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THE METHOD OF CENTERED SYSTEM OF SMOOTH FUZZY TOPOLOGICAL SPACE VIA t- OPEN SETS

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ABSTRACT

In this paper, we introduce maximal smooth fuzzy t-centered system, the smooth fuzzy space $\theta(R)$. The concept of t-absolute $\omega(R)$ of a smooth fuzzy topological space is studied.

Key Words: *Maximal smooth fuzzy t-centered system, the smooth fuzzy space $\theta(R)$ and t-absolute $\omega(R)$.*

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INTRODUCTION AND PRELIMINARIES:

The concept of fuzzy set was introduced by Zadeh [11]. Since then the concept has invaded nearly all branches of Mathematics. In 1985, a fuzzy topology on a set X was defined as a fuzzy subset T of the family I^X of fuzzy subsets of X satisfying three axioms, the basic properties of such a topology were represented by Sostak [9]. In 1992, Ramadan [6], studied the concepts of smooth topological spaces. The method of centered systems in the theory of topology was introduced in [5]. In 2007, the above concept was extended to fuzzy topological spaces by Uma, Roja and Balasubramanian [10]. In this paper, t-absolute $\omega(R)$ is studied in the theory of smooth fuzzy topology. The concept of fuzzy compactness was found in [3]. The fundamental theorem on smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping is also studied.

Definition 1.1: [9] A function $T: I^X \to I$ is called a smooth fuzzy topology on X if it satisfies the following conditions:
(a) $T(\bar{0}) = T(\bar{1}) = 1$
(b) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$ for any $\mu_1, \mu_2 \in I^X$
(c) $T\left(\bigvee_{i\in\Gamma} \mu_i\right) \geq \bigwedge_{i\in\Gamma} T(\mu_i)$ for any $\{ \mu_i \}_{i\in\Gamma} \in I^X$

The pair (X, T) is called a smooth fuzzy topological space.

Definition 1.2: [10] Let R be a fuzzy Hausdroff space. A system $p = \{\lambda_\alpha\}$ of fuzzy open sets of R is called fuzzy centered system if any finite collection of fuzzy sets of the system has a non-zero intersection. The system p is called maximal fuzzy centered system or a fuzzy end if it cannot be included in any larger fuzzy centered system.

Definition 1.3: [10] Let $\theta(R)$ denote the collection of all fuzzy ends belonging to R. We introduce a fuzzy topology in $\theta(R)$ in the following way: Let P_λ be the set of all fuzzy ends that include λ as an element, where λ is a fuzzy open set of R. Now P_λ is a fuzzy neighbourhood of each fuzzy end contained in P_λ. Thus to each fuzzy open set of R, there corresponds a fuzzy neighbourhood P_λ in $\theta(R)$.

Definition 1.4: [10] A fuzzy Hausdroff space R is extremally disconnected if the closure of an open set is open.

Definition 1.5: [3] The fuzzy real line R (L) is the set of all monotone decreasing elements $\lambda \in L^R$ satisfying $\vee \{ \lambda(t)/ t \in R \} = 1$ and $\wedge \{ \lambda(t)/ t \in R \} = 0$, after the identification of $\lambda, \mu \in L^R$ iff $\lambda(t-) = \mu(t-)$ and $\lambda(t+) = \mu(t+)$ for all $t \in R$,

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where $\lambda(t-) = \wedge \{ \lambda(s) : s < t \}$ and $\lambda(t+) = \vee \{ \lambda(s) : s > t \}$. The natural L-fuzzy topology on R (L) is generated from the sub-basis $\{L_t, R_t\}$ where $L_t(\lambda) = \lambda(t-)'$ and $R_t(\lambda) = \lambda(t+)$.

Definition 1.6: [4] The L-fuzzy unit interval I(L) is a subset of R(L) such that $[\lambda] \in I(L)$ if $\lambda(t) = 1$ for $t < 0$ and $\lambda(t) = 0$ for $t > 1$.

Definition 1.7: [6] A fuzzy set λ is quasi-coincident with a fuzzy set μ, denoted by $\lambda\ q\ \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise $\lambda \not{q}\ \mu$.

2. THE SPACES OF MAXIMAL SMOOTH FUZZY t-centered SYSTEMS

Definition 2.1: A smooth fuzzy topological space (X,T) is said to be smooth fuzzy t-Hausdorff iff for any two distinct fuzzy points x_{t_1}, x_{t_2} in X, there exists r-fuzzy t-open sets $\lambda, \mu \in I^X$ such that, $x_{t_1} \in \lambda$ and $x_{t_2} \in \mu$ with $\lambda\ q\ \mu$.

Notation 2.1: A smooth fuzzy t-Hausdorff space is denoted by R.

Definition 2.2: Let R be a smooth fuzzy t-Hausdorff space. A system $p = \{\lambda_i\}$ of r-fuzzy t-open sets of R is called a smooth fuzzy t-centered system if any finite collection of $\{\lambda_i\}$ is such that $\lambda_I \not{q}\ \lambda_j$ for $i \neq j$. The system p is called maximal smooth fuzzy t-centered system or a smooth fuzzy t-end if it cannot be included in any larger smooth fuzzy t-centered system of r-fuzzy t-open sets.

Definition 2.3: Let (X, T) be a smooth fuzzy topological space. Its Q* t-neighbourhood structure is a mapping $Q^*: X \times I^X \to I$ (X denotes the totality of all fuzzy points in X), defined by
$Q^*(x_0^t, \lambda) = \sup \{ \mu : \mu$ is an r-fuzzy t-open set, $\mu \leq \lambda,\ x_0^t \in \mu \}$ and $\lambda = \inf_{x_0^t q \lambda} Q^*(x_0^t, \lambda)$ is r-fuzzy t-open set.

We note the following properties of maximal smooth fuzzy t-centered system.

(1) If $\lambda_i \in p$ (i = 1, 2, 3…n), then $\bigwedge_{i=1}^{n} \lambda_i \in p$.

Proof: If $\lambda_i \in p$ (i = 1, 2, 3…n), then $\lambda_i \not{q}\ \lambda_j$ for $i \neq j$. If $\bigwedge_{i=1}^{n} \lambda_i \notin p$, then $p \cup \{ \bigwedge_{i=1}^{n} \lambda_i \}$ will be a larger smooth fuzzy t-end than p. This contradicts the maximality of p. Therefore, $\bigwedge_{i=1}^{n} \lambda_i \in p$.

(2) If $\overline{0} \neq \lambda < \mu$, $\lambda \in p$ and μ is an r-fuzzy t-open set, then $\mu \in p$.

Proof: If $\mu \notin p$, then $p \cup \{ \mu \}$ will be a larger smooth fuzzy t-end than p. This contradicts the maximality of p.

(3) If λ is r-fuzzy t-open set, then $\lambda \notin p$ iff there exists $\mu \in p$ such that $\lambda\ q\ \mu$.

Proof: Let us suppose that $\lambda \notin p$ for each r-fuzzy t-open set. If there exists no $\mu \in p$ such that $\lambda\ q\ \mu$, then $\lambda \not{q}\ \mu$ for all $\mu \in p$. That is, $p \cup \{ \lambda \}$ will be a larger smooth fuzzy t-end than p. This contradicts the maximality of p.

Conversely, suppose that there exists $\mu \in p$ such that $\lambda\ q\ \mu$. If $\lambda \in p$, then $\lambda \not{q}\ \mu$, which is a contradiction. Hence $\lambda \notin p$.

(4) If $\lambda_1 \vee \lambda_2 = \lambda_3 \in p$, $\lambda_1$and λ_2 are r-fuzzy t-open sets with $\lambda_1\ q\ \lambda_2$, then either $\lambda_1 \in p$ or $\lambda_2 \in p$.

Proof: Let us suppose that both $\lambda_1 \in p$ and $\lambda_2 \in p$. Then $\lambda_1 \not{q}\ \lambda_2$, which is a contradiction. Hence either $\lambda_1 \in p$ or $\lambda_2 \in p$.

Note 2.1: Every smooth fuzzy t-centered system of r-fuzzy t-open sets can be extended in atleast one way to a maximum one.

3. THE SMOOTH FUZZY MAXIMAL STRUCTURE IN θ(R)

Let $\theta(R)$ denote the collection of all smooth fuzzy t-ends belonging to R. We introduce a smooth fuzzy maximal structure in $\theta(R)$ in the following way:

Let P_λ be the set of all smooth fuzzy t-ends that include λ as an element, where λ is a r-fuzzy t-open set of R. Now, P_λ is a smooth fuzzy Q*t-neighbourhood structure of each smooth fuzzy t-end contained in P_λ. Thus to each r-fuzzy t-open set λ of R corresponds to a smooth fuzzy Q*t-neighbourhood structure P_λ in $\theta(R)$.

Proposition 3.1: If λ and μ are r-fuzzy t-open sets, then
(a) $P_{\lambda\vee\mu} = P_\lambda \cup P_\mu$.
(b) $P_\lambda \cup P_{\bar{1}-C_{T(R)}(\lambda,r)} = \theta(R)$.

Proof:
(a) Let $p \in P_\lambda$. That is, $\lambda \in p$. Then by property (2), $\lambda \vee \mu \in p$. That is, $p \in P_{\lambda\vee\mu}$. Hence $P_\lambda \cup P_\mu \subseteq P_{\lambda\vee\mu}$. Let $p \in P_{\lambda\vee\mu}$. That is, $\lambda \vee \mu \in p$. By the definition of P_λ, $\lambda \in p$ or $\mu \in p$. That is, $p \in P_\lambda$ or $p \in P_\mu$, therefore, $p \in P_\lambda \cup P_\mu$. This shows that $P_\lambda \cup P_\mu \supseteq P_{\lambda\vee\mu}$. Hence, $P_{\lambda\vee\mu} = P_\lambda \cup P_\mu$.

(b) If $p \notin P_{\bar{1}-C_{T(R)}(\lambda,r)}$, then $\bar{1} - C_{T(R)}(\lambda, r) \notin p$. That is, $\lambda \in p$ and $p \in P_\lambda$. Hence,
$\theta(R) - P_{\bar{1}-C_{T(R)}(\lambda,r)} \subset P_\lambda$. If $p \in P_\lambda$, then $\lambda \in p$. That is, $\bar{1} - C_{T(R)}(\lambda, r) \notin p$, $p \notin P_{\bar{1}-C_{T(R)}(\lambda,r)}$.
Therefore,
$p \in \theta(R) - P_{\bar{1}-C_{T(R)}(\lambda,r)}$. That is, $P_\lambda \subset \theta(R) - P_{\bar{1}-C_{T(R)}(\lambda,r)}$. Hence, $P_\lambda \cup P_{\bar{1}-C_{T(R)}(\lambda,r)} = \theta(R)$.

Proposition 3.2: $\theta(R)$ with the smooth fuzzy maximal structure described above is a smooth fuzzy t-compact space and has a base of smooth fuzzy Q*t-neighbourhoods $\{P_\lambda\}$ that are both smooth fuzzy t-open and smooth fuzzy t-closed ends.

Proof: It follows from the definition above that $\theta(R)$ is a smooth fuzzy T_1 space. Each P_λ in $\theta(R)$ is smooth fuzzy t-open end by definition and by (b) of Proposition 3.1. it follows that it is smooth fuzzy t-closed. Thus $\theta(R)$ has Q*t-neighbourhoods $\{P_\lambda\}$ that are both smooth fuzzy t-open and smooth fuzzy t-closed. We now show that $\theta(R)$ is smooth fuzzy t-compact. Let $\{P_{\lambda_\alpha}\}$ be a covering of $\theta(R)$ where each P_{λ_α} is smooth fuzzy t-open. If it is impossible to pick a finite subcovering from the covering, then no set of the form $\bar{1} - \bigvee_{i=1}^{n}$ t-$C_{T(R)}(\lambda_{\alpha_i}, r)$ is $\bar{0}$, since otherwise the sets $P_{\lambda_{\alpha_i}}$ would form a finite covering of $\theta(R)$. Hence the sets $\bar{1} - \bigvee_{i=1}^{n}$ t-$C_{T(R)}(\lambda_{\alpha_i}, r)$ form a smooth fuzzy t-centered system. It may be extended to a maximal smooth fuzzy t-centered system p. This maximal smooth fuzzy t-centered system is not contained in $\{P_{\lambda_\alpha}\}$ since it contains in particular, all the $\bar{1}$ – t-$C_{T(R)}(\lambda_\alpha, r)$. This contradiction proves that $\theta(R)$ is smooth fuzzy t-compact.

4. THE ABSOLUTE ω(R) OF A SMOOTH FUZZY TOPOLOGICAL SPACE R.

The maximal smooth fuzzy t-centered system of r-fuzzy t-open sets of R regarded as elements of the space $\theta(R)$, fall into two classes, those smooth fuzzy t-ends each of which contain all r-fuzzy t-open sets containing a fuzzy point of R and the smooth fuzzy t-ends not containing such smooth fuzzy t-system of r-fuzzy t-open sets. The space of all smooth fuzzy t-ends of first type of $\theta(R)$ is called the smooth fuzzy t-absolute of R and is denoted by $\omega(R)$. In $\omega(R)$ each fuzzy point α of R is represented by smooth fuzzy t-ends containing all r-fuzzy t-open sets containing α.

Now $\omega(R) = \cup \{\lambda(\alpha) / \alpha$ is a fuzzy point of R, where $\lambda(\alpha)$ denotes the set of all smooth fuzzy t-ends containing all r-fuzzy t-open sets containing $\alpha\}$. The smooth fuzzy t-absolute space $\omega(R)$ is mapped in a natural way onto R. If $p \in \omega(R)$, then we define $\pi_R(p) = \alpha$, where α is the fuzzy point such that all r-fuzzy t-open sets containing α belongs to p. π_R is called smooth fuzzy natural mapping of $\omega(R)$ onto R.

Definition 4.1: Let R_1 and R_2 be any two smooth fuzzy t-Hausdorff spaces. A mapping f: $R_1 \to R_2$ is called smooth fuzzy t-irreducible* if there is no proper r-fuzzy t-closed set λ of R_1 such that $f(\lambda) = \bar{1}_{R_2}$

Definition 4.2: Let R_1 and R_2 be any two smooth fuzzy t-Hausdorff spaces. A mapping f: $R_1 \to R_2$ is called smooth fuzzy t-perfect if the image of a r-fuzzy t-closed set is r-fuzzy t-closed and the inverse image of each fuzzy point is smooth fuzzy t-compact.

Definition 4.3: Let R_1 and R_2 be any two smooth fuzzy t-Hausdorff spaces. A mapping $f: R_1 \to R_2$ is called smooth fuzzy t-compact if the inverse image of each λ is smooth fuzzy t-compact.

Proposition 4.1: The natural mapping π_R of $\omega(R)$ onto R is smooth fuzzy t-irreducible* and smooth fuzzy t-compact.

Proof: Let β be a fuzzy point of R. If $\pi_R(P) = \beta$, $\pi^{-1}{}_R(\beta)$ is a set of all smooth fuzzy t-ends p which contain all the r-fuzzy t-open sets containing β. Since $\theta(R)$ has a base of smooth fuzzy Q*t-neighbourhood structure $\{P_\lambda\}$ that are both smooth fuzzy t-open and smooth fuzzy t-closed, $\pi_R^{-1}(\beta)$ is a r-fuzzy t-closed set in $\theta(R)$. Since $\theta(R)$ is smooth fuzzy t-compact, $\pi_R^{-1}(\beta)$ is smooth fuzzy t-compact. Therefore π_R is smooth fuzzy t-compact. To Prove π_R is smooth fuzzy t-irreducible* it is enough to show that every r-fuzzy t-open set in $\omega(R)$ contains whole of some set $\pi_R^{-1}(\beta)$, where β is a fuzzy point of R. But this follows, because each P_λ contains the whole of $\pi^{-1}{}_R(\beta)$, where $\beta \leq \lambda$, and because $\{P_\lambda\}$ is a Q*t-neighbourhood in $\theta(R)$.

Proposition 4.2: If $f : R_1 \to R_2$ is a smooth fuzzy t-irreducible* and t-closed, then the image of every r-fuzzy t-open set $\lambda \neq \overline{0}$ in R_1 is a r-fuzzy t-open set in R_2 with $f(\lambda) \neq \overline{0}$.

Proof: Let λ be a r-fuzzy t-open set with $\lambda \neq \overline{0}$ in R_1. Since f is smooth fuzzy t-closed. $f(\overline{1} - \lambda)$ is also an r-fuzzy t-closed. Since f is onto, $f(\overline{1} - \lambda) = \overline{1} - f(\lambda)$. Therefore $f(\lambda)$ is a r-fuzzy t-open set in R_2. Since f is smooth fuzzy t-irreducible* $f(\overline{1} - \lambda) \neq \overline{1}$. That is, $\overline{1} - f(\lambda) \neq \overline{1} \Rightarrow f(\lambda) \neq \overline{0}$.

Notation: $t\text{-}Int_T(\lambda, r)$ denotes the interior of an fuzzy set λ.

Proposition 4.3: If f is a smooth fuzzy t-irreducible* mapping of R_1 onto R_2, $Int_{T_{R_1}}(f^{-1}(\lambda), r) \neq \overline{0}$ for every r-fuzzy t-open set $\lambda \neq \overline{0}$ in R_2.

Proof: Since f is smooth fuzzy t-closed and smooth fuzzy t-irreducible*, $f(\overline{1} - t\text{-}Int_{T_{R_1}}(f^{-1}(\lambda), r)) \neq \overline{1}$. Since f is onto $f(t\text{-}Int_{T_{R_1}}(f^{-1}(\lambda), r)) \neq \overline{0}$. By Proposition 4.2 it follows that $t\text{-}Int_{T_{R_1}}(f^{-1}(\lambda), r) \neq \overline{0}$.

5. The fundamental theorem on smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping.

Theorem 5.1: Let R_1 and R_2 be smooth fuzzy t-Hausdorff spaces. Let f be a smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping of R_1 onto R_2. Then there exists a smooth fuzzy t-homeomorphism ψ of $\omega(R_1)$ onto $\omega(R_2)$ such that $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$.

$$\begin{tikzcd} R_1 \arrow[r, "f"] & R_2 \\ \omega(R_1) \arrow[u, "\pi_{R_1}"] \arrow[r, "\psi"'] & \omega(R_2) \arrow[u, "\pi_{R_2}"'] \end{tikzcd}$$

Proof: Let $\{\lambda\}$ be a maximal smooth fuzzy t-centered system of r-fuzzy t-open sets in R_1. In R_2 consider the system $\{t\text{-}Int_{R_2}(f(\lambda),r)\}$, where $t\text{-}Int_{R_2}(f(\lambda), r)$ is an r-fuzzy t-open, by Proposition 4.3 each of its sets is non-zero. Clearly the system is smooth fuzzy t-centered. Extend it to a maximal smooth fuzzy t-centered system of r-fuzzy t-open sets in R_2 and prove that this extension is unique. Suppose that there exist two r-fuzzy t-open sets $\lambda_1, \lambda_2 \in R_2$ with $\lambda_1 \, q \, \lambda_2$, such that $\lambda_1 \not q \; t\text{-}Int_{R_2}(f(\lambda), r)$ and $\lambda_2 \not q \; t\text{-}Int_{R_2}(f(\lambda), r)$ for every λ in $\{\lambda\}$. Now, $t\text{-}Int_{R_1}(f^{-1}(\lambda_1), r) \, q \; t\text{-}Int_{R_1}(f^{-1}(\lambda_2), r)$. But this is impossible, because $\{\lambda\}$ is maximal smooth fuzzy t-centred system. Thus $\{t\text{-}Int_{R_2}(f(\lambda), r)\}$ can be extended in only one way to a maximal smooth fuzzy t-centered system $\{\gamma_i\}$ where γ_i is an r-fuzzy t-open set.

Assume that $\{\lambda\}$ contains all r-fuzzy t-open sets λ_i, containing the fuzzy point α in R_1 and show that $\{\gamma\}$ contains all r-fuzzy t-open sets λ_i in R_2 containing the fuzzy point β in R_2 such that $\beta = f(\alpha)$. Let δ_β be r-fuzzy t-open set containing the fuzzy point β. Because f is smooth fuzzy t-irreducible* and smooth fuzzy t-closed, $t\text{-Int}_{R_1}(f^{-1}(\delta_\beta), r)$ is r-fuzzy t-open set containing the fuzzy point α, $t\text{-Int}_{R_1}(f^{-1}(\delta_\beta), r) \in \{\lambda\}$.

The set $t\text{-Int}_{R_2}(f(t\text{-Int}_{R_1}(f^{-1}(\delta_\beta), r), r)) \le \delta_\beta$ and belongs to $\{\gamma\}$. Hence $q = \{\gamma\}$ is a point of $\omega(R_2)$. Let $\psi(p) = q$, to show that ψ is a mapping of $\omega(R_1)$ onto $\omega(R_2)$. Let $q = \{\gamma\} \in \omega(R_2)$.

Consider the system $\{t\text{-Int}_{R_1}(f^{-1}(\gamma), r)\}$ of r-fuzzy t-open sets in R_1. The system is smooth fuzzy t-centered. We extend it to a maximal smooth fuzzy t-centered system of r-fuzzy t-open sets $p = \{\lambda\}$ and consider the point $\psi(p)$. As we have shown, $t\text{-Int}_{R_2}(f(\lambda), r)$ may be extended in a unique way to a maximal system $\{\gamma_1\}$. To show that $\psi(p) = q$, it is sufficient to show that $\{\gamma\} < \{\gamma_1\}$ and for this, it is enough to show that $\gamma \not{q}$ $t\text{-Int}_{R_2}(f(\lambda),r)$ for each $\gamma \in \{\gamma\}$ and each $t\text{-Int}_{R_2}(f(\lambda), r) \in \{t\text{-Int}_{R_2}(f(\lambda), r)\}$. Clearly $\lambda \not{q}$ $t\text{-Int}_{R_1}(f^{-1}(\gamma), r)$. Let $\eta \ne \bar{0}$ be r-fuzzy t-open set such that $\eta \le \lambda \wedge t\text{-Int}_{R_1}(f^{-1}(\gamma), r)$ and let $\alpha \le \eta$ be such that $f(\alpha) \le \gamma$. $t\text{-Int}_{R_2}(f(\eta), r)\ q\ \gamma$ and $t\text{-Int}_{R_2}(f(\eta), r) \le t\text{-Int}_{R_2}(f(\lambda),r)$. On the other hand, $t\text{-Int}_{R_2}(f(\eta), r)\ \not{q}\ \gamma$. That is $t\text{-Int}_{R_2}(f(\lambda), r)\ \not{q}\ \gamma$. Therefore $\psi(p) = q$. ψ is onto. The mapping ψ is one to one. For if $p_1 \ne p_2$ then there are r-fuzzy t-open sets λ_1 and λ_2, $\lambda_1 \in p_1$ and $\lambda_2 \in p_2$ such that $\lambda_1\ q\ \lambda_2$, but then $f(\lambda_1)\ q\ f(\lambda_2)$. That is $t\text{-Int}_{R_2}(f(\lambda_1), r)\ q\ t\text{-Int}_{R_2}(f(\lambda_2), r)$. Hence $\psi(p_1) \ne \psi(p_2)$. The mapping ψ is one - one of $\theta(R_1)$ into $\theta(R_2)$ taking $\omega(R_1)$ onto $\omega(R_2)$. Let $p' = \{\lambda\}$ be an arbitrary smooth fuzzy t-end in R_1, that is an element of $\theta(R_1)$ and let $q' = \psi(p') = \{\gamma\}$. Now prove $\psi(p_\lambda) \subset p_\gamma = P_{t\text{-Int}_{R_2}(f(\lambda), r)}$. If $p'' \in P_\lambda$, then $\lambda \in p''$. So $t\text{-Int}_{R_2}(f(\lambda),r) \in \psi(p'')$ which means that $\psi(p'') \subset P_{t\text{-Int}_{R_2}(f(\lambda), r)}$. This proves that ψ is a smooth fuzzy t-homeomorphism. To prove the theorem we have to show that $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$. Consider the mapping ψ only on $\omega(R_1) \subset \theta(R_1)$. From the construction of ψ it follows that every smooth fuzzy t-end containing all r-fuzzy t-open sets λ_i containing α is mapped by ψ into a smooth fuzzy t-end containing λ_i with r-fuzzy t-open sets containing fuzzy point β, $\psi(\pi^{-1}_{R_1}(\alpha)) \subset (\pi^{-1}_{R_2}(\beta))$. Hence $f \circ \pi_{R_1} = \pi_{R_2} \circ \psi$. Thus the theorem proved.

Corollary 5.2: The smooth fuzzy t-absolute of R_1 and R_2 are smooth fuzzy t-homeomorphism if there exists a smooth fuzzy topological space R such that R can be mapped onto both R_1 and R_2 by smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping.

$$\begin{tikzcd}
R_1 & R \arrow[l, dash, "f_1"'] \arrow[r, "f_2"] & R_2 \\
\omega(R_1) \arrow[u, "\pi_{R_1}"] & \omega(R) \arrow[u, "\pi_R"] \arrow[l, dash, "\psi_1"] \arrow[r, "\psi_2"'] & \omega(R_2) \arrow[u, "\pi_{R_2}"']
\end{tikzcd}$$

Proof: Let f_1 be a smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping from R onto R_1 and let f_2 be smooth fuzzy t-irreducible* and smooth fuzzy t-perfect mapping from R into R_2. By theorem 5.1 there exists a smooth fuzzy t-homeomorphism ψ of $\omega(R)$ onto $\omega(R_1)$ such that $f_1 \circ \pi_R = \pi_{R_1} \circ \psi_1$ and there exists a smooth fuzzy t-homeomorphism ψ_2 of $\omega(R)$ onto $\omega(R_2)$ such that $f_2 \circ \pi_R = \pi_{R_1} \circ \psi_2$. Therefore $\omega(R_1)$ and $\omega(R_2)$ are smooth fuzzy t-homeomorphic.

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