X / F(x) \subseteq V\} \text{ and } F^-(V) = \{x \in X / F(x) \cap V = \phi\}.
\]

Definition 2: A multifunction \( F: X \rightarrow Y \) is said to be

i) upper rarely gp-continuous (briefly u.r.g.p.c) at \( x \in X \) if for each \( V \subseteq O(Y, F(x)) \), there exist a rare set \( R_V \) with \( V \cap C(R_V) = \phi \) and \( U \in GPO(X, x) \), such that \( F(U) \subseteq V \cup R_V \),
lower rarely $g$-continuous (briefly l.r.g.p.c) at $x \in X$ if for each $V \in O(Y)$ with $f(x) \cap V = \phi$ there exist a rare set $R_V$ with $V \cap C(R_V) = \phi$ and $U \in GPO(X, x)$ such that $F(U) \subset C(V)$,

(ii) lower weakly gp-continuous at $x \in X$ if for each $V \in O(Y)$ with $f(x) \cap V \neq \phi$ there exists $U \in GPO(X, x)$ such that 

$F(U) \cap C(V) \neq \phi$ for every $u \in U$,

(iii) upper/ lower rarely gp-continuous if it is upper/ lower rarely gp-continuous at each point of $X$.

**Definition 3:** A multifunction $F : X \to Y$ is said to be

i) upper weakly gp-continuous at $x \in X$ if for each $V \in O(Y)$, there exist $U \in GPO(X, x)$ such that $F(U) \subset C(V)$,

ii) lower weakly gp-continuous at $x \in X$ if for each $V \in O(Y)$ with $f(x) \cap V \neq \phi$ there exists $U \in GPO(X, x)$ such that 

$F(U) \cap C(V) \neq \phi$ for every $u \in U$,

iii) upper/ lower weakly gp-continuous if it is upper/ lower weakly gp-continuous at each point of $X$.

**Theorem 1:** The following statements are equivalent for a multifunction $F : X \to Y$:

i) $F$ is u.r.g.p.c at $x \in X$,

ii) For each $V \in O(Y, F(x))$, there exists $U \in GPO(X, x)$ such that $\operatorname{Int}[F(U) \cap (Y - V)] = \phi$.

iii) For each $V \in O(Y, F(x))$, there exists $R_V \in GPO(X, x)$ such that $\operatorname{Int}[F(U) \cap (Y - V)] = \phi$.

**Proof:** (i) $\Rightarrow$ (ii): Let $V \in O(Y, F(x))$. By $f(x) \subset V \subset \operatorname{Int}(C(V))$ and the fact that $\operatorname{Int}(C(V)) \in O(Y)$, there exist a rare set $R_V$ with $\operatorname{Int}(C(V)) \cap C(R_V) = \phi$ and a gp-open set $U \subset X$ containing $x$ such that $F(U) \subset \operatorname{Int}(C(V)) \cup R_V$. We have $\operatorname{Int}[F(U) \cap (Y - V)] = \operatorname{Int}(F(U)) \cap \operatorname{Int}(Y - V) \subset \operatorname{Int}(C(V)) \cup R_V \cap (Y - C(V)) \subset (C(V) \cup \operatorname{Int}(R_V)) \cap (Y - C(V)) = \phi$.

(ii) $\Rightarrow$ (iii): Obvious.

(iii) $\Rightarrow$ (i): Let $V \in O(Y, F(x))$. Then, by (iii) there exists $U \in GPO(X, x)$ such that $\operatorname{Int}[F(U)] \subset C(V)$. Thus $F(U) = [F(U) - \operatorname{Int}(F(U))] \cup \operatorname{Int}[F(U)] \subset [F(U) - \operatorname{Int}(F(U))] \cup \operatorname{Int}(V) = [F(U) - \operatorname{Int}(F(U))] \cup V \cup \operatorname{Int}(C(V) - V)$. Put $P = (F(U) - \operatorname{Int}(F(U))) \cap (Y - V)$ and $G = C(V) - V$, then $P$ and $G$ are rare sets. Moreover, $R_V = P \cup G$ is a rare set such that $C(R_V) \cap V = \phi$ and $F(U) \subset C(V) \cup R_V$. Hence $F$ is u.r.g.p.c.

**Theorem 2:** The following are equivalent for a multifunction $F : X \to Y$:

i) $F$ is l.r.g.p.c at $x \in X$,

ii) For each $V \in O(Y)$ such that $f(x) \cap V \neq \phi$ there exists a rare set $R_V$ with $V \cap C(R_V) = \phi$ such that $x \in \operatorname{Int}_g(F(V \cup R_V))$,

iii) For each $V \in O(Y)$ such that $f(x) \cap V \neq \phi$ there exists a rare set $R_V$ with $C(V) \cap R_V = \phi$ such that $x \in \operatorname{Int}_g(F(C(V) \cup R_V))$,

iv) For each $V \in RO(Y)$ such that $f(x) \cap V \neq \phi$ there exists a rare set $R_V$ with $V \cap C(R_V) = \phi$ such that $x \in \operatorname{Int}_g(F(V \cup R_V))$.

**Proof:** (i) $\Rightarrow$ (ii): Let $V \in O(Y)$ such that $f(x) \cap V \neq \phi$. By (i), there exist a rare set $R_V$ with $V \cap C(R_V) = \phi$ and $U \in GPO(X, x)$ such that $f(x) \cap (V \cup R_V) \neq \phi$ for each $u \in U$. Therefore, $u \in F(V \cup R_V)$ for each $u \in U$ and hence $U \subset F(V \cup R_V)$. Since $U \subset GPO(X, x)$, we obtain $x \in U \subset \operatorname{Int}_g(F(V \cup R_V))$.

(ii) $\Rightarrow$ (iii): Let $V \in O(Y)$ such that $f(x) \cap V \neq \phi$. By (ii), there exists a rare set $R_V$ with $V \cap C(R_V) = \phi$ such that $x \in \operatorname{Int}_g(F(V \cup R_V))$. We have $R_V \subset Y - V = (Y - C(V)) \cup (C(V) - V)$ and hence $R_V \subset [R_V \cap (Y - C(V))] \cup (C(V) - V)$. Now, put $P = R_V \cap (Y - C(V))$. Then $P$ is a rare set and $P \cap C(V) = \phi$. Moreover, we have $x \in \operatorname{Int}_g(F(V \cup R_V)) \subset \operatorname{Int}_g(F(P \cup C(V)))$.

(iii) $\Rightarrow$ (iv): Let $V$ be any regular open set of $Y$ such that $f(x) \cap V \neq \phi$. By (iii), there exists a rare set $R_V$ with $C(V) \cap R_V = \phi$ such that $x \in \operatorname{Int}_g(F(C(V) \cup R_V))$. Put $P = R_V \cup (C(V) - V)$, then $P$ is a rare set and $V \cap C(P) = \phi$. Moreover, we have $x \in \operatorname{Int}_g(F(C(V) \cup R_V)) = \operatorname{Int}_g(F(R \cup ((C(V) - V) \cup V)) = \operatorname{Int}_g(F(P \cup V))$.

(iv) $\Rightarrow$ (i): Let $V \in O(Y)$ such that $f(x) \cap V \neq \phi$. Then $F(x) \cap \operatorname{Int}(C(V)) \neq \phi$ and $\operatorname{Int}(C(V))$ is regular open in $Y$. By (iv), there exists a rare set $R_V$ with $V \cap C(R_V) = \phi$ such that $x \in \operatorname{Int}_g(F(V \cup R_V))$. Therefore, there exists $U \in$
GPO(X, x) such that U ⊂ F(V ∪ R_v); hence F(u) ∩ (V ∪ R_v) ≠ ø; for each u ∈ U. This shows that F is lower rarely g-continuous at x.

**Corollary 1:** (([2], Theorem 2)) The following statements are equivalent for a function f : X → Y :

i) f is rarely gp-continuous at x ∈ X,

ii) For V ∈ O(Y, f(x)), there exists U ∈ GPO(X, x) such that Int[f(U) ∩ (Y - V)] = ø,

iii) For each V ∈ O(Y, f(x)), there exists U ∈ GPO(X, x) such that Int[f(U)] ⊂ Cl(V).

**REFERENCES**


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