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ON DECOMPOSABILITY OF THE CURVATURE TENSOR IN SECOND ORDER RECURRENT CONFORMAL FINSLER SPACES

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ABSTRACT

The decomposability of curvature tensor in Finsler manifold was studied by Pandey[2] and decomposability of curvature tensor in recurrent conformal Finsler spaces have studied by Mishra and Lodhi[1]. The purpose of the present paper is to decomposition of curvature tensor in second order recurrent conformal Finsler space and study the properties of conformal decomposition tensor.

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1. INTRODUCTION

Let the two distinct function $F(x,\dot{x})$ and $\bar{F}(x,\dot{x})$ are defined over a n-dimensional Finsler space F_n. Then the two metrices resulting from the function are called conformal, if the corresponding metric tensor $gij\,(x,\dot{x})$ and $\bar{g}ij\,(x,\dot{x})$ are proportional to each other. Knebelman [4] has proved that the factor of proportionality between them is at most point function. Thus we have

(1.1) $$\bar{g}_{ij}\,(x,\dot{x}) = e^{2\sigma}\, g_{ij}\,(x,\dot{x}),$$
where
(1.2) $$\sigma = \sigma\,(x).$$

Hence,
(1.3) $$\bar{g}^{ij}\,(x,\dot{x}) = e^{-2\sigma}\, g^{ij}\,(x,\dot{x}),$$
and
(1.4) $$\bar{F}(x,\dot{x}) = e^{2\sigma} F\,(x,\dot{x}).$$

The space equipped with such quantities $\bar{F}(x,\dot{x})$ and $\bar{g}_{ij}\,(x,\dot{x})$ etc is called a conformal Finsler space [3] and usually denoted by $\bar{F}_n$.

The decomposition of curvature tensor H^i_{jkh} is defined by P. N. Pandey [2]

(1.5) $$H^i_{jkh} = X^i_j A_{kh},$$
where X^i_j is non zeor tensor and A_{kh} is skew symmetric decomposition tensor.

The recurrent curvature tensor H^i_{jkh} is characterized by the condition

(1.6) $$H^i_{jkh(l)} = V_l H^i_{jkh},$$
where
(1.8) $$H^i_{jkh} \neq 0.$$

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The covariant vector V_l is called recurrence vector. The space equipped with such recurrent curvature tensor is called recurrent Finsler space and it denoted by $R - F_n$.

The covariant derivative of a vector $X^i(x,\dot{x})$ with respect to $\bar{x}^j$ in the sense of Berwald's is given by

(1.9) $$X^i_j(x,\dot{x}) = \partial_j X^i - \left(\dot{\partial}_j X^i\right) G^m_j + X^i G^i_{mj},$$
where $G^i_{mj}(x,\dot{x})$ are the Berwald's connection coefficients. They satisfy

(1.10) $$\dot{\partial}_m G^i_j(x,\dot{x}) = G^i_{mj}.$$

The curvature tensor H^i_{jkh} under the conformal change (1.1) as

(1.11)
$$\bar{H}^i_{jkh} = H^i_{jkh}(x,\dot{x}) - 2\sigma_m \dot{\partial}_j \left\{\dot{\partial}_{[k} B^{im}\right\}_{(h)]} + 2\sigma_{m[(k)}\dot{\partial}_{h]}\dot{\partial}_j B^{im} + 2\sigma_r \left(\dot{\partial}_{[k} B^{im}\right) G^r_{h]mj} + + 2\sigma_m \sigma_r \dot{\partial}_j \left(\dot{\partial}_{[k} B^{sm}\right) \dot{\partial}_{h]} \dot{\partial}_s B^{ir},$$

and the skew symmetric decomposition tensor A_{kh} under the conformal change (1.1) as [1]

(1.12) $$\bar{A}_{kh} = e^{\sigma} A_{kh} - e^{\sigma} V^j y_i 2\left[\sigma_m \dot{\partial}_j \left(\dot{\partial}_{[k} B^{im}\right)_{(h)]} - \sigma_{m[(k)}\dot{\partial}_{h]}\dot{\partial}_j B^{im} - \sigma_m \sigma_r \dot{\partial}_j \left\{\left(\dot{\partial}_{[k} B^{sm}\right) \dot{\partial}_{h]} \dot{\partial}_s B^{ir}\right\}\right],$$
where
(1.13) $$B^{im}(x,\dot{x}) = \tfrac{1}{2} F^2 g^{im} - \dot{x}^i \dot{x}^m.$$

The function B^{im} is homogenous of second order degree in its directional arguments.

2. DECOMPOSITION OF CURVATURE TENSOR IN SECOND ORDER RECURRENT CONFORMAL FINSLER SPACE($R - \bar{F}_n{}^*$)

The decomposition of conformal curvature tensor $\bar{H}^i_{jkh}$ is defined by C. K. Mishra and Gautam Lodhi[1]

(2.1) $$\bar{H}^i_{jkh} = \bar{X}^i_J \bar{A}_{kh},$$
where $\bar{X}^i_j$ is non zeor conformal tensor and $\bar{A}_{kh}$ is skew symmetric conformal decomposition tensor.

The recurrent conformal curvature tensor $\bar{H}^i_{jkh}$ is characterized by the condition

(2.2) $$\bar{H}^i_{jkh(l)} = \bar{V}_l H^i_{jkh}$$
and

(2.3) $$\bar{H}^i_{jkh(l)(m)} = (\bar{V}_{l(m)} + \bar{V}_l \bar{V}_m)\bar{H}^i_{jkh},$$
where
(2.4) $$\bar{H}^i_{jkh} \neq 0.$$

The covariant vectors $\bar{V}_l$ and $\bar{V}_m$ are called conformal recurrence vectors and $\bar{V}_{l(m)}$ is a conformal recurrence tensor.

The space equipped with such recurrent conformal curvature tensor is called second order recurrent conformal Finsler space and we denote it by $R - \bar{F}_n{}^*$.

Differentiating (2.1) covariantly with respect to $\bar{x}^l$ in the sense of Berwald's, we get

(2.5) $$\bar{H}^i_{jkh(l)} = \bar{X}^i_{J(l)} \bar{A}_{kh} + \bar{A}_{kh(l)} \bar{X}^i_J.$$

Let us assume that the conformal tensor $\bar{X}^i_J$ is covariant constant, then (2.6) reduces to

(2.6) $$\bar{H}^i_{jkh(l)} = \bar{A}_{kh(l)} \bar{X}^i_J.$$

Using (2.1) and (2.2) in (2.6), we get

(2.7) $$\bar{A}_{kh(l)} = \bar{V}_l \bar{A}_{kh}.$$

Differentiating (2.7) covariantly with respect to $\bar{x}^m$ in the sense of Berwald's and using (2.7), we get

(2.8) $$\bar{A}_{kh(l)(m)} = \left(\bar{V}_{l(m)} + \bar{V}_l\bar{V}_m\right)\bar{A}_{kh}.$$

Conversely, we assume equation (2.7) and (2.8) are true.

Differentiating (2.5) covariantly with respect to $\bar{x}^m$ in the sense of Berwald's, we get

(2.9) $$\bar{H}^i_{jk\,h(l)(m)} = \bar{X}^i_{j(l)(m)}\bar{A}_{kh} + \bar{A}_{kh(m)}\,\bar{X}^i_{j(l)} + \bar{A}_{kh(l)(m)}\,\bar{X}^i_j + \bar{A}_{kh(l)}\,\bar{X}^i_{j(m).}$$

Applying (2.3), (2.7) and (2.8) in (2.9), we have

(2.10) $$\bar{X}^i_{j(l)(m)}\bar{A}_{kh} + \bar{V}_m\bar{A}_{kh}\,\bar{X}^i_{j(l)} + \bar{V}_l\bar{A}_{kh}\,\bar{X}^i_{j(m)} = 0.$$

In view of (2.1) and (2.7) the equation (2.5), yields

(2.11) $$\bar{X}^i_{j(l)}\bar{A}_{kh} = 0.$$

Since $\bar{A}_{kh}$ is non zero, It implies

(2.12) $$\bar{X}^i_{j(l)} = 0.$$

In view of equation (2.12), equation (2.10) immediately reduces to

(2.13) $$\bar{X}^i_{j(l)(m)}\bar{A}_{kh} = 0,$$

which shows that $\bar{X}^i_j$ (or $\bar{X}^i_{j(l)}$ is covariant constant).

Theorem 2.1: In $R - \bar{F}_n{}^*$, the necessary and sufficient condition for the skew symmetric conformal decomposition tensor $\bar{A}_{kh}$ to be recurrent is that the conformal tensor $\bar{X}^i_j$ is covariant constant in the sense of Berwald's.

Interchanging the indices l and m in (2.8), we have

(2.14) $$\bar{A}_{kh(m)(l)} = \left(\bar{V}_l\bar{V}_m + \bar{V}_{m(l)}\right)\bar{A}_{kh}.$$

Subtracting equation (2.14) from (2.8), we get

(2.15) $$\bar{A}_{kh(l)(m)} - \bar{A}_{kh(m)(l)} = \left(\bar{V}_{l(m)} - \bar{V}_{m(l)}\right)\bar{A}_{kh}.$$

Accordingly, we have the

Theorem 2.2: In $R - \bar{F}_n{}^*$, the conformal recurrence tensor $\bar{V}_{l(m)}$ is non symmetric if $\bar{X}^i_j$ is covariant constant in the sense of Berwald's.

Adding equation (2.14) and (2.8), we have

(2.16) $$\bar{A}_{kh(l)(m)} + \bar{A}_{kh(m)(l)} = \left(\bar{k}_{(l)(m)} + \bar{k}_{(m)(l)}\right)\bar{A}_{kh},$$

where $$\bar{k}_{(l)(m)} = (\,\bar{V}_l\bar{V}_m + \bar{V}_{l(m)})\ \neq 0.$$

Accordingly we have the

Theorem 2.3: Every recurrent conformal Finsler space for which the conformal recurrence vector $\bar{V}_l$ satisfies $\bar{V}_l\bar{V}_m + \bar{V}_{l(m)} \neq 0$ is a second order conformal recurrent Finsler space$(R - \bar{F}_n{}^*)$ if $\bar{X}^i_j$ is covariant constant.

Transvecting equation (2.8) by $\bar{X}^i_j$ and using (2.1), we have

(2.17) $$\bar{X}^i_j\bar{A}_{kh(l)(m)} = (\bar{V}_{l(m)} + \bar{V}_l\bar{V}_m)\bar{H}^i_{jk\,h}.$$

From equation (2.3) and (2.17), we get

(2.18) $$\bar{H}^i_{jkh(l)(m)} = \bar{X}^i_j \bar{A}_{kh(l)(m)}.$$

Thus, we have the

Theorem 2.4: In $R - \bar{F}_n^{\ *}$, the conformal curvature tensor $\bar{H}^i_{jkh}$ decomposed in the form of equation (2.18) if $\bar{X}^i_j$ is covariant constant.

Differentiating (2.8) with respect to $\bar{x}^n$ in the sense of Berwald's and using (2.7), we get

(2.19) $$\bar{A}_{kh(m)(l)(n)} = \bar{k}_{(l)(m)(n)}\bar{A}_{kh} + \bar{V}_n\bar{k}_{(l)(m)}\bar{A}_{kh}.$$

Adding the expression obtained by cyclic change of (2.19) with respect to the indices l, m and n, we have

(2.20) $$\bar{A}_{kh[(m)(l)(n)]} = \left(\bar{k}_{[(l)(m)(n)]} + \bar{V}_{[n}\bar{k}_{(l)(m)]}\right)\bar{A}_{kh}.$$

Theorem 2.5: In $R - \bar{F}_n^{\ *}$, If $\bar{X}^i_j$ is covariant constant then the conformal decomposition tensor $\bar{A}_{kh}$ satisfies the relation (2.20) .

C. K. Mishra and Gautam Lodhi[1] proved the Bianchi identity for conformal decomposition tensor $\bar{A}_{kh}$ is given by

(2.21) $$\bar{A}_{kh(l)} + \bar{A}_{hl(k)} + \bar{A}_{lk(h)} = 0.$$

Differentiating (2.21) with respect to $\bar{x}^m$ in the sense of Berwald's, we get

(2.22) $$\bar{A}_{kh(l)(m)} + \bar{A}_{hl(k)(m)} + \bar{A}_{lk(h)(m)} = 0.$$

Transvecting equation (2.22) by $\bar{X}^i_j$ and using equation (2.18), we get

(2.23) $$\bar{H}^i_{jkh(l)(m)} + \bar{H}^i_{jhl(k)(m)} + \bar{H}^i_{jlk(h)(m)} = 0,$$

$$\bar{H}^i_{j[kh(l)](m)} = 0.$$

Accordingly we have the

Theorem 2.6: In $R - \bar{F}_n^{\ *}$, under the decomposition (2.1) for homothetic mapping, if $\bar{X}^i_j$ is covariant constant then the conformal curvature tensor $\bar{H}^i_{jkh}$ satisfies the identity (2.23).

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