THE EFFECT OF MAGNETIC FIELD ON SEPARATION OF BINARY MIXTURE OF VISCOUS FLUIDS BY BARO DIFFUSION AND THERMAL DIFFUSION NEAR A STAGNATION POINT – A NUMERICAL STUDY

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ABSTRACT

Effects of pressure gradient and temperature gradient on separation of a binary mixture of incompressible viscous fluids in two dimensional magnetohydrodynamics stagnation point flow have been investigated. The numerical solutions of non-linear coupled ordinary differential equations have been obtained and exhibited graphically. It has been found that the effects of the pressure gradient and temperature gradient are to separate the components of the binary mixture. The lighter component gets collected near the stationary plate.

Keywords: barodiffusion, thermal diffusion, magnetohydrodynamics, binary fluid mixture.

1. NOMENCLATURE:

- \( B_d = \frac{A_j b}{\rho} \) is the Baro diffusion coefficient (Dimensionless number)
- \( C_1 \) the concentration of the rarer and lighter component of the binary fluid mixture
- \( C_p \) the specific heat at constant pressure (kJ kg\(^{-1}\) K\(^{-1}\))
- \( D \) is the diffusion coefficient (m\(^2\) s\(^{-1}\))
- \( H_x \) and \( H_y \) the magnetic field strength along \( x \) and \( y \) directions respectively (Gauss)
- \( \vec{H} \) is the magnetic intensity vector (Gauss)
- \( \vec{J} \) is the conduction current density vector (Gauss / m)
- \( K \) the thermal conductivity (W m\(^{-1}\) K\(^{-1}\))
- \( k_p \) is baro diffusion ratio (m s\(^2\) kg\(^{-1}\))
- \( k_pD \) is the baro diffusion coefficient (m\(^3\) s kg\(^{-1}\))
- \( k_T \) is the thermal diffusion ratio (1 / \( \Theta c \))
- \( k_TD \) is the thermal diffusion coefficient (m\(^2\) \( \Theta c^{-1} \) s\(^{-1}\))
- \( M = \frac{\mu H_0^2 \rho}{\sigma \rho} \) is the Hartmann number (Dimensionless number)
- \( p \) is the pressure (Nm\(^{-2}\))
- \( Pr = \frac{\kappa}{\mu C_p} \) is the Prandtl number (Dimensionless number)
- \( Sc = \frac{\rho}{\mu} \) is the Schmidt number (Dimensionless number)
- \( T \) is the temperature (\( \Theta c \))
- \( T_i = \frac{T - T_0}{\Theta c} \) is the temperature (Dimensionless number)
- \( \tau_d = \frac{c_p}{c_v} \) is the Thermal diffusion number (Dimensionless number)
- \( u \) and \( v \) denote the velocity components along \( x \) and \( y \) directions respectively (m s\(^{-1}\))
- \( \mu_e \) is magnetic permeability [kg m (coulomb)\(^{-2}\)]
- \( \nu_H \) is magnetic viscosity or magnetic diffusivity [(coulomb)\(^2\) mho\(^{-1}\) kg\(^{-1}\)]
- \( \mu \) is the coefficient of viscosity of the binary fluid mixture (kg s\(^{-1}\) m\(^{-1}\))
- \( \sigma \) is the electric conductivity (mho / m)
- \( \gamma = \frac{\Theta c_{p}}{\Theta c_{v}} \) ratio of specific heats (Dimensionless number)
- \( \nu \) is the kinematic viscosity (m\(^2\) s\(^{-1}\))
- \( \rho \) is the density of the binary mixture (kg / m\(^3\))
- \( \phi \) represents the heat due to viscous dissipation (1/ s\(^2\))

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2. INTRODUCTION:

The process of separation of components of a binary mixture of viscous incompressible fluids where in one of the components is present in extremely small proportion is important due to its varieties of applications such as in mechanical, chemical and aerospace engineering; physics; chemistry and biology. The separation of isotopes in its naturally occurring mixture is one such example. The composition of binary mixture is described by the concentration $C_1$, defined as the ratio of mass of rarer and lighter component to the total mass of the mixture in a given volume. The concentration $C_2$ of heavier and abundant component is given by $C_2 = 1 - C_1$. A binary mixture subject to the temperature gradient can generate thermal diffusion i.e the temperature gradients cause solute fluxes. This phenomenon is known as Soret effect. In the flow of such a mixture the diffusion of individual species takes place by three mechanisms namely concentration gradient, pressure gradient and temperature gradient. The diffusion flux $\mathbf{\Gamma}$ is given by Landau and Lifshitz [1] as:

$$\mathbf{\Gamma} = -\rho D \nabla C_1 + k_p \nabla p + k_T \nabla T$$  

In equation (1) the first term in the right hand side represents the ordinary diffusion whose contribution to the mass flux depends in a complicated way on the concentration gradients of the substances present in the binary mixture. The second one representing the pressure diffusion term indicates that there may be a net movement of the $i^{th}$ species in the mixture if there is a pressure gradient imposed on the system. The last one representing the thermal diffusion term describes the tendency for species to diffuse under the influence of a temperature gradient. The effects of the last two terms are quite small, but devices can be arranged to produce very steep pressure gradients and temperature gradients so that separation of mixtures may be effected. Caldwell [2, 3, 4] studied Benard convection in sea water and found a stabilizing effect of the thermal mass diffusion on the static state of heat conduction. Shah [5] has discussed the effect of pressure gradient and temperature gradient on separation of a binary mixture of incompressible viscous fluids confined between two parallel plates. Srivastava [6, 7] has studied Separation of a binary mixture of viscous fluids by baro diffusion and thermal diffusion near a stagnation point. Sharma and Singh [8, 9] have studied the effect of temperature gradient on separation of species in hydromagnetic flow of a binary mixture of incompressible viscous fluids between two parallel plates, first taking the plates horizontal and second by taking the plates vertical. They found that the effect of temperature gradient is to separate the components of the binary mixture and the magnetic field increases the effect of species separation. Sharma and Singh [10, 11, 12] and Sharma et al. [13] have studied the effect of magnetic field on separation of a binary mixture fluid. Ho-Ming Yeh [14] has studied separation of heavy water from water-isotopes mixture in flat-plate thermal diffusion columns with transverse sampling streams and optimal plate spacing.

In this piece of work we have discussed numerically the mass transfer in a binary mixture of thermally and electrically conducting incompressible viscous fluids which impinges perpendicularly on a wall maintained at a temperature higher than that of the fluid at a large distance from the wall. It is assumed that concentration $C_1$ is small so that its square is negligible.

3. FORMULATION OF THE PROBLEM:

We consider two dimensional steady fully developed stagnation point flow of a binary mixture of thermally and electrically conducting incompressible viscous fluids over an insulated plate situated at $y = 0$. The stagnation point is at origin and x-axis is along the plate. There is an external applied magnetic field of constant strength $H_0$ in the direction of the y-axis i.e. perpendicular to the plate.

For the case under consideration the equation of continuity, equations of motion, magnetic field equations, energy equation and the equation for the species conservation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\mu_e}{2 \rho} \frac{\partial}{\partial x} \left( H_x^2 + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\mu_e}{\rho} \left( H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_y}{\partial y} \right),$$  

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\mu_e}{2 \rho} \frac{\partial}{\partial y} \left( H_x^2 + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\mu_e}{\rho} \left( H_x \frac{\partial H_x}{\partial x} + H_y \frac{\partial H_y}{\partial y} \right),$$  

$$u \frac{\partial H_x}{\partial x} + v \frac{\partial H_y}{\partial y} = \nu_h \left( \frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} \right),$$  

$$u \frac{\partial H_x}{\partial x} + v \frac{\partial H_y}{\partial y} = \nu_h \left( \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} \right).$$
\[
\rho c_p \left( \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi + \frac{f^2}{\sigma} \tag{7}
\]

and
\[
u \frac{\partial C_1}{\partial x} + v \frac{\partial C_1}{\partial y} = \nu \left[ \frac{\partial^2 C_1}{\partial x^2} + \frac{\partial^2 C_1}{\partial y^2} + \frac{\partial}{\partial x} \left( k_p \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_p \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial x} \left( k_T \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_T \frac{\partial T}{\partial y} \right) \right]. \tag{8}
\]

The boundary conditions of the problem are
\[
\begin{align*}
u = 0, & \quad v = 0, \quad T = T_1, \quad \rho V_n C_1 - \rho \left( \frac{\partial C_1}{\partial n} + k_p \frac{\partial P}{\partial n} + k_T \frac{\partial T}{\partial n} \right) = 0, \quad \text{at } y = 0 \quad \text{(9)}
\end{align*}
\]

\[
\begin{align*}
\nu = bx, & \quad T = T_0, \quad C = C_0, \quad \text{as } y \to \infty. \quad \text{(10)}
\end{align*}
\]

The coefficients \( k_p \) and \( k_T \) may be determined from the thermodynamic properties alone. Landau and Lifshitz [15] have given the explicit expression for the baro diffusion ratio \( k_p \) as
\[
k_p = (m_2 - m_1) \frac{C_1}{C_2 P_1} + \frac{C_2}{P_2 P_\infty} \tag{11}
\]

Since \( C_2 = 1 - C_1 \) and we have assumed \( C_1 \) to be very small so \( C_2 \) may be neglected and hence (11) becomes
\[
k_p = (m_2 - m_1) \frac{C_1 C_2}{P_2 P_\infty} = \mathcal{A} C_1 \tag{12}
\]

where
\[
\mathcal{A} = \frac{m_2 - m_1}{m_2 P_\infty} \tag{13}
\]

The expression \( k_T \) has been suggested by Hurle and Jakeman [15] as
\[
k_T = S_T C_1 C_2 \tag{14}
\]

For small value of \( C_1 \) (since \( C_2 = 1 - C_1 \) and we have assumed \( C_1 \) is very small, so \( C_2 \) may be neglected) (14) becomes
\[
k_T = S_T C_1 \tag{15}
\]

The set of equations (2) to (8) are coupled partial differential equations. To reduce then to ordinary differential equations in non-dimensional form we make the following substitutions
\[
\eta = \frac{\rho b}{\mu} y, \quad r = \frac{\rho b}{\mu} x, \quad u = xb \phi'(\eta), \quad v = xb \psi'(\eta), \quad H_x = xb \frac{\psi'(\eta)}{\mu c}, \quad H_y = -\psi(\eta) \frac{\rho b}{\mu c},
\]

\[
p = p_0 - \mu b [p_1'(\eta) + r^2 p_2'(\eta)], \quad y = \frac{\mu v}{\mu c} [H_1(\eta) + r^2 H_2(\eta)], \quad C_1 = C_0 [G_1(\eta) + r^2 G_2(\eta)].
\]

For above form of velocity components the equation of continuity is satisfied identically. Substituting the corresponding values of the quantities from equation (15) into equations (3) to (8) and equating the like powers of \( r \) from both sides, we get
\[
\phi'' + \phi \phi'' - \phi^2 = \psi \psi'' - \psi^2 + k, \tag{16}
\]

\[
\phi \phi' = p_1' - \phi', \tag{17}
\]

\[
p_2'' = \psi \psi'', \tag{18}
\]

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Thus we conclude that the separation of the binary mixture can be enhanced by increasing the values of all the parameters.

The boundary conditions (9) and (10) become

\[ G_1 = \frac{1}{S_c^2} (2G_2 + G_1) + \frac{R_a}{S_c} \left( -G_1 \phi^2 + G_1 k - G_1 \phi \phi' - G_1 \phi' - G_1 \phi'' - G_1 \phi''' - G_1 \phi'''' \right) + \frac{t_d}{S_c} (2G_1 H_2 + G_1 H_2') \]

\[ G_2 = \frac{1}{S_c^2} G_2 + \frac{R_a}{S_c} \left( -3G_2 \phi^2 + 3G_2 k - G_2 \phi \phi' - G_2 \phi' - \frac{1}{\gamma} G_1 \phi \phi'' - \frac{1}{\gamma} G_1 \phi' \phi'' - G_2 \phi' - \phi'' - G_2 \phi'' - G_2 \phi'''' - G_2 \phi'''''' + t_d S_c G_2 H_2 + G_2 H_2' + G_2 H_2'' + G_2 H_2''' + G_2 H_2'''' \right) \]

where the constant \( k \) is obtained by integrating the inviscid flow equation along the plate and comparing the pressure equation of (15) and is given by

\[ k = 1 + \frac{\phi^2(0)}{\gamma} = 1 + M^2 \]

The boundary conditions (9) and (10) become

\[ \phi = 0, \phi' = 0, p_1 = 0, p_2 = \frac{1}{2} \left( 1 + \frac{\phi^2(0)}{\gamma} \right), \psi = -\sqrt{\left( \frac{R_a H_2^2}{\mu b} \right)}, \psi' = 0, H_1 = N, \text{ at } \eta = 0 \]

\[ H_2 = 0, G_1 = B_d G_1 \phi'' + t_d G_1 H_1' = 0, G_2 = B_d G_2 \phi'' + t_d (G_1 H_2' + G_2 H_2') = 0 \]

\[ \phi' \to 1, H_1 \to 0, H_2 \to 0, G_1 \to 1, G_2 \to 0 \text{ as } \eta \to \infty. \]

Since the solutions of the set of non-linear coupled ordinary differential equations (16) – (23) under the boundary conditions (24) cannot be obtained in a closed form therefore we have solved these equations numerically with MATLAB’s built-in solver bvp4c.

4. RESULT AND DISCUSSION:

Numerical calculations have been carried out for various values of \( Pr, Prm, \gamma, B_d, t_d, S_c \) and \( M \), and these numerical results for velocity, temperature and concentration profile are displayed in Figs. 1 – 3. From Fig. 1 it is observed that \( \phi \) and \( \phi' \) both are zero at \( \eta = 0 \) and then increase with the increase in the value of \( \eta \) and \( \phi' \) attains its maximum value at about \( \eta = 4 \). From this we conclude that there is a boundary layer type flow near the plate. Figs. (2a), (2b) represent the variation in temperature against \( \eta \) for various values of \( Pr \) and \( Prm \) respectively. It is observed that the temperature decreases as \( \eta \) increases. The temperature decreases sharply with increase in the values of these parameters. The temperature decreases sharply in the region \( 0 \leq \eta \leq 2.5 \) for increase in the values of the parameter \( Prm \) but a reverse effect is observed for \( \eta > 2.5 \). The temperature remains constant for \( \eta \geq 4 \).

In Fig. 3 the concentration profile of the lighter component of the binary fluid mixture is drawn against \( \eta \) for various values of the parameters \( \gamma, B_d, t_d, S_c \) and \( M \). It is observed from the figure that the concentration of the lighter component of the binary mixture is more at the plate and decreases exponentially as \( \eta \) increases to 2.5. Thereafter in the region \( \eta > 2.5 \) no various in \( \phi(0) \) is observed. The concentration of the lighter and rarer component of the binary mixture increases sharply near the plate with increase in the values of all the parameters namely \( \gamma, B_d, t_d, S_c \) and \( M \). Thus we conclude that the separation of the binary mixture can be enhanced by increasing the values of all the parameters.
Fig. 1: The graphs of $\phi$ and $\phi'$ against $\eta$.

Fig. 2(a): The graph of $T_1(\eta)$ against $\eta B_d = 0.001$, $t_d = 0.001$, $S_c = 3.4$, $Pr = 0.00001$, $\gamma = 1$, $N = 2$, $M = 0$, and for various values of Pr.

Fig. 2(b): The graph of $T_1(\eta)$ against $\eta B_d = 0.001$, $t_d = 0.001$, $S_c = 3.4$, $M = 0$, and for various values of Pm.

Fig. 3(a): The graph of $\frac{C_1(\eta)}{C_0}$ against $\eta$ for $B_d = 0.001$, $t_d = 0.001$, $S_c = 3.5$, $M = 0$, and for various values of $\gamma$.

Fig. 3(b): The graph of $\frac{C_1(\eta)}{C_0}$ against $\eta$ for $B_d = 0.001$, $t_d = 0.001$, $S_c = 3.5$, $M = 0$, and for various values of Pm.

Fig. 3(c): The graph of $\frac{C_1(\eta)}{C_0}$ against $\eta$ for $B_d = 0.001$, $t_d = 0.001$, $S_c = 3.5$, $M = 0$, and for various values of $t_d$. 

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Finally, the effects of the rate of heat and mass transfer are shown in Table 1. The behaviour of these parameters is self-evident from the Table 1 and hence any further discussion about them seems to be redundant.

Table 1: Numerical values of $\phi(0), \phi'(0), \phi''(0), \psi(0), \psi'(0), H_1(0), H_1'(0)$, $H_2(0), H_2'(0), G_1(0), G_1'(0), G_2(0)$ and $G_2'(0)$ for $Pr = 0.07, Pm = 0.00001$ and $N = 2$.

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