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ON ρ-CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, we introduce a new class of continuous functions called ρ-continuous functions by utilizing ρ-closed sets. Moreover, we study their properties in topological space. It turns out, among others, the ρ-continuous weaker than perfect continuity and stronger than both gp-continuity and πgp-continuity.

Keywords: *ρ-closed set, ρ-open set, ρ-continuous function, ρ-compact space, ρ-connected space.*

2000 AMS Classification: *54A05, 54C05, 54C08.*

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1. INTRODUCTION AND PRELIMINARIES

Continuous functions in topology found a valuable place in the applications of mathematics as it has applications to engineering especially to digital signal processing and neural networks. Topologist studied weaker and stronger forms of continuous functions in topology using the sets stronger and weaker than open and closed sets. Balachandran et.al [4], Levine [13], Mashhour et.al [15], Rajesh et.al [21], Gnanambal et.al[10], Park et.al[19] have introduced g - continuity, semi-continuity, pre-continuity, $\tilde{g}$-continuity, gpr-continuity and πgp-continuity respectively. As generalizations of closed sets, ρ-closed sets were introduced and studied by the same author[5]. The aim of this paper is to introduce new classes of functions called ρ-continuous functions. Moreover, the relationships and properties of ρ-continuous functions are obtained.

Throughout this paper (X, τ), (Y, σ), (Z, η) represent non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

We recall the following definitions in the sequel.

Definition 1.1: Let (X, τ)be a topological space. A subset A of the space X is said to be
1. Preopen [15] if $A\subseteq (cl(A))$ and preclosed if $cl(int(A))\subseteq A$.
2. Semi-open [13] if $A\subseteq(int(A))$ and semi-closed if $int(cl(A))\subseteq A$.
3. Semi-preopen [1] if $A\subseteq(int(cl(A)))$ and semi-preclosed if $int(cl(int(A))) \subseteq A$.
4. Regular open if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.
5. π-open [32] if it a finite union of regular open sets.

Recall that the intersection of all semi-closed (resp. preclosed, semi-preclosed) sets containing A is called the semi-closure of A and is denoted by scl(A) (resp. pcl(A), spcl(A)).

Definition 1.2: Let (X, τ) be a topological space. A subset $A\subseteq X$ is said to be
1. generalized closed (briefly g-closed)[14] if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.
2. generalized preclosed (briefly gp-closed)[17] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.
3. generalized preregular closed (briefly gpr-closed)[9] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is regular open in X.
4. pregeneralized closed (briefly pg-closed)[17] if $pcl(A)\subseteq U$ whenever $A\subseteq U$ and U is preopen in X.

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5. g*-preclosed (briefly g*p-closed)[30] if pcl(A)⊆U whenever A⊆U and U is g-open in X.
6. generalized semi-preclosed (briefly gsp-closed)[6] if spcl(A)⊆U whenever A⊆U and U is open in X.
7. pre semi closed [31] if spcl(A)⊆U whenever A⊆U and U is g-open in X.
8. $\hat{g}$-closed [26] if cl(A)⊆U whenever A⊆U and U is semi open in X.
9. πgp-closed [18] if pcl(A)⊆U whenever A⊆U and U is \pi-open in X.
10. ω-closed [24] if cl(A)⊆U whenever A⊆U and U is semi open in X.
11. *g-closed [29] if cl(A)⊆U whenever A⊆U and U is $\hat{g}$-open in X.
12. #g- semi closed (briefly #gs-closed)[28] if scl(A)⊆U whenever A⊆U and U is *g-open in X.
13. $\tilde{g}$-closed set [12] if cl(A)⊆U whenever A⊆U and U is #gs-open in X.
14. ρ-closed set [5] if pcl(A)⊆(U) whenever A⊆U and U is $\tilde{g}$-open in X.
15. ρ_s-closed set [5] if pcl(A)⊆(cl(U)) whenever A⊆U and U is $\tilde{g}$-open in X.

The complements of the above mentioned sets are called their respective open sets.

Definition 1.3: A function *f : (X, τ)→ (Y, σ)* is called
1. Semi-continuous [13] if $f^{-1}(V)$ is semi-open in (X, τ) for every open set V in (Y, σ).
2. Pre-continuous [15] if $f^{-1}(V)$ is preclosed in (X, τ) for every closed set V in (Y, σ).
3. g-continuous [4] if $f^{-1}(V)$ is g-closed in (X, τ) for every closed set V in (Y, σ).
4. ω-continuous [23] if $f^{-1}(V)$ is ω-closed in (X, τ) for every closed set V in (Y, σ).
5. gsp-continuous [6] if $f^{-1}(V)$ is gsp-closed in (X, τ) for every closed set V in (Y, σ).
6. gp-continuous [2] if $f^{-1}(V)$ is gp-closed in (X, τ) for every closed set V in (Y, σ).
7. gpr-continuous [10] if $f^{-1}(V)$ is gpr-closed in (X, τ) for every closed set V in (Y, σ).
8. Semi-Pre-continuous [1] if $f^{-1}(V)$ is semi-preopen in (X, τ) for every open set V in (Y, σ).
9. Pre-semi-continuous [31] if $f^{-1}(V)$ is Pre-semiclosed in (X, τ) for every closed set V in (Y, σ).
10. πgp-continuous [19] if $f^{-1}(V)$ is πgp-closed in (X, τ) for every closed set V in (Y, σ).
11. pg-continuous [17] if $f^{-1}(V)$ is pg-closed in (X, τ) for every closed set V in (Y, σ).
12. g*p-continuous [30] if $f^{-1}(V)$ is g*p-closed in (X, τ)for every closed set V in (Y, σ).
13. #g-semi-continuous [28] if $f^{-1}(V)$ is #gs-closed in (X, τ) for every closed set V in (Y, σ).
14. $\tilde{g}$-continuous [21] if $f^{-1}(V)$ is $\tilde{g}$-closed in (X, τ) for every closed set V in (Y, σ).
15. Contra-continuous [7] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ).
16. Perfectly-continuous [3] if $f^{-1}(V)$ is both open and closed in (X, τ) for every open set V in (Y, σ).
17. Contra-Pre-continuous [11] if $f^{-1}(V)$ is Preclosed in (X, τ) for every open set V in (Y, σ).
18. $\tilde{g}$-irresolute [22] if $f^{-1}(V)$ is $\tilde{g}$-closed in (X, τ) for every $\tilde{g}$-closed set V in (Y, σ).
19. M-Preclosed [16] if f(V) is preclosed in (Y, σ) for every preclosed set V in (X,\tau).
20. RC-continuous [8] if $f^{-1}(V)$ is regular closed in (X, τ)for every open set V in (Y, σ).

Definition 1.4: A space (X, τ) is called
1. a $T_{1/2}$ space [14] if every g-closed set is closed.
2. a Tω space [23] if every ω-closed set is closed.
3. a $g_sT^{\#}_{1/2}$ space [28] if every #g-semi-closed set is closed.
4. a $T\tilde{g}$ -space [30] if every $\tilde{g}$ -closed set is closed.

Theorem 1.5: [5] (1) Every open and preclosed subset of (X, τ)is ρ-closed. Converse need not be true.
(2) Every ρ-closed set is ρ_s-closed (resp. gp-closed, gpr-closed, gsp-closed, πgp-closed)set. Converse need not be true.
(3) If D[A]⊆D_p[A] for each subset A of a space (X, τ), then the union of two ρ-closed sets is ρ-closed.
(4) A subset A of (X, τ)is regular open if A is both open and ρ-closed.

2. ρ-CONTINUOUS FUNCTIONS

Definition 2.1:
i) A function f: (X, τ) → (Y, σ) is said to be ρ*-continuous (resp.ρ_s-continuous) if $f^{-1}(V)$ is* ρ-closed *(resp.ρ_s-closed) in (X, τ) for every closed set V in (Y, σ).*
ii) A function f : (X, τ)→ (Y, σ) is said to be ρ*-irresolute (resp. ρ_s-irresolute) if $f^{-1}(V)$ is* ρ-closed *(resp.ρ_s-closed) in (X, τ) for every* ρ*-closed (resp.ρ_s-closed) V in (Y, σ).*

Example 2.2: 1. *Let X=Y={a, b, c, d},τ = {φ, { b}, {c}, {b, c}, {b, c, d}, X} and σ = {φ, {a, b},X}. Define a function f: (X, τ)→ (X, σ) by f(a) = f(b) = c, f(c) = d, f(d) = a. Then f is* ρ*-continuous.*

2. *Let X={a, b, c},τ={φ,{c},{a, b}, X} and σ={φ,{a, b},X}. Define f: (X, τ) →(X, σ) by f(a)=c , f(b)=b and f(c)=a. Then the inverse image of every* ρ*-closed set is* ρ*-closed under f. Hence f is* ρ*-irresolute.*

Proposition 2.3: *Every Contra-continuous and Pre-continuous is ρ-continuous.*

Proof: Let f: (X, τ) → (Y, σ) be Contra- continuous and pre-continuous. Let V be closed in (Y, σ). Then $f^{-1}(V)$ is preclosed and also open in (X, τ).Hence by Theorem 1.5(1), $f^{-1}(V)$ is ρ-*closed* in (X, τ). Therefore f is ρ-continuous.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.4: *Let X ={a, b, c, d, e}, Y={a, b, c, d}, τ = {ϕ, {a, b}, {a, b, d}, {a, b, c, d}, {a b, d, e}, X} and σ= {ϕ, {b, c}, {a, b, c}, {b, c, d}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = d; f(b) = a; f(c) = a; f(d) = b; f(e) = d. Then f is ρ-continuous but neither contra-continuous nor pre-continuous. Observe that for the closed set V = {a, d} in (Y, σ), $f^{-1}(V)$ = {a, b, c, e} is ρ-closed and it is neither preclosed nor open in (X, τ).*

Proposition 2.5: *Every ρ-continuous is gp-continuous.*

Proof: By Theorem 1.5(2), every ρ-closed set is gp-closed, the Proof follows.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.6: *Let X = Y = {a, b, c, d, e},τ = {ϕ,{ b, c, d},{a, b, c, d},{b, c, d, e},X} and σ= {ϕ,{a, b, e},X}. Define f: (X, τ) → (X, σ) by f (a) = e, f (b) = c, f(c) = d, f (d) = a, f (e) = b. Then f is gp-continuous but not ρ-continuous. Observe that for the closed set V = {c, d} in (X, σ), $f^{-1}(V)$ = {b, c} is gp-closed but not ρ-closed in (X, τ).*

Proposition 2.7: *Every ρ-continuous is gpr-continuous.*

Proof: By Theorem 1.5(2), every ρ-*closed* set is gpr-closed, the proof follows.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.8: *Let X= {a, b, c, d, e}, Y= {a, b, c, d}, τ = {ϕ, {c}, {e}, {a, b}, {c, e}, {a, b, c}, {a, b, e},{a, b, c, e}, X} and σ= {ϕ,{c ,d},Y}. Define f: (X, τ) → (Y, σ) by f(a) = f(c) = a; f(b) = f(e) = b; f(d) = c. Then f is gpr-continuous but not ρ-continuous. Observe that for the closed set V = {a, b} in(Y, σ), $f^{-1}(V)$ = {a, b, c, e} is gpr-closed but not ρ-closed in (X, τ).*

Proposition 2.9: *Every ρ-continuous is gsp-continuous.*

Proof: By Theorem 1.5(2), every ρ-*closed* set is gsp-closed, the proof follows.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.10: *Let X= {a, b, c, d, e}, Y={a, b, c}, τ = {ϕ,{a, b},{c, d},{a, b, c, d},X} and σ= {ϕ,{c},{b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = a; f(c) = a; f(d) = c; f(e) = c. Then f is gsp-continuous but not ρ-continuous. Observe that for the closed set V = {a, b} in (Y, σ), $f^{-1}(V)$ = {a ,b, c} is gsp-closed but not ρ-closed in (X, τ).*

Proposition 2.11: *Every ρ-continuous is πgp-continuous.*

Proof: By Theorem 1.5(2), every ρ-*closed* set is πgp-closed, the proof follows.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.12: *Let X= {a, b, c, d, e}, Y= {a, b, c, d},τ = {ϕ, {a, b}, {c, d}, {a, b, c, d}, X} and σ = {ϕ, {b, c}, {a, b, c}, {b, c, d}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = d; f(b) = c; f(c) = c; f(d) = a; f(e) = b. Then f is πgp-continuous but not ρ-continuous. Observe that for the closed set V = {a, d} in (Y, σ), $f^{-1}(V)$ = {a, d} is πgp -closed but not ρ-closed in (X, τ).*

Remark 2.13: ρ-*continuous and pre-continuous are independent concepts as we illustrate by means of the following examples.*

Example 2.14:

1. *Let X = {a, b, c, d, e}, Y={a, b, c, d}, τ = {ϕ, {a, b}, {a, b, d}, {a, b, c, d}, {a, b, d, e}, X} and σ= {ϕ,{b, c},{a, b, c},{b, c, d},Y}. Define f: (X, τ) → (Y, σ) by f(a) = d; f(b) = a; f(c) = a; f(d) = b; f(e) = d. Then f is ρ-continuous but not pre-continuous. It is clear that for the closed set V = {a, d} in (Y, σ), $f^{-1}(V)$ = {a, b, c, e} is ρ-closed but not preclosed in (X, τ).*
2. *Let X = {a, b, c, d, e}, Y ={a, b, c, d}, τ= {ϕ,{a, b},{a, b, c},{a, b, d},{a, b, c, d},X} and σ= {ϕ,{a, b},{a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = d; f(b) = b; f(c) = a; f(d) = c; f(e) = c. Then f is pre-continuous but not ρ-continuous. Observe that for the closed set V = {d} in (Y, σ), $f^{-1}(V)$ = {a} is preclosed but not ρ-closed in (X, τ).*

Remark 2.15: ρ-*continuous is independent concept of semi-continuous and semi-pre-continuous as we illustrate by means of the following examples.*

Example 2.16:

1. *Let* $X=\{a, b, c, d\}=Y$, $\tau = \{\phi,\{b\},\{c\},\{b, c\},\{b, c, d\},X\}$ *and* $\sigma= \{\phi,\{a, b\},X\}$. *Define* $f: (X, \tau) \to (X, \sigma)$ *by* $f(a) = c = f(b)$; $f(c) = d$; $f(d) = a$. *Then f is* ρ-*continuous but neither semi-continuous nor semi-pre-continuous. Since for the closed set* $V = \{c, d\}$ *in* (X, σ), $f^{-1}(V) = \{a, b, c\}$ *is* ρ-*closed but neither semi-closed nor semi-pre-closed in* (X, τ).
2. *Let* $X= \{a, b, c, d\} =Y$, $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ *and* $\sigma= \{\phi,\{c, d\}, X\}$. *Define* $f: (X, \tau) \to(X, \sigma)$ *by* $f (a) = d$; $f(b) = c$; $f(c) = a$; $f(d) = b$. *Then f is both semi-continuous and semi-pre-continuous but not* ρ-*continuous. Observe that for the closed set* $V = \{a, b\}$ *in* (X, σ), $f^{-1}(V) =\{c, d\}$ *is both semi-closed and semi-pre-closed but not* ρ-*closed in* (X, τ).

Remark 2.17: ρ-*continuous and pre-semi-continuous are independent concepts as we illustrate by means of the following examples.*

Example 2.18:

1. *Let* $X= \{a, b, c\} =Y$, $\tau = \{\phi,\{a\}, \{c\}, \{c, a\}, X\}$ *and* $\sigma=\{\phi,\{a, b\}, X\}$. *Define* $f: (X, \tau)\to(X, \sigma)$ *by* $f(a) = c$; $f(b) = a$; $f(c) = b$. *Then f is pre-semi-continuous but not* ρ-*continuous. For the closed set* $V=\{c\}$ *in* (X, σ), $f^{-1}(V) = \{a\}$ *is pre-semi-closed but not* ρ-*closed in* (X, τ).
2. Let $X = \{a, b, c\} = Y$, $\tau = \{\phi,\{c\},X\}$ and $\sigma = \{\phi,\{a, b\},X\}$. Define $f: (X, \tau) \to (X, \sigma)$ by $f(a) = a$; $f(b) = f(c) = c$. Then f is ρ-continuous but not pre-semi-continuous. Observe that for the closed set $V=\{c\}$ in (X, σ), $f^{-1}(V) = \{b, c\}$ is ρ-*closed* but not pre-semi-closed in (X, τ).

Remark 2.19: ρ-*continuous and pg-continuous are independent concepts as we illustrate by means of the following examples.*

Example 2.20:

1. *Let* $X= \{a, b, c, d, e\} =Y$, $\tau = \{\phi,\{a\},\{d, e\},\{a, d, e\}, X\}$ *and* $\sigma = \{\phi, \{b\}, \{d, e\}, \{b, d, e\}, \{a, c, d, e\}, X\}$. *Define* $f: (X, \tau) \to(X, \sigma)$ *by* $f (a) = d$; $f (b) = b$; $f(c) = a$; $f (d) = e$; $f (e) =c$. *Then f is* ρ-*continuous but not pg-continuous. So for the closed set* $V = \{a, c, d, e\}$ *in* (X, σ), $f^{-1}(V) = \{a, c, d, e\}$ *is* ρ-*closed but not pg-closed in* (X, τ).
2. Let $X = \{a, b, c, d, e\}$, $Y = \{a, b, c, d\}$, $\tau= \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ and $\sigma = \{\phi, \{a, b\}, \{c, d\}, Y\}$. Define $f: (X, \tau) \to(Y, \sigma)$ by $f(a) = c$; $f(b) = b$; $f(c) = a$; $f(d\} = d$; $f(e) = a$. Then f is pg-continuous but not ρ-continuous. Observe that for the closed set $V = \{c, d\}$ in (Y, σ), $f^{-1}(V) = \{a, d\}$ is pg-closed but not ρ-*closed* in (X, τ).

Remark 2.21: ρ-*continuous and g*p-continuous are independent concepts as we illustrate by means of the following examples.*

Example 2.22:

1. *Let* $X= \{a, b, c, d, e\} =Y$, $\tau = \{\phi, \{b\}, \{d, e\}, \{b, d, e\}, \{a, c, d, e\}, X\}$ *and* $\sigma = \{\phi,\{a\},\{d, e\},\{a, d, e\}, X\}$. *Define* $f: (X, \tau) \to (X, \sigma)$ *by* $f (a) = b$; $f (b) = c$; $f(c) = a$; $f(d) = e$; $f(e) = d$. *Then f is* ρ-*continuous but not g*p-continuous. Obviously for the closed set* $V = \{b, c, d, e\}$ *in* (X, σ), $f^{-1}(V) = \{a, b, d, e\}$ *is* ρ-*closed but not g*p-closed in* (X, τ).
2. Let $X= \{a, b, c, d, e\}$, $Y= \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{d, e\}, \{b, d, e\}, \{a, c, d, e\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Define $f: (X, \tau) \to(Y, \sigma)$ by $f(a) = f(b) = a$; $f(c)= f(e) = b$; $f(d) = c$. Then f is g*p-continuous but not ρ-continuous. Observe that for the closed set $V = \{c\}$ in (Y, σ), $f^{-1}(V) = \{d\}$ is g*p-closed but not ρ-*closed* in (X, τ).

Remark 2.23: ρ-*continuous and g -continuous are independent concepts as we illustrate by means of the following examples.*

Example 2.24:

1. *Let* $X= \{a, b, c, d, e\} =Y$, $\tau = \{\phi, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, \{a, b, d, e\}, X\}$ *and* $\sigma = \{\phi,\{a, b\},\{c, d\},\{a, b, c, d\},X\}$. *Define* $f: (X, \tau) \to(X, \sigma)$ *by* $f(a) = a$; $f(b) = c$; $f(c) = b$; $f(d) = d$; $f(e) = e$. *Then f is* ρ-*continuous but not g-continuous. It is clear that for the closed set* $V = \{c, d, e\}$ *in* (X, σ), $f^{-1}(V) = \{b, d, e\}$ *is* ρ-*closed but not g-closed in* (X, τ).
2. Let (X, κ) be digital topology and (X, τ) be usual topology. Define $f: (X, \kappa) \to(X, \tau)$ by $f(x) = x$. Let $\{4\}$ be closed in (X, τ). Then $f^{-1}(\{4\}) = \{4\}$ is closed in (X, κ) and thus g-closed but not ρ-*closed* in (X, κ).Because there is a $\tilde{g}$-open set $U=\{1,2,3,4\}$ containing $\{4\}$, is not open in (X, κ) such that $pcl(\{4\})=\{4\} \not\subseteq int(U)=\{1,2,3\}$. Hence f is g-continuous but not ρ-continuous.

Remark 2.25: ρ-*continuous and continuous are independent concepts as we illustrate by means of the following examples. .*

Example 2.26: *By Example 2.16(1), f is* ρ-*continuous but not continuous. Since for the closed set* $V =\{c, d\}$ *in* (X, σ), $f^{-1}(V) = \{a, b, c\}$ *is* ρ-*closed but not closed in* (X, τ) *and by Example 2.24(2), f is continuous but not* ρ-*continuous. Since for the closed set* $V= \{4\}$ *in* (X, τ), $f^{-1}(V)=\{4\}$ *is closed but not* ρ-*closed in* (X, κ).

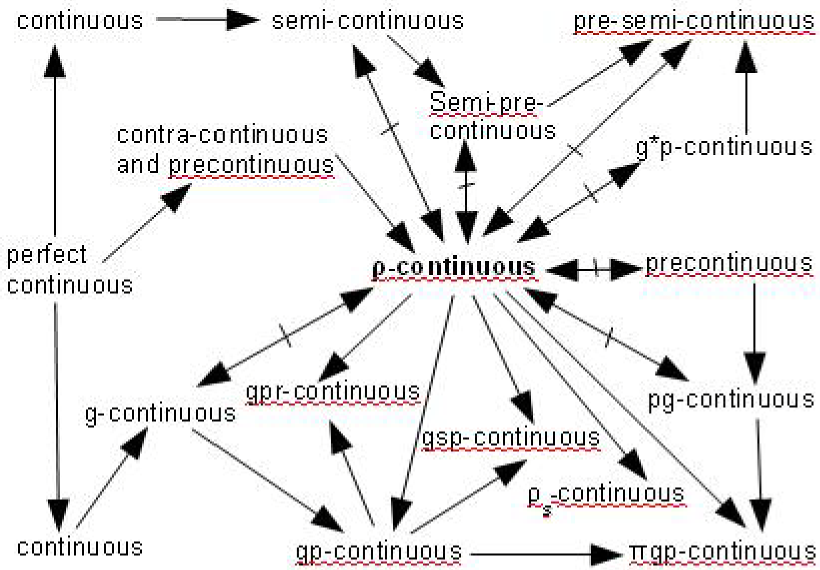
Proposition 2.27: *Every* ρ-*continuous is* ρ_s-*continuous.*

Proof: By Theorem 1.5(2), every ρ-*closed* set is ρ_s-closed, the proof follows.

The converse of the above proposition need not be true as it is seen from the following example.

Example 2.28: *Let* $X= \{a, b, c, d, e\}$, $Y=\{a, b, c, d\}$, $\tau = \{\phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$ *and* $\sigma= \{\phi, \{b, c\}, \{a, b, c\}, \{b, c, d\}, Y\}$. *Define* $f: (X, \tau) \rightarrow (Y, \sigma)$ *by* $f(a) = d$; $f(b) = c$; $f(c) = e$; $f(d) = a$; $f(e) = b$. *Then f is* ρ_s-*continuous but not* ρ-*continuous. For the closed set* $V = \{a, d\}$ *in* (Y, σ),$f^{-1}(V) = \{a,d\}$ *is* ρ_s-*closed but not* ρ-*closed in* (X, τ).

Remark 2.29: *We have the following relationship between* ρ-*continuous and other related generalized continuous.* $A\rightarrow B$ $(A \not\leftrightarrow B)$ *represent A implies B but not conversely (A and B are independent of each other).*



3. CHARACTERIZATION OF ρ-CONTINUOUS FUNCTIONS

Now we shall obtain characterization of ρ-continuous functions in the sense of Definition 2.1.

Theorem 3.1: *A function* $f: (X, \tau) \rightarrow (Y, \sigma)$ *is* ρ-*continuous if and only if* $f^{-1}(U)$ *is* ρ-*open in* (X, τ), *for every open set U in* (Y, σ).

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be ρ-continuous and U an open set in (Y, σ). Then $f^{-1}(U^c)$ is ρ-*closed* in (X, τ). But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is ρ-open in (X, τ). The converse is analogous.

Remark 3.2: *The composition of two* ρ-*continuous function need not be* ρ-*continuous and this is shown by the following example.*

Example 3.3: *Let* $X= \{a, b, c\}$, $\tau =\{\phi, \{b\}, X\}$, $\sigma=\{\phi, \{a, b\}, X\}$ *and* $\eta=\{\phi, \{a\}, \{a, b\}, X\}$. *Define* $f: (X, \tau)\rightarrow(X, \sigma)$ *by* $f(a)=c$; $f(b)=a$; $f(c)=b$ *and define* $g: (X, \sigma)\rightarrow(X, \eta)$ *by* $g(a)=c$; $g(b)=a$ *and* $g(c)=b$. *Then f and g are* ρ-*continuous but* $g \circ f$ *is not* ρ-*continuous. Since* $\{c\}$ *is closed in* (X, η). $(g \circ f)^{-1}(\{c\}) = f^{-1} (g^{-1}(\{c\})) = f^{-1}(\{a\}) =\{b\}$ *which is not* ρ-*closed in* (X, τ).

Definition 3.4:
1) A space (X, τ) *is said to be* ρ-T_s *space if every* ρ_s-*closed set is closed.*
2) A space (X, τ) is said to be ρ-$T_{1/2}$ space if every ρ_s-closed set is preclosed.

Theorem 3.5: *If* (X, τ) *and* (Z, η) *be topological spaces and* (Y, σ) *be* ρ-T_s *space then the composition* $g\circ f: (X, \tau)\rightarrow(Z, \eta)$ *of* ρ-*continuous(resp. continuous) function* $f: (X, \tau)\rightarrow (Y, \sigma)$ *and the* ρ_s -*continuous function* $g: (Y, \sigma)\rightarrow (Z, \eta)$ *is* ρ-*continuous(resp. continuous).*

Proof: Let G be any closed set of (Z, η). Then by assumption $g^{-1}(G)$ is closed in (Y, σ). Since f is ρ-continuous, then $f^{-1}(g^{-1}(G)) = (g\circ f)^{-1}(G)$ is ρ-*closed* (resp. closed)in (X, τ). Thus $g\circ f$ is ρ-continuous (resp. continuous).

Theorem 3.6: *Let (X, τ) and (Z, η) be any topological spaces and (Y, σ) be $T_{1/2}$ space (resp. T_ω space, $T_{\tilde{g}}$ space, $g_sT^{\#}_{½}$ space). Then the composition g∘f: (X, τ)→(Z, η) of* ρ*-continuous function f: (X, τ)→(Y, σ) and the g-continuous (resp. ω-continuous, $\tilde{g}$-continuous, $\#g_S$-continuous) function g : (Y, σ)→ (Z, η) is* ρ*-continuous.*

Proof: Let G be any closed set of (Z, η). Then $g^{-1}(G)$ is g-closed (resp.ω-closed, $\tilde{g}$-closed, $\#g_s$-closed) in (Y, σ) and by assumption, $g^{-1}(G)$ is closed in (Y, σ). Since f is ρ-continuous, $f^{-1}(g^{-1}(G))=(g∘f)^{-1}(G)$ is ρ-closed in (X,τ). Thus g∘f is *ρ*-continuous.

Theorem 3.7: *If f: (X, τ) →(Y, σ) is ρ-continuous and g: (Y, σ) → (Z, η) is continuous. Then their composition g∘f: (X,τ)→(Z,η) is ρ-continuous.*

Proof: Let G be any closed set of (Z, η). Then $g^{-1}(G)$ is closed in (Y, σ). Since f is *ρ*-continuous, $f^{-1}(g^{-1}(G)) = (g∘f)^{-1}(G)$ is *ρ-closed* in (X, τ). Thus g∘f is *ρ*-continuous.

Theorem 3.8: *If f: (X, τ) → (Y, σ) is continuous and contra-pre-continuous and g: (Y, σ) →(Z, η) is contra-continuous. Then their composition g∘f :(X, τ)→(Z, η) is ρ-continuous.*

Proof: Let G be any closed set of (Z, η). Since g is contra-continuity, then $g^{-1}(G)$ is open in (Y, σ). Since f is continuity and contra-pre-continuity, $f^{-1}(g^{-1}(G)) = (g∘f)^{-1}(G)$ is open and preclosed in (X, τ). Then by Theorem 1.5(1), $(g∘f)^{-1}(G)$ is *ρ-closed* in (X, τ). Then g∘f is *ρ*-continuous.

Theorem 3.9: *If f: (X, τ) → (Y, σ) is ρ-irresolute and g: (Y, σ) → (Z, η) is ρ-continuous then g∘f: (X, τ)→ (Z, η) is ρ-continuous.*

Proof: Let G be any closed set of (Z, η). Since g is *ρ*-continuous, $g^{-1}(G)$ is *ρ-closed* in (Y, σ). Since f is irresolute, $f^{-1}(g^{-1}(G)) = (g∘f)^{-1}(G)$ is *ρ-closed* in (X, τ). Thus g∘f is *ρ*-continuous.

Theorem 3.10: *Let f: (X, τ)→(Y, σ) be a ρ_s-continuous function. Then f is continuous if (X, τ) is ρ-T_s.*

Proof: Let G be any closed set of (Y, σ).Since f is ρ_s-continuous and by assumption $f^{-1}(G)$ is closed in (X, τ).Thus f is continuous.

Definition 3.11:
1. *Let x be a point of (X, τ) and V be a subset of X. Then V is called a ρ-neighbourhood of x in (X, τ) if there exists a ρ-open set U of (X, τ) such that x∈ U⊆V.*
2. [5] The intersection of all ρ-closed sets each containing a set A in a topological space X is called the ρ-closure of A and is denoted by ρ-cl(A).

Theorem 3.12: *Let A be a subset of (X, τ). Then x∈* ρ*-cl(A) if and only if for any ρ-neighbourhood N_x of x in (X, τ), A∩ N_x ≠ Ø.*

Proof: Necessity- Assume that *x*∈ρ-cl(A). Suppose that there exists a *ρ*-neighbourhood N_x of x such that A∩ N_x = Ø. Since N_x is a ρ-neighbourhood of *x* in (X, τ), by Definition 3.11, there exists a *ρ*-open set V_x such that $x∈V_x ⊆N_x$, Therefore, we have $A∩V_x$ = Ø and so $A⊆(V_x)^c$. Since $(V_x)^c$ is a *ρ-closed* set containing A, we have $ρ\text{-cl}(A)⊆(V_x)^c$ and therefore *x*∉*ρ*-cl(A), which is a contradiction.

Sufficiency- Assume that for each *ρ*-neighbourhood N_x of *x* in (X, τ), $A∩N_x$ ≠ Ø. Suppose *x*∉ρ-cl(A). Then there exists a *ρ-closed* set V of (X, τ) such that A⊆V and *x*∉V. Thus, $x ∈V^c$ and V^c is *ρ*-open in (X,τ). But $A∩V^c$ = Ø which is a contradiction.

Theorem 3.13: *Let f: (X, τ)→(Y, σ) be a function. Then the following statements are equivalent.*
1. The function f is ρ-continuous.
2. The inverse of each open set in (Y, σ) is ρ-open in (X, τ).
3. The inverse of each closed set in (Y, σ) is ρ-closed in (X, τ).
4. For each x in (X, τ), the inverse of every neighbourhood of f(x) is a ρ-neighbourhood of x.
5. For each x in (X, τ) and each neighbourhood N of f(x), there is a ρ-neighbourhood W of x such that f (W)⊆ N.
6. For each subset A of (X, τ), f (ρ-cl(A))⊆ cl(f(A)).
7. For each subset B of (Y,σ), $ρ\text{-cl}(f^{-1}(B))⊆f^{-1}(cl(B))$.

Proof:
(1)⟷ (2) This follows from Theorem 3.1.

(2)⟷ (3) The Proof is clear from the result $f^{-1}(A^c) = (f^{-1}(A))^c$.

(2)⟷ (4) For x in(X, τ), Let N be a neighbourhood of f(x). Then there exists an open set V in (Y, σ) such that f(x)∈V⊆N. Consequently, f^{-1}(V) is ρ-open set in (X,τ) and x∈f^{-1}(V) ⊆f^{-1}(N). Thus, f^{-1}(N) is a ρ-neighbourhood of x.

(4)⟷ (5) Let x∈X and let N be a neighbourhood of f(x). Then by assumption, W = f^{-1}(N) is a ρ-neighbourhood of x and f(W) = f (f^{-1}(N))⊆N.

(5)⟷ (6) Suppose that (5) holds and let y∈f(ρ-cl(A))and let N be any neighbourhood of y. Then there exists a x∈X and a ρ-neighbourhood W of x, such that f(x) = y, x∈W, x∈ρ-cl(A) and f(W)⊆N,

By Theorem 3.12, ∩W A ≠ Ø and hence f(A) ∩ N ≠ Ø . Hence y = f(x)∈cl(f(A)).Therefore, f (ρ-cl(A))⊆cl(f(A)).Conversely, suppose that (6) holds and let x∈X and N be any neighbourhood of f(x). Let A = f^{-1}(Y \ N), since f(ρ-cl(A)) ⊆ cl(f(A)) ⊆ Y \ N, ρ-cl(A) ⊆ A. Then ρ-cl(A) = A. Since x ∉ ρ-cl(A), there exists a ρ-neighbourhood W of x such that W∩A = Ø and hence f(W)⊆f(X-A)⊆N.

(6)⟷ (7) Suppose that (6) holds and B be any subset of (Y, σ), Then replacing A by f^{-1}(B) in (vi), We obtain f(ρ-cl (f^{-1}(B))) ⊆ cl(f(f^{-1}(B))) ⊆ cl(B). That is ρ-cl(f^{-1}(B))⊆f^{-1}(cl(B)). Conversely, suppose that(7) holds, let B = f(A). Where A is a subset of (X, τ). Then we have, ρ-cl(A)⊆ρ-cl(f^{-1}(B))⊆(f^{-1}(cl(f(A)))) and so f(ρ-cl(A)⊆cl(f(A)) This completes the Proof of the theorem.

Definition 3.14:[22] A function f : *(X, τ)→(Y, σ) is called $\tilde{g}$-irresolute if* f^{-1}(V) is a *$\tilde{g}$-closed set of (X, τ) for every $\tilde{g}$-closed set V of (Y, σ)* .

Proposition 3.15: *If f: (X, τ) →(Y, σ) is $\tilde{g}$-irresolute and M-preclosed function, then f(A) is ρ-closed in (Y, σ) for every ρ-closed set A of (X, τ).*

Proof: Let U be any $\tilde{g}$-open in (Y, σ) such that f (A)⊆U. Then A⊆f^{-1}(U). Since f is $\tilde{g}$-irresolute then f^{-1}(U) is $\tilde{g}$-open in (X, τ) and A is *ρ-closed* in (X, τ), we have pcl(A)⊆Int(f^{-1}(U)).Thus f(pcl(A))⊆Int(U).Since f is M-preclosed and pcl(A) is preclosed in (X, τ) then f(pcl(A)) is a Preclosed set in (Y, σ). Now pcl(f(A)) ⊆ pcl(f(pcl(A))) = f(pcl(A)) ⊆ Int(U) and so f(A) is *ρ-closed* in (Y, σ).

Theorem 3.16: *If the bijective function f : (X, τ)→ (Y, σ) is pre-irresolute and $\tilde{g}$-open mappings, then f is ρ-irresolute.*

Proof: Let A be *ρ-closed* in (Y, σ) and let U be any $\tilde{g}$ -open in (X, τ) such that f^{-1}(A)⊆U. Then A⊆f(U). Since f is $\tilde{g}$-open then f (U) is $\tilde{g}$-open in (Y, σ) and A is *ρ-closed* in (Y, σ), we have pcl(A)⊆Int(f(U)) and thus f^{-1}(pcl(A))⊆f^{-1} (Intf((U))) ⊆ Int (f^{-1}(f(U))=Int(U). Since f is pre-irresolute and pcl(A) is preclosed in (Y, σ) then f^{-1}(pcl(A)) is a Pre-closed set in (X, τ). Now pcl(f^{-1} (A))⊆pcl(f^{-1}(pcl(A)))= f^{-1}(pcl(A))⊆ Int(U). Hence f^{-1}(A) is *ρ-closed* in (X, τ) and f is ρ-irresolute.

Theorem 3.17:
1. If f: (X, τ)→(Y, σ) is gp-continuous and contra-continuous then f is ρ-continuous.
2. If f: (X, τ) → (Y, σ) is gpr-continuous(resp. πgp-continouous) and RC-continuous then f is ρ-continuous.

Proof:
(1) Let V be any closed set in (Y, σ).Since f is gp-continuous and contra-continuous, f^{-1}(V) is gp-closed and open in (X, τ). By Proposition 2.2[20], f^{-1}(V) is Preclosed in (X, τ) and by Theorem 1.5(2), f^{-1}(V) is *ρ-closed* in (X, τ) and so f is ρ-continuous.
(2) Let V be any closed set in (Y, σ). Since f is gpr-continuous (resp. πgp-continuous) and RC-continuous, f^{-1}(V) is gpr-closed (resp. πgp-closed) and regular open (resp. regular open is π-open) in (X, τ). By Theorm 3.10[9], f^{-1}(V) is Preclosed in (X, τ).(resp. By Theorem 2.4[18], f^{-1}(V) is preclosed in (X, τ)). Since regular open is open, by Theorem 1.5(1), f^{-1}(V) is *ρ-closed* in (X, τ) and so f is ρ-continuous.

Theorem 3.18: *If f: (X, τ) →(Y, σ) is ρ-irresolute and g: (Y, σ) → (Z, η) is ρ-irresolute then g∘f: (X, τ) →(Z, η) is ρ-irresolute.*

Proof: Let G be any *ρ-closed* in (Z, η). Since g is ρ-irresolute, g^{-1}(G) is *ρ-closed* in (Y, σ), since f is ρ-irresolute, f^{-1}(g^{-1}(G)) = $(g∘f)^{-1}$(G) is *ρ-closed* in (X, τ). Thus g∘f is ρ-irresolute.

Regarding the restriction of a ρ-continuous function, we have the following.

Lemma 3.19[12]: 1. Let A be a $\tilde{g}$-closed set of (X, τ). If A is regular closed, then pcl(A) is also $\tilde{g}$ -closed.
2. If A ⊆Y⊆ X where A is $\tilde{g}$-open in Yand Y is $\tilde{g}$-open in X then A is $\tilde{g}$-open in X.
3. Let A ⊆Y⊆ X and suppose that A is $\tilde{g}$-closed in X then A is $\tilde{g}$-closed in Y.

Theorem 3.20: *Let $f: (X, \tau) \to (Y, \sigma)$ be a ρ-continuous function and H be a open ρ-closed subset of X. Assume that $\rho C(X,\tau)$ (the class of all ρ-closed sets of (X,τ)) be closed under finite intersections. Then the restriction $f\,|\,H :(H, \tau/H) \to (Y,\sigma)$ is ρ-continuous.*

Proof: Let F be a closed subset of Y. By hypothesis and assumption, $f^{-1}(F) \cap H = H_1$(say) is *ρ-closed* in X. since $(f|H)^{-1}(F) = H_1$, it is sufficient to show that H_1 is *ρ-closed* in H. Let G_1 be $\tilde{g}$-open set in H such that $H_1 \subseteq G_1$. Then by hypothesis and by Lemma 3.19(2),G_1 is $\tilde{g}$-open.in X. Since H_1 is *ρ-closed* in X, $pcl_X(H_1) \subseteq Int(G_1)$. Since H is open and By Lemma 2.10[10], $pcl_H(H_1) = pcl_X(H_1) \cap H \subseteq Int(G_1) \cap H = Int(G_1) \cap Int(H) = Int(G_1 \cap H) \subseteq Int(G_1)$ and so $H_1 = (f|H)^{-1}(F)$ is *ρ-closed* in H. Thus f|H is *ρ*-continuous.

Theorem 3.21: *Let A and Y be subsets of (X, τ) such that $A \subseteq Y \subseteq X$. Let A be $\tilde{g}$-closed and regular closed in (X, τ). If A is an ρ-closed set in (Y,σ) and Y is open and ρ-closed set in (X, τ) then A is ρ-closed in (X, τ).*

Proof: Let U be $\tilde{g}$-open set of (X,τ) such that $A \subseteq U$. since Y is open in (X, τ) and A is *ρ-closed* set in (Y, σ) , then we have $pcl_Y (A) \subseteq Int_Y (U \cap Y)$. Thus we have, $pcl(A) \cap Y \subseteq pcl_Y(A) \subseteq Int_Y(U \cap Y) = Int(U \cap Y)$. By Lemma 3.19(1), X-pcl(A) is $\tilde{g}$-open in (X, τ). Hence $Int(U \cap Y) \cup (X\text{-}pcl(A))$ is $\tilde{g}$-open in (X, τ) and it contains Y. Since Y is *ρ-closed* in (X, τ). we have, $pcl(A) \subseteq pcl(Y) \subseteq Int[Int(U \cap Y) \cup (X\text{-}pcl(A))] \subseteq Int(U) \cup (X\text{-}pcl(A))$. Thus $pcl(A) \subseteq Int(U)$ and so A is *ρ-closed* in (X, τ).

Now we have the following results which concerns pasting Lemma for *ρ*-continuous functions.

Theorem 3.22: *Let $X=G \cup H$ be a topological space with topology τ and Y be a topological space with topology σ. Let $f: (G,\tau/G) \to (Y, \sigma)$ and $g: (H, \tau/H) \to (Y, \sigma)$ be ρ-continuous functions such that $f(x) = g(x)$ for every $x \in G \cap H$. Assume that $D[E] \subseteq Dp[E]$, for any $E \subseteq X$. Suppose that both G and H are open and ρ-closed in (X, τ).Then their combination $f \Delta g : (X, \tau) \to (Y, \sigma)$, defined by $(f \Delta g)(x) = f(x)$ if $x \in G$ and $(f \Delta g)(x) = g(x)$ if $x \in H$, is ρ-continuous.*

Proof: Let F be a closed subset of (Y, σ).Then $f^{-1}(F)$ is *ρ-closed* in$(G, \tau|G)$ and $g^{-1}(F)$ is closed set in $(H, \tau|H)$.Since G, H are both open and *ρ-closed* subsets of (X, τ). By Theorem 3.19, $f^{-1}(F)$ and $g^{-1}(F)$ are both *ρ-closed* sets in (X, τ).By Theorem 1.5(3), $f^{-1}(F) \cup g^{-1}(F)$ is *ρ-closed* in (X, τ). By definition $(f \Delta g)^{-1}(F) = f^{-1}(F) \cup g^{-1}(F)$, is *ρ-closed* in (X, τ). Hence, $f \Delta g$ is ρ-continuous.

4. STRONGLY *P*-CONTINUOUS AND PERFECTLY *P*-CONTINUOUS FUNCTIONS

Definition 4.1: *A function $f: (X, \tau) \to (Y, \sigma)$ is called*

1. Strongly gp-continuous [20] if $f^{-1}(V)$ is closed (resp. open) in (X, τ) for every gp-closed set(resp. gp-open set) V of (Y, σ).
2. Strongly πgp-continuous [19] if $f^{-1}(V)$ is closed (resp. open) in (X, τ) for every *πgp*-closed set(resp. *πgp*-open set) V of (Y, σ).
3. Strongly *ρ*-continuous if $f^{-1}(V)$ is closed (resp. open) in (X, τ) for every *ρ-closed* set (resp. *ρ*-open set) V of (Y, σ).
4. Perfectly *ρ*-continuous if $f^{-1}(V)$ is clopen in (X, τ) for every *ρ-closed* set(resp. *ρ*-open set) V of (Y, σ).
5. Pre-*ρ*-continuous if $f^{-1}(V)$ is *ρ-closed* in (X, τ) for every preclosed set V of (Y, σ).

Remark 4.2: *From the above definition and ρ-closed→gp-closed (resp. πgp-closed) we have*

Strongly gp-continuous ↘
Perfectly ρ-continuous → *Strongly ρ-continuous*
Strongly πgp-continuous ↗

Theorem 4.3:

1. If $f: (X, \tau) \to (Y, \sigma)$ is perfectly ρ-continuous then f is strongly ρ-continuous and also ρ-irresolute.
2. If $f: (X, \tau) \to (Y, \sigma)$ is pre-ρ-continuous then f is ρ-continuous.
3. If $f: (X, \tau) \to (Y, \sigma)$ is strongly ρ-continuous and $g: (Y, \sigma) \to (Z, \eta)$ is ρ-continuous $g \circ f: (X, \tau) \to (Z, \eta)$ is continuous.
4. If $f: (X, \tau) \to (Y, \sigma)$ is strongly ρ-continuous and $g: (Y, \sigma) \to (Z, \eta)$ is perfectly ρ-continuous, then $g \circ f: (X, \tau) \to (Z, \eta)$ is strongly ρ-continuous.
5. If $f: (X, \tau) \to (Y, \sigma)$ is perfectly ρ-continuous and $g: (Y, \sigma) \to (Z, \eta)$ is pre ρ-continuous, then $g \circ f: (X, \tau) \to (Z, \eta)$ is ρ-continuous.

Proof:

1. Let V be *ρ-closed* in (Y, σ). Then $f^{-1}(V)$ is clopen in (X, τ) and hence $f^{-1}(V)$ is closed in (X, τ) and so f is strongly *ρ*-continuous. Since closed set implies preclosed set and by Theorem 1.5(1), $f^{-1}(V)$ is *ρ-closed* in (X, τ). Thus f is *ρ*-irresolute.
2. Closed set implies preclosed set and the proof is obvious.

3. Let V be closed in (Z, η). Then $g^{-1}(V)$ is *ρ-closed* in (Y, σ) and $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is closed in (X, τ). Thus g∘f is continuous.
4. Let V be *ρ-closed* in (Z, η). Then $g^{-1}(V)$ is clopen in (Y, σ). since closed implies preclosed and by Theorem 1.5(1), $g^{-1}(V)$ is *ρ-closed* in (Y, σ) and hence $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is closed in (X, τ). Thus g∘f is strongly *ρ*-continuous.
5. Let V be closed in (Z,η). since closed implies preclosed. Then $g^{-1}(V)$ is *ρ-closed* in (Y, σ). Hence $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is clopen in (X, τ) and so $(g\circ f)^{-1}(V)$ is *ρ-closed* in (X, τ). Therefore g∘f is *ρ*-continuous.

Theorem 4.4: *Let f: (X, τ) →(Y, σ) be a surjective, ρ-irresolute and M-preclosed. If (X, τ) is a ρ-$T_{1/2}$ space, then (Y, σ) is also ρ-$T_{1/2}$ space.*

Proof: Let A be *ρ-closed* in (Y, σ). Since f is *ρ*-irresolute, then $f^{-1}(A)$ is *ρ-closed* in (X, τ). Since (X,τ) is a *ρ*-$T_{1/2}$ space and by Theorem 1.5(2), $f^{-1}(A)$ is preclosed in (X, τ). Since f is M-preclosed, then $f(f^{-1}(A)) = A$ is preclosed in (Y, σ). Hence by Theorem 1.5(2), A is ρ_s-closed in (Y, σ) which is preclosed in (Y, σ). Therefore (Y,σ) is *ρ*-$T_{1/2}$ space.

5 *ρ*-Compact and *ρ*-Connected

Definition 5.1: *A topological space (X, τ) is ρ-compact if every ρ-open cover of X has a finite subcover.*

A subset A of *(X, τ) is regular open if A is open and* ρ-closed [Theorem 1.5[4]]. This suggest that a space is S-closed if it is strongly S-closed and ρ-compact. A space *(X,τ)* is S-closed[25] if every regular closed cover of X has a finite subcover. A space *(X, τ)* is strongly S-closed[7] if every cover of *(X, τ)* by closed sets has a finite subcover.

Definition 5.2: *A topological space (X, τ) is ρ-connected if X cannot be written as the disjoint union of two non-empty ρ-open sets.*

Theorem 5.3: *Let f: (X, τ)→(Y, σ) be a surjective, ρ-continuous function .If X is ρ-compact, then Y is compact.*

Proof: Let $\{A_i : i\in I\}$be an open cover of Y. Then $\{ f^{-1}(A_i) : i\in I \}$ is a *ρ*-open cover of X. Since X is ρ-compact, it has a finite sub cover, say $\{f^{-1}(A_1),\ f^{-1}(A_2), ..., f^{-1}(A_n)\}$. The Surjectiveness of f implies $\{A_1, A_2, ..., A_n\}$ is a finite sub cover of Y and hence Y is compact.

Theorem 5.4: *Let f: (X, τ)→(Y, σ) be a surjective, ρ-continuous (resp. ρ-irresolute) function .If X is ρ-connected, then Y is connected(resp. ρ-connected).*

Proof: Suppose Y is not connected (resp. not ρ-connected) Then Y = A∪B. Where A∩B = Ø, A≠Ø, B≠Ø and A, B are open (resp. *ρ*-open) sets in Y. Since f is surjective, f(X) =Y since f is *ρ*-continuous (resp. *ρ*-irresolute),
$X = f^{-1}(A)\cup f^{-1}(B)$, is the disjoint union of two non-empty *ρ*-open sets. This contradicts the fact that X is *ρ*-connected.

Definition 5.5: *A subset A of a space X is called ρ-compact relative to X if every collection $\{U_i : i\in I\}$ of ρ-open subsets of X such that $A\subseteq\cup\{U_i : i\in I\}$, there exists a finite subset I_0 of I such that $A\subseteq\cup\{U_i : i\in I_0\}$.*

Theorem 5.6: *Every ρ-closed subset of a ρ-compact space X is ρ-compact relative to X .*

Proof: Let A be a *ρ-closed* subset of a *ρ*-compact space X. Let $\{ U_i : i\in I \}$ be a cover of A by *ρ*-open subsets of X. So $A \subseteq \cup\{ U_i : i\in I \}$ and then $(X \setminus A)\cup (\cup\{ U_i : i\in I \}) = X$. since X is *ρ*-compact there exists a finite subset I_0 of I such that $(X \setminus A)\cup(\cup\{ U_i : i\in I_0\}) = X$. Then $A\subseteq\cup \{ U_i : i\in I_0\}$ and hence A is *ρ*-compact relative to X.

Theorem 5.7: *If f: (X,τ)→(Y,σ) is ρ-irresolute and a subset A of X is ρ-compact relative to X then its image f(A) is ρ-compact relative to Y.*

Proof: Let $\{f(U_i): i\in I\}$ be a cover of f(A) by *ρ*-open subset of (Y,σ). Since f is *ρ*-irresolute, then $\{U_i : i\in I\}$is a cover of A by *ρ*-open subsets of (X,τ). Since A is compact relative to X, there exists a finite subcover I_0 of I such that $A\subseteq\cup\{U_i : i\in I_0\}$. Hence $f(A)\subseteq\cup\{ f(U_i): i\in I_0\}$ and so f(A) is compact relative to Y.

Theorem 5.8: *If P: X×Y→X be a projection. Then p is irresolute.*

Proof: Let A be a *ρ-closed* subset X. since p is a Projection, then $p^{-1}(A) = A\times Y$ is subset of X×Y.

Now to show $p^{-1}(A) = A\times Y$ is *ρ-closed* in X×Y. Let U be $\tilde{g}$-open subset of X×Y such that $A\times Y\subseteq U$.

Then V×Y = U,for some open subset V of X containing A. Since A is *ρ-closed* in X, We have, $pcl_X(A)\subseteq Int(V)$ and $pcl_X(A)\times Y\subseteq Int(V)\times Y$, i.e, $pcl_{X\times Y}(A\times Y)\subseteq Int(V\times Y)=Int(U)$. Hence $p^{-1}(A) = A\times Y$ is *ρ-closed* in X×Y. □

Theorem 5.9: *If the Product space X×Y is ρ-compact then each of the spaces X and Y is ρ-compact.*

Proof: Let X×Y be *ρ*-compact. By Theorem 5.8, the projection p : X×Y→ X is *ρ*-irresolute and then by Theorem 5.7, p(X×Y) = X is *ρ*-compact. The proof for the space Y is similar to the case of X. □

Lemma 5.10: (The tube lemma) consider the product space XxY, where Y is compact. If N is an open set of XxY containing the slice x_0xY of XxY, then N contains some tube WxY about x_0xY, where W is a neighbourhood of $x_0$in X.

Theorem 5.11: *Let A be any subset of Y.*
1. If X×A is ρ-closed in the Product space X×Y and Y is $T_{\tilde{g}}$ space then A is ρ-closed in Y.
2. If X is compact and A is ρ-closed in Y and X×Y is $T_{\tilde{g}}$ space then X×A is ρ-closed in X×Y.

Proof:
1. Let U be $\tilde{g}$-open set of Y such that A⊆U. Then X × A ⊆ X × U. Since Y is $T_{\tilde{g}}$,therefore U is open in Y and X×U is open in X×Y, hence X×U is $\tilde{g}$-open in X×Y. Since X×A is *ρ-closed* in X×Y, Therefore pcl(X×A) ⊆ Int(X×U)= X×U. By proposition 2.8[20], X × pcl(A) ⊆ X × Int(U). Thus pcl(A) ⊆ Int(U) and so A is *ρ-closed* in Y.
2. Let U be $\tilde{g}$ -open set of X×Y such that X×A⊆U. since X is compact and X×Y is $T_{\tilde{g}}$ and by the generalization of lemma 5.10,there exists an open set V in Y containing A such that X×V⊆U. Since A is *ρ-closed* in Y, pcl(A)⊆Int(V), X×pcl(A)⊆X×Int(V) = Int(X) × Int(V)⊆Int(X×V), By proposition 2.8[20], pcl (X×A)⊆Int (X×V)⊆Int (U). Therefore X×A is *ρ-closed* in X×Y.

Theorem 5.12: *If the Product space X×Y is ρ-connected then each of the spaces X and Y is ρ-connected.*

Proof: Let X×Y be *ρ*-connected. By Theorem 5.8, the projection p : X×Y→X is *ρ*-irresolute and then by Theorem 5.4, p(X×Y) = X is *ρ*-connected. The proof for the space Y is similar to the case of X.

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