THE METHOD OF CENTERED SYSTEM OF SMOOTH FUZZY TOPOLOGICAL SPACE VIA t-OPEN SETS

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ABSTRACT

In this paper, we introduce maximal smooth fuzzy t-centered system, the smooth fuzzy space \( \theta(R) \). The concept of t-absolute \( \omega(R) \) of a smooth fuzzy topological space is studied.

Key Words: Maximal smooth fuzzy t-centered system, the smooth fuzzy space \( \theta(R) \) and t-absolute \( \omega(R) \).

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INTRODUCTION AND PRELIMINARIES:

The concept of fuzzy set was introduced by Zadeh [11]. Since then the concept has invaded nearly all branches of Mathematics. In 1985, a fuzzy topology on a set \( X \) was defined as a fuzzy subset \( T \) of the family \( I^X \) of fuzzy subsets of \( X \) satisfying three axioms, the basic properties of such a topology were represented by Sostak [9]. In 1992, Ramadan [6], studied the concepts of smooth topological spaces. The method of centered systems in the theory of topology was introduced in [5]. In 2007, the above concept was extended to fuzzy topological spaces by Uma, Roja and Balasubramanian [10]. In this paper, t-absolute \( \omega(R) \) is studied in the theory of smooth fuzzy topology. The concept of fuzzy compactness was found in [3]. The fundamental theorem on smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping is also studied.

Definition 1.1: [9] A function \( T: I^X \rightarrow I \) is called a smooth fuzzy topology on \( X \) if it satisfies the following conditions:

(a) \( T(0) = T(1) = 1 \)
(b) \( T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2) \) for any \( \mu_1, \mu_2 \in I^X \)
(c) \( T(\bigvee_{i \in I} \mu_i) \geq \bigwedge_{i \in I} T(\mu_i) \) for any \( \{ \mu_i \}_{i \in I} \in I^X \)

The pair \((X, T)\) is called a smooth fuzzy topological space.

Definition 1.2: [10] Let \( R \) be a fuzzy Hausdorff space. A system \( p = \{ \lambda_{\alpha} \} \) of fuzzy open sets of \( R \) is called fuzzy centered system if any finite collection of fuzzy sets of the system has a non-zero intersection. The system \( p \) is called maximal fuzzy centered system or a fuzzy end if it cannot be included in any larger fuzzy centered system.

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where \( \lambda(t) = \bigwedge \{ \lambda(s) : s < t \} \) and \( \lambda(t^+) = \bigvee \{ \lambda(s) : s > t \} \). The natural L-fuzzy topology on R (L) is generated from the sub-basis \( \{ L_0, R_0 \} \) where \( L_0(\lambda) = \lambda(t^-) \) and \( R_0(\lambda) = \lambda(t^+) \).

**Definition 1.6:** [4] The L-fuzzy unit interval \( I(L) \) is a subset of R(L) such that \( \{ \lambda \} \in I(L) \) if \( \lambda(t) = 1 \) for \( t < 0 \) and \( \lambda(t) = 0 \) for \( t > 1 \).

**Definition 1.7:** [6] A fuzzy set \( \lambda \) is quasi-coincident with a fuzzy set \( \mu \), denoted by \( \lambda \sqcap \mu \), if there exists \( x \in X \) such that \( \lambda(x) + \mu(x) > 1 \). Otherwise \( \lambda \not\sqcap \mu \).

2. THE SPACES OF MAXIMAL SMOOTH FUZZY \( t \)-centered SYSTEMS

**Definition 2.1:** A smooth fuzzy topological space \( (X, T) \) is said to be smooth fuzzy \( t \)-Hausdorff if for any two distinct points \( x, y \in X \), there exists \( r \)-fuzzy \( t \)-open sets \( \lambda, \mu \in I^X \) such that, \( x \in \lambda \) and \( y \in \mu \) with \( \lambda \sqcap \mu \).

**Notation 2.1:** A smooth fuzzy \( t \)-Hausdorff space is denoted by \( R \).

**Definition 2.2:** Let \( R \) be a smooth fuzzy \( t \)-Hausdorff space. A system \( p = \{ \lambda_i \} \) of \( r \)-fuzzy \( t \)-open sets of \( R \) is called a smooth fuzzy \( t \)-centered system if any finite collection of \( \{ \lambda_i \} \) is such that \( \lambda_i \not\sqcap \lambda_j \) for \( i \neq j \). The system \( p \) is called maximal smooth fuzzy \( t \)-centered system or a smooth fuzzy \( t \)-end if it cannot be included in any larger smooth fuzzy \( t \)-centered system of \( r \)-fuzzy \( t \)-open sets.

**Definition 2.3:** Let \( (X, T) \) be a smooth fuzzy topological space. Its \( Q^* \) \( t \)-neighbourhood structure is a mapping \( Q^* : X \times I^X \to I \) (\( X \) denotes the totality of all fuzzy points in \( X \)), defined by \( Q^*(X^t_0, \lambda) = \sup \{ \mu : \mu \) is an \( r \)-fuzzy \( t \)-open set, \( \mu \leq \lambda \), \( X^t_0 \in \mu \} \) and \( \lambda = \inf_{x \in \phi} Q^*(X^t_0, \lambda) \) is \( r \)-fuzzy \( t \)-open set.

We note the following properties of maximal smooth fuzzy \( t \)-centered system.

1. If \( \lambda_i \in p \) (\( i = 1, 2, 3 \ldots, n \)), then \( \bigwedge_{i=1}^n \lambda_i \in p \).

**Proof:** If \( \lambda_i \in p \) (\( i = 1, 2, 3 \ldots, n \)), then \( \lambda_i \not\sqcap \lambda_j \) for \( i \neq j \). If \( \bigwedge_{i=1}^n \lambda_i \not\in p \), then \( p \cup \{ \bigwedge_{i=1}^n \lambda_i \} \) will be a larger smooth fuzzy \( t \)-end than \( p \). This contradicts the maximality of \( p \). Therefore, \( \bigwedge_{i=1}^n \lambda_i \in p \).

2. If \( \emptyset \neq \lambda \sqcap \mu \), \( \lambda \in p \) and \( \mu \) is an \( r \)-fuzzy \( t \)-open set, then \( \mu \in p \).

**Proof:** If \( \mu \not\in p \), then \( p \cup \{ \mu \} \) will be a larger smooth fuzzy \( t \)-end than \( p \). This contradicts the maximality of \( p \).

3. If \( \lambda \) is \( r \)-fuzzy \( t \)-open set, then \( \lambda \not\in p \) iff there exists \( \mu \in p \) such that \( \lambda \not\sqcap \mu \).

**Proof:** Let us suppose that \( \lambda \not\in p \) for each \( r \)-fuzzy \( t \)-open set. If there exists no \( \mu \in p \) such that \( \lambda \not\sqcap \mu \), then \( \lambda \not\sqcap \mu \) for all \( \mu \in p \). That is, \( p \cup \{ \lambda \} \) will be a larger smooth fuzzy \( t \)-end than \( p \). This contradicts the maximality of \( p \).

Conversely, suppose that there exists \( \mu \in p \) such that \( \lambda \not\sqcap \mu \). If \( \lambda \in p \), then \( \lambda \not\sqcap \mu \), which is a contradiction. Hence \( \lambda \not\in p \).

4. If \( \lambda_1 \lor \lambda_2 = \lambda_3 \in p \), \( \lambda_1 \) and \( \lambda_2 \) are \( r \)-fuzzy \( t \)-open sets with \( \lambda_1 \not\sqcap \lambda_2 \), then either \( \lambda_1 \in p \) or \( \lambda_2 \in p \).

**Proof:** Let us suppose that both \( \lambda_1 \in p \) and \( \lambda_2 \in p \). Then \( \lambda_1 \not\sqcap \lambda_2 \), which is a contradiction. Hence either \( \lambda_1 \in p \) or \( \lambda_2 \in p \).

**Note 2.1:** Every smooth fuzzy \( t \)-centered system of \( r \)-fuzzy \( t \)-open sets can be extended in at least one way to a maximum one.

3. THE SMOOTH FUZZY MAXIMAL STRUCTURE IN \( \theta(R) \)

Let \( \theta(R) \) denote the collection of all smooth \( t \)-ends belonging to \( R \). We introduce a smooth fuzzy maximal structure in \( \theta(R) \) in the following way:
Let $P_\lambda$ be the set of all smooth fuzzy $t$-ends that include $\lambda$ as an element, where $\lambda$ is a r-fuzzy $t$-open set of $R$. Now, $P_\lambda$ is a smooth fuzzy $Q^t$-neighbourhood structure of each smooth fuzzy $t$-end contained in $P_\lambda$. Thus to each r-fuzzy $t$-open set $\lambda$ of $R$ corresponds to a smooth fuzzy $Q^t$-neighbourhood structure $P_\lambda$ in $\theta(R)$.

**Proposition 3.1:** If $\lambda$ and $\mu$ are r-fuzzy $t$-open sets, then

(a) $P_{\lambda \vee \mu} = P_\lambda \cup P_\mu$.

(b) $P_\lambda \cup P_{T-C_{T(R)}(\lambda, r)} = \theta(R)$.

**Proof:**

(a) Let $p \in P_\lambda$. That is, $\lambda \in p$. Then by property (2), $\lambda \vee \mu \in p$. That is, $p \in P_{\lambda \vee \mu}$. Hence $P_\lambda \cup P_\mu \subseteq P_{\lambda \vee \mu}$. Let $p \in P_{\lambda \vee \mu}$. That is, $\lambda \vee \mu \in p$. By the definition of $P_\lambda$, $\lambda \in p$ or $\mu \in p$. That is, $p \in P_\lambda$ or $p \in P_\mu$, therefore, $p \in P_\lambda \cup P_\mu$. This shows that $P_\lambda \cup P_\mu \supseteq P_{\lambda \vee \mu}$. Hence, $P_{\lambda \vee \mu} = P_\lambda \cup P_\mu$.

(b) If $p \notin P_{T-C_{T(R)}(\lambda, r)}$, then $\overline{\lambda} - C_{T(R)}(\lambda, r) \notin p$. That is, $\lambda \notin p$ and $p \notin P_\lambda$. Hence,

$$\theta(R) - P_{T-C_{T(R)}(\lambda, r)} \subseteq P_\lambda.$$ If $p \in P_\lambda$, then $\lambda \in p$. That is, $\overline{\lambda} - C_{T(R)}(\lambda, r) \notin p$, $p \notin \theta(R) - P_{T-C_{T(R)}(\lambda, r)}$. Hence, $P_\lambda \cup P_{T-C_{T(R)}(\lambda, r)} = \theta(R)$.

**Proposition 3.2:** $\theta(R)$ with the smooth fuzzy maximal structure described above is a smooth fuzzy t-compact space and has a base of smooth fuzzy $Q^t$-neighbourhoods $\{P_\lambda\}$ that are both smooth fuzzy t-open and smooth fuzzy t-closed ends.

**Proof:** It follows from the definition above that $\theta(R)$ is a smooth fuzzy $T_1$ space. Each $P_\lambda$ in $\theta(R)$ is smooth fuzzy t-open end by definition and by (b) of Proposition 3.1. it follows that it is smooth fuzzy t-closed. Thus $\theta(R)$ has $Q^t$-neighbourhoods $\{P_\lambda\}$ that are both smooth fuzzy t-open and smooth fuzzy t-closed. We now show that $\theta(R)$ is smooth fuzzy t-compact. Let $\{P_{\lambda_i}\}$ be a covering of $\theta(R)$ where each $P_{\lambda_i}$ is smooth fuzzy t-open. If it is impossible to pick a finite subcovering from the covering, then no set of the form $F = \bigcap_{i=1}^n t-C_{T(R)}(\lambda_{\alpha_i}, r)$ is $\lambda$, since otherwise the sets $P_{\lambda_{\alpha_i}}$ would form a finite covering of $\theta(R)$. Hence the sets $F = \bigcap_{i=1}^n t-C_{T(R)}(\lambda_{\alpha_i}, r)$ form a smooth fuzzy t-centered system.

It may be extended to a maximal smooth fuzzy t-centered system $p$. This maximal smooth fuzzy t-centered system is not contained in $\{P_{\alpha_i}\}$ since it contains in particular, all the $t-C_{T(R)}(\lambda_{\alpha_i}, r)$. This contradiction proves that $\theta(R)$ is smooth fuzzy t-compact.

4. THE ABSOLUTE $\omega_0(R)$ OF A SMOOTH FUZZY TOPOLOGICAL SPACE $R$.

The maximal smooth fuzzy t-centered system of r-fuzzy t-open sets of $R$ regarded as elements of the space $\theta(R)$, fall into two classes, those smooth fuzzy $t$-ends each of which contain all r-fuzzy t-open sets containing a fuzzy point of $R$ and the smooth fuzzy $t$-ends not containing such smooth fuzzy t-system of r-fuzzy t-open sets. The space of all smooth fuzzy $t$-ends of first type of $\theta(R)$ is called the smooth fuzzy t-absolute of $R$ and is denoted by $\omega_0(R)$. In $\omega_0(R)$ each fuzzy point $\alpha$ of $R$ is represented by smooth fuzzy t-ends containing all r-fuzzy t-open sets containing $\alpha$.

Now $\omega_0(R) = \{\lambda(\alpha) / \alpha$ is a fuzzy point of $R$, where $\lambda(\alpha)$ denotes the set of all smooth fuzzy $t$-ends containing all r-fuzzy $t$-open sets containing $\alpha$. The smooth fuzzy $t$-absolute space $\omega_0(R)$ is mapped in a natural way onto $R$. If $p \in \omega_0(R)$, then we define $\pi_p(p) = \alpha$, where $\alpha$ is the fuzzy point such that all r-fuzzy $t$-open sets containing $\alpha$ belongs to $p$. $\pi_p$ is called smooth fuzzy natural mapping of $\omega_0(R)$ onto $R$.

**Definition 4.1:** Let $R_1$ and $R_2$ be any two smooth fuzzy $t$-Hausdorff spaces. A mapping $f: R_1 \rightarrow R_2$ is called smooth fuzzy $t$-irreducible* if there is no proper r-fuzzy $t$-closed set $\lambda$ of $R_1$ such that $f(\lambda) = \overline{\lambda}_{R_2}$.

**Definition 4.2:** Let $R_1$ and $R_2$ be any two smooth fuzzy $t$-Hausdorff spaces. A mapping $f: R_1 \rightarrow R_2$ is called smooth fuzzy $t$-perfect if the image of a r-fuzzy $t$-closed set is r-fuzzy $t$-closed and the inverse image of each fuzzy point is smooth fuzzy $t$-compact.
Definition 4.3: Let $R_1$ and $R_2$ be any two smooth fuzzy $t$-Hausdorff spaces. A mapping $f: R_1 \rightarrow R_2$ is called smooth fuzzy $t$-compact if the inverse image of each $\lambda$ is smooth fuzzy $t$-compact.

Proposition 4.1: The natural mapping $\pi_0$ of $\omega(R)$ onto $R$ is smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-compact.

Proof: Let $\beta$ be a fuzzy point of $R$. If $\pi_0(P) = \beta$, $\pi_0^{-1}(\beta)$ is a set of all smooth fuzzy $t$-ends $p$ which contain all the $r$-fuzzy $t$-open sets containing $\beta$. Since $\theta(R)$ has a base of smooth fuzzy $Q^t$-neighbourhood structure $\{P_\lambda\}$ that are both smooth fuzzy $t$-open and smooth fuzzy $t$-closed, $\pi_0^{-1}(\beta)$ is a $r$-fuzzy $t$-closed set in $\theta(R)$. Since $\theta(R)$ is smooth fuzzy $t$-compact, $\pi_0^{-1}(\beta)$ is smooth fuzzy $t$-compact. Therefore $\pi_0$ is smooth fuzzy $t$-compact. To prove $\pi_0$ is smooth fuzzy $t$-irreducible* it is enough to show that every $r$-fuzzy $t$-open set in $\omega(R)$ contains whole of some set $\pi_0^{-1}(\beta)$, where $\beta \leq \lambda$, and because $\{P_\lambda\}$ is a $Q^t$-neighbourhood in $\theta(R)$.

Proposition 4.2: If $f : R_1 \rightarrow R_2$ is a smooth fuzzy $t$-irreducible* and $t$-closed, then the image of every $r$-fuzzy $t$-open set $\lambda \neq 0$ in $R_1$ is a $r$-fuzzy $t$-open set in $R_2$ with $f(\lambda) \neq 0$.

Proof: Let $\lambda$ be a $r$-fuzzy $t$-open set with $\lambda \neq 0$ in $R_1$. Since $f$ is smooth fuzzy $t$-closed, $f(\lambda - r)$ is also a $r$-fuzzy $t$-closed. Since $f$ is onto, $f(\lambda - r) = \overline{f(\lambda)}$. Therefore $f(\lambda)$ is a $r$-fuzzy $t$-open set in $R_2$. Since $f$ is smooth fuzzy $t$-irreducible* $f(\overline{\lambda - r}) \neq 0$. That is, $\overline{f(\lambda)} = \overline{f(\overline{\lambda})} \Rightarrow f(\lambda) \neq 0$.

Notation: $t$-$\text{Int}(\lambda, r)$ denotes the interior of an fuzzy set $\lambda$.

Proposition 4.3: If $f$ is a smooth fuzzy $t$-irreducible* mapping of $R_1$ onto $R_2$, $\text{Int}_{R_1}(f^{-1}(\lambda), r) \neq 0$ for every $r$-fuzzy $t$-open set $\lambda \neq 0$ in $R_2$.

Proof: Since $f$ is smooth fuzzy $t$-closed and smooth fuzzy $t$-irreducible*, $f(\overline{\lambda - r}) \neq 0$. By Proposition 4.2 it follows that $\text{Int}_{R_1}(f^{-1}(\lambda), r) \neq 0$.

5. The fundamental theorem on smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-perfect mapping.

Theorem 5.1: Let $R_1$ and $R_2$ be smooth fuzzy $t$-Hausdorff spaces. Let $f$ be a smooth fuzzy $t$-irreducible* and smooth fuzzy $t$-perfect mapping of $R_1$ onto $R_2$. Then there exists a smooth fuzzy $t$-homeomorphism $\psi$ of $\omega(R_1)$ onto $\omega(R_2)$ such that $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$.

Proof: Let $\{\lambda\}$ be a maximal smooth fuzzy $t$-centered system of $r$-fuzzy $t$-open sets in $R_1$. In $R_2$ consider the system $\{t$-$\text{Int}_{R_2}(f(\lambda), r)\}$, where $t$-$\text{Int}_{R_2}(f(\lambda), r)$ is an $r$-fuzzy $t$-open, by Proposition 4.3 each of its sets is non-zero. Clearly the system is smooth fuzzy $t$-centered. Extend it to a maximal smooth fuzzy $t$-centered system of $r$-fuzzy $t$-open sets in $R_2$, and prove that this extension is unique. Suppose that there exist two $r$-fuzzy $t$-open sets $\lambda_1, \lambda_2 \in R_2$ with $\lambda_1 \leq \lambda_2$, such that $\lambda_1 \leq \lambda_2$ $t$-$\text{Int}_{R_2}(f(\lambda), r)$ and $\lambda_2 \leq \lambda_2$ $t$-$\text{Int}_{R_2}(f(\lambda), r)$ for every $\lambda$ in $\{\lambda\}$. Now, $t$-$\text{Int}_{R_2}(f^{-1}(\lambda_1), r) \leq t$-$\text{Int}_{R_2}(f^{-1}(\lambda_2), r)$. But this is impossible, because $\{\lambda\}$ is maximal smooth fuzzy $t$-centered system. Thus $\{t$-$\text{Int}_{R_2}(f(\lambda), r)\}$ can be extended in only one way to a maximal smooth fuzzy $t$-centered system $\{\gamma_i\}$ where $\gamma_i$ is an $r$-fuzzy $t$-open set.
Assume that \( \{ \lambda \} \) contains all r-fuzzy t-open sets \( \lambda_\alpha \) containing the fuzzy point \( \alpha \) in \( R_1 \) and show that \( \{ \gamma \} \) contains all r-fuzzy t-open sets \( \gamma_\beta \) containing the fuzzy point \( \beta \) in \( R_2 \) such that \( \beta = f(\alpha) \). Let \( \delta_\beta \) be r-fuzzy t-open set containing the fuzzy point \( \beta \). Because \( f \) is smooth fuzzy t-irreducible* and smooth fuzzy t-closed, t-Int_{f^{-1}(\delta_\beta)}(r) is r-fuzzy t-open set containing the fuzzy point \( \alpha \), \( t-\text{Int}_{f^{-1}(\delta_\beta)}(r) \in \{ \lambda \} \).

The set \( t-\text{Int}_{f^{-1}(\delta_\beta)}(r) \) is r-fuzzy t-open set such that \( \gamma = \{ \gamma \} \) is a point of \( o(R_2) \). Let \( \psi(p) = q \), to show that \( \psi \) is a mapping of \( o(R_1) \) onto \( o(R_2) \). Let \( q = \{ \gamma \} \in o(R_2) \).

Consider the system \( \{ t-\text{Int}_{f^{-1}(\delta_\beta)}(r) \} \) of r-fuzzy t-open sets in \( R_1 \). The system is smooth fuzzy t-centered. We extend it to a maximal smooth fuzzy t-centered system of r-fuzzy t-open sets \( p = \{ \lambda \} \) and consider the point \( \psi(p) \). As we have shown, \( t-\text{Int}_{f^{-1}(\lambda)}(r) \) may be extended in a unique way to a maximal system \( \{ \gamma \} \). To show that \( \psi(p) = q \), it is sufficient to show that \( \{ \gamma \} < \{ \gamma \} \) and for this, it is enough to show that \( \gamma \) \( t-\text{Int}_{f^{-1}(\lambda)}(r) \) for each \( \gamma \in \{ \gamma \} \) and each \( t-\text{Int}_{f^{-1}(\lambda)}(r) \in \{ t-\text{Int}_{f^{-1}(\lambda)}(r) \} \). Clearly \( \gamma \in t-\text{Int}_{f^{-1}(\lambda)}(r) \). Let \( \eta = \emptyset \) be r-fuzzy t-open set such that \( \eta \leq \lambda \) \( t-\text{Int}_{f^{-1}(\lambda)}(r) \) and let \( \alpha \leq \eta \) be such that \( f(\alpha) \leq \gamma \). Then \( t-\text{Int}_{f^{-1}(\lambda)}(r) \) is r-fuzzy t-open set containing the fuzzy point \( \beta \), \( f(\alpha) \in \{ \lambda \} \) and \( t-\text{Int}_{f^{-1}(\lambda)}(r) \leq t-\text{Int}_{f^{-1}(\lambda)}(r) \). On the other hand, \( t-\text{Int}_{f^{-1}(\lambda)}(r) \subseteq t-\text{Int}_{f^{-1}(\lambda)}(r) \). Therefore \( \psi(p) = q \). It is onto. The mapping \( \psi \) is one to one. For \( p_1 \neq p_2 \) then there are r-fuzzy t-open sets \( \lambda_1 \) and \( \lambda_2 \) such that \( \lambda_1 \leq \lambda_2 \) and \( f(\lambda_1) \neq f(\lambda_2) \). Thus \( \psi(p_1) \neq \psi(p_2) \). The mapping \( \psi \) is one to one of \( R_1 \) into \( R_2 \) taking \( o(R_1) \) onto \( o(R_2) \). Let \( \rho = \{ \lambda \} \) be an arbitrary smooth fuzzy t-end in \( R_1 \), that is an element of \( o(R_1) \) and let \( q = \psi(p) = \{ \gamma \} \). Now prove \( \psi(p) = \{ \gamma \} \) and \( p = o(R_1) \). Then \( \lambda = \gamma \). So \( t-\text{Int}_{f^{-1}(\lambda)}(r) \in \psi(p) \) which means that \( \psi(p) \subseteq \psi(t-\text{Int}_{f^{-1}(\lambda)}(r)) \). This proves that \( \psi \) is a smooth fuzzy t-homeomorphism. To prove the theorem we have to show that \( f \circ o_{\pi_{R_1}} = \pi_{R_2} \circ o \psi \). Consider the mapping \( \psi \) only on \( o(R_1) \) \( \subseteq o(R_1) \). From the construction of \( \psi \) it follows that every smooth fuzzy t-end containing all r-fuzzy t-open sets \( \lambda_1 \) containing \( \alpha \) is mapped by \( \psi \) into a smooth fuzzy t-end containing \( \lambda_1 \) with r-fuzzy t-open sets containing fuzzy point \( \beta \), \( \psi \in (\pi_{R_1}^{-1}(\alpha)) \subseteq (\pi_{R_2}^{-1}(\beta)) \). Hence \( f \circ o_{\pi_{R_1}} = \pi_{R_2} \circ o \psi \). Thus the theorem proved.

**Corollary 5.2:** The smooth fuzzy t-absolute of \( R_1 \) and \( R_2 \) are smooth fuzzy t-homeomorphism if there exists a smooth fuzzy topological space \( R \) such that \( R \) can be mapped onto both \( R_1 \) and \( R_2 \) by smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping.

**Proof:** Let \( f_1 \) be a smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping from \( R \) onto \( R_1 \) and let \( f_2 \) be smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping from \( R \) into \( R_2 \). By theorem 5.1 there exists a smooth fuzzy t-homeomorphism \( \psi \) of \( o(R) \) onto \( o(R_2) \) such that \( f_1 \circ o_{\pi_{R_1}} = \pi_{R_1} \circ o \psi \) and there exists a smooth fuzzy t-homeomorphism \( \pi_2 \) of \( o(R) \) onto \( o(R_2) \) such that \( f_2 \circ o_{\pi_{R_1}} = \pi_{R_2} \circ o \psi \). Therefore \( o(R_1) \) and \( o(R_2) \) are smooth fuzzy t-homeomorphic.

**REFERENCE**


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