EFFECT OF THERMO DIFFUSION AND CHEMICAL REACTION ON NON DARCY CONVECTIVE HEAT AND MASS TRANSFER FLOW IN A VERTICAL CHANNEL WITH RADIATION

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ABSTRACT

We discuss the effect of radiation, diffusion and chemical reaction on the mixed convective heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium in a vertical channel in the presence of heat generating sources. By employing a regular perturbation technique with a porous parameter \( \delta \) acting on the flow phenomena is analyzed graphically. The rate of heat and mass transfer are evaluated numerically.

Key Words: Thermo diffusion, Chemical reaction, Non darcy, Vertical Channel, Radiation, Porous medium.

1. INTRODUCTION

Heat transfer in the case of homogeneous fluid saturated porous medium has been studied with relation to different applications like dynamics of hot underground springs, terrestrial heat flow through aquifers, hot fluid and ignition from displacements in reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials and heat exchange with fluidized beds. Mass transfer in isothermal conditions has been studied with applications to problems of mixing of fresh ans salt water in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils etc. Prevention of salt dissolution into lake waters near the sea shores has become a serious problem for research.

Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage , evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed . Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[2],Lai and Kulacki[4] and Murthy and Singh[5].Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai[3].The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al[1]. The combined effects of thermal and mass diffusion in channel flows has been studied in recent times by a few authors, notably Nelson and Wood[6,7], and others[11,12].

In almost all these works the boundary layer formulation od Darcy’s law, the energy and diffusion equations were used. Non-Darcy effects on natural convection in porous media have received a great deal of attention in recent years because of the experiments conducted with several combinations of solids and fluids covering wide range of governing parameters which indicates that the experimental data for systems other than glass water at low Rayleigh numbers do not agree with theoretical predictions based on the Darcy flow model. This divergence in the heat transfer results has been reviewed in detail in the works Prasad et al [8] among others.

For some industrial applications such as glass production and furnace design in space technology applications , Cosmical flight aerodynamics, rocket propulsions systems, plasma physics which effects can be significant. Raptis and Perdikis[9] have studied the effects of thermal radiation and free convection flow past a moving vertical plate.

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In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing.

In this paper we deal with the non-darcy effects on two-dimensional laminar simultaneous heat and mass transfer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid through a porous medium confined in a vertical channel. The equations of continuity, linear momentum, energy and diffusion which govern the flow fields are solved by employing a regular perturbation technique. The behaviour of the velocity, temperature and concentration, skin friction, Nusselt number and Sherwood Number has been discussed for variations in the governing parameters.

2. FORMULATION OF THE PROBLEM

We consider a coupled heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel bounded by porous flat walls in the presence of heat generating sources, transverse magnetic field effects and a first order chemical reaction. The flow is assumed to be steady, laminar and two-dimensional and the surface is maintained at constant temperature and concentration. As the plates are sufficiently long, the flow variables does not depend on the vertical and axial co-ordinates. It is also assumed that the applied magnetic field is uniform and that magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected. All thermophysical properties are constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation. Under these assumptions, the equations describing the physical situation are given by

\[ \frac{\partial u}{\partial y} = 0 \]  
\[ \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + \beta g(T - T_e) + \beta^* g(C - C_e) - \frac{\sigma B_0^2 u}{\rho} \frac{(V_k) u - \delta F}{\sqrt{k}} u^2 \]  
\[ \rho_0 C_p v \frac{\partial T}{\partial y} = \lambda \frac{\partial T}{\partial y^2} + Q(T - T_e) - \frac{\partial (q_R)}{\partial y} \]  
\[ v \frac{\partial C}{\partial y} = D \frac{\partial C}{\partial y^2} - \gamma_1 C + K_{11} \frac{\partial^2 T}{\partial y^2} \]

where \( y \) is the horizontal or transverse coordinate, \( u \) is the axial velocity, \( v \) is the transverse velocity, \( T \) is the fluid temperature, \( C \) is the concentration, \( T_e \) is the ambient temperature, \( C_e \) is the ambient concentration and \( \rho, g, \beta, \beta^*, \mu, \sigma, B_0, Q, D, \gamma_1 \) are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation/absorption coefficient, mass diffusion coefficient, chemical reaction parameter and \( K_{11} \) is the cross diffusivity respectively. The physical boundary conditions for the problem are

\[ u(-L) = 0, v(-L) = v_w, T(-L) = T_1, C(-L) = C_1 \]
\[ u(+L) = 0, v(+L) = v_w, T(+L) = T_2, C(+L) = C_2 \]

where \( v_w > 0, T_1, T_2 \) and \( C_1, C_2 \) are the suction velocity, surface temperature and concentration on \( y = \pm L \) respectively.

Invoking Rosseland approximation for radiative heat flux [9].

\[ q_r = -\frac{4\sigma^*}{3}\frac{\partial T'^4}{\partial y} \]

Expanding \( T'^4 \) in Taylor series about \( T_e \) and neglecting higher order terms we get

\[ T'^4 \approx 4TT_e^3 - 3T_e^4 \]

\( q_r \) represents the radiation heat flux in the y direction, \( \sigma^* \) the Stefan–Boltzmann constant and \( \beta_R \) the mean absorption coefficient.

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In order to write the governing equations and boundary conditions in the dimensionless form the following non-dimensional quantities are introduced

\[ y' = \frac{y}{L}, u' = \frac{u}{(v/L)}, \theta = \frac{T-T_1}{T_2-T_1}, C' = \frac{C-C_1}{C_2-C_1} \]

the equations after dropping the dashes are

\[ \frac{d^2 u}{dy^2} + S \frac{du}{dy} + G \delta (\theta + N C) - \delta (M^2 + D^{-1})u - (\delta^2 \Lambda)u^2 \]

\[ \frac{d^2 \theta}{dy^2} + SP \frac{d\theta}{dy} - \alpha P \theta = 0 \]

\[ \frac{d^2 C}{dy^2} + SSC \frac{dC}{dy} - k Sc C = - \frac{Sc So}{N} \frac{\partial^2 \theta}{\partial y^2} \]

where

\[ P = \frac{\mu C_p}{\lambda} \] (Prandtl Number)

\[ Sc = \frac{V}{D} \] (Schmidt Number)

\[ k = \frac{\gamma_1 L^2}{\nu} \] (Chemical reaction parameter)

\[ M^2 = \frac{\sigma B_1^2 L^2}{\rho_0 \nu} \] (Hartman Number)

\[ N = \frac{\beta^* \Delta T}{\beta \Delta r} \] (Buoyancy ratio)

\[ D^{-1} = \frac{L^2}{k_1} \] (Darcy parameter)

\[ N_1 = \frac{\lambda \beta_{r}}{4 \sigma^* T_e} \] (radiation parameter)

\[ \alpha = \frac{QL^2}{\lambda} \] (Heat source parameter)

\[ So = \frac{K_{11} \beta^*}{\gamma^*} \] (soret parameter)

\[ \Lambda = FD^{-1/2} \] (Inertia parameter or Foechhimer Number)

\[ N_2 = \frac{3N_1}{3N_1 + 4} \]

The non-dimensional boundary conditions are

\[ u(\pm 1) = 0, \theta (-1) = 0, C (-1) = 0 \]

\[ \theta (+1) = 1, C (+1) = 1 \]

3. ANALYSIS OF THE FLOW

The governing equations of the flow, temperature and concentration are coupled non-linear differential equations. Assuming the porosity \( \delta \) to be small, we write

\[ u(y) = u_0(y) + \delta u_1(y) + \delta^2 u_2(y) + \ldots \] (3.1a)

\[ \theta(y) = \theta_0(y) + \delta \theta_1(y) + \delta^2 \theta_2(y) + \ldots \] (3.1b)

\[ C(y) = C_0(y) + \delta C_1(y) + \delta^2 C_2(y) + \ldots \] (3.1c)

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Substituting the above expansions (3.1a)-(3.1c) in the equations (2.6)-(2.8) and equating the like powers of $\delta$, we obtain equations to the zeroth order as

\[ \frac{d^2 u_0}{dy^2} + S \frac{du_0}{dy} = \pi \]  \hspace{1cm} (3.2)

\[ \frac{d^2 \theta_0}{dy^2} + SP \frac{d\theta_0}{dy} - \alpha P \theta_0 = 0 \]  \hspace{1cm} (3.3)

\[ \frac{d^2 C_0}{dy^2} + SS_C \frac{dC_0}{dy} - \gamma C_0 = - \frac{ScSo}{N} \frac{\partial^2 \theta_0}{\partial y^2} \]  \hspace{1cm} (3.4)

The first order equations are

\[ \frac{d^2 u_1}{dy^2} + S \frac{du_1}{dy} = -G(\theta_0 + NC_0) + (D^{-1} + M^2)u_1 - Au_0^2 \]  \hspace{1cm} (3.5)

\[ \frac{d^2 \theta_1}{dy^2} + SP \frac{d\theta_1}{dy} - \alpha P \theta_1 = 0 \]  \hspace{1cm} (3.6)

\[ \frac{d^2 C_1}{dy^2} + SS_C \frac{dC_1}{dy} - \gamma C_1 = - \frac{ScSo}{N} \frac{\partial^2 \theta_1}{\partial y^2} \]  \hspace{1cm} (3.7)

The second order equations are

\[ \frac{d^2 u_2}{dy^2} + S \frac{du_2}{dy} = -G(\theta_1 + NC_1) + (D^{-1} + M^2)u_2 - Au_1^2 \]  \hspace{1cm} (3.8)

\[ \frac{d^2 \theta_2}{dy^2} + SP \frac{d\theta_2}{dy} - \alpha P \theta_2 = 0 \]  \hspace{1cm} (3.9)

\[ \frac{d^2 C_2}{dy^2} + SS_C \frac{dC_2}{dy} - \gamma C_2 = - \frac{ScSo}{N} \frac{\partial^2 \theta_2}{\partial y^2} \]  \hspace{1cm} (3.10)

\[ \gamma = KSc \]

The corresponding boundary conditions are

\[ u_0(\pm 1) = 0, \theta_0(-1) = 0, C_0(-1) = 0 \]
\[ \theta_0(+1) = 1, C_0(+1) = 1 \]  \hspace{1cm} (3.11a)

\[ u_1(\pm 1) = 0, \theta_1(\pm 1) = 0, C_1(\pm 1) = 0 \]  \hspace{1cm} (3.11b)

\[ u_2(\pm 1) = 0, \theta_2(\pm 1) = 0, C_2(\pm 1) = 0 \]  \hspace{1cm} (3.11c)

4. SOLUTION OF THE PROBLEM

Solving the equations (3.2)-(3.10) subject to the boundary conditions (3.11a, b, c) we obtain

\[ u_0 = a_2 \left( \exp(-sy) - \cosh(sy) \right) \]
\[ u_1 = a_{19} + a_{20} \exp(-s \ y) + \phi_1(y) \]

\[ \phi_1(y) = a_{11}y - a_{12} y^2 - a_{13} y \exp(-s \ y) - (a_{14} + a_{16}) \exp(-m_2 y) - \]
\[ - a_{15} \exp(-m_1 y) - a_{18} \exp(-m_4 y) \]

\[ u_2 = a_{25} + a_{26} \exp(-sy) + \phi_2(y) \]

\[ \phi_2(y) = a_{58} y + a_{59} y^2 + a_{60} y^3 + \frac{1}{2s^2} \exp(-2sy) + \frac{1}{6s^2} \exp(2sy) + a_{53} \exp(-m_1 y) + \]
\[ + a_{54} \exp(m_2 y) + a_{55} \exp(m_3 y) + a_{56} \exp(m_4 y) + (a_{61} + a_{43} y^2 + a_{45}) \exp(-sy) + \]
\[ + a_{57} \exp(sy) \]

\[ \theta_0 = a_3 \exp(m_1 y) + a_4 \exp(m_2 y) \]
\[ \theta_1 = 0 \]
\[ \theta_2 = 0 \]

\[ C_0 = a_2 \exp(-m_2 y) + a_6 \exp(m_4 y) + a_7 \exp(-m_1 y) + a_8 \exp(m_2 y) \]

\[ C_1 = 0, \quad C_2 = 0 \]

5. NUSSELT NUMBER AND SHERWOOD NUMBER

The rate of heat transfer (Nusselt Number) on the boundaries \( y = \pm 1 \) are given by

\[ (Nu)_{y=\pm 1} = \left( \frac{d\theta_0}{dy} + \delta \frac{d\theta_1}{dy} + \delta^2 \frac{d\theta_2}{dy} \right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (Nu)_{y=+1} = b_8 \]
\[ (Nu)_{y=-1} = b_8 \]

The rate of mass transfer (Sherwood Number) on the boundaries \( y = \pm 1 \) are given by

\[ (Sh)_{y=\pm 1} = \left( \frac{dC_0}{dy} + \delta \frac{dC_1}{dy} + \delta^2 \frac{dC_2}{dy} \right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (Sh)_{y=+1} = b_8 \]
\[ (Sh)_{y=-1} = b_{10} \]

For \( S_0 = 0 \) the results are in good agreement with Sudha [10]

6. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we investigate the soret effect on the convective heat and mass transfer of a viscous electrically conducting, chemically reacting fluid in vertical channel with uniform suction. We consider three different cases \( k > 0, k = 0 \) and \( k < 0 \), where \( k \) is the chemical reaction parameter. \( k > 0 \) represents destructive chemical reaction, \( k = 0 \) represents no chemical reaction and \( k < 0 \) is for generative chemical reaction.

The velocity \( u \) is represented in figs. 1-7 for different values of the governing parameters \( S_0, N_1, \alpha \) and \( k \). \( u > 0 \) is the actual flow and \( u < 0 \) is the reversal flow. It is found that the velocity profiles rise from its value zero on the boundary \( y = -1 \) attains maximum at \( y = 0.8 \) and then falls to zero on \( y = 1 \). The axial velocity enhances with increase in \( G \leq 2 \times 10^3 \) and reduces with higher \( G \geq 3 \times 10^5 \). It reduces with increase in the strength of the heat generating source (fig. 1). From
The non-dimensional temperature ($\theta$) is shown in figs 8-15. The temperature is found to be positive for all variations. The non-dimensional temperature enhances with increase in $G$ and $\alpha \leq 4$ and experience a depreciation with higher $\alpha \geq 6$ (fig-8). From fig-9, we find that higher the Lorentz force larger the temperature $\theta$. The variation of $\theta$ with buoyancy ratio $N$ shows that when molecular buoyancy force dominates over the thermal buoyancy force the temperature experience an enhancement when the buoyancy forces act in the same direction while for the forces acting in opposite directions it reduces in the flow region (fig-10). An increase in the radiation parameter $N_1$ reduces $\theta$ everywhere in the flow region (Fig-11). From fig-12 we notice that an increase in the suction parameter leads to an increment in the temperature in the region $-0.8 \leq y \leq 0.8$ and is almost linear in the neighborhood of $y=1$. The variation of $\theta$ with Schmidt number $S_c$ shows that lesser the molecular diffusivity smaller the temperature in the flow region. The variation of $\theta$ with $S_0$ reveals that $\theta$ enhances with increase $S_0 \geq 0$ and depreciates with $|S_0| < 0$ (Fig-14). Fig-15 represents, $\theta$ with chemical reaction parameter $k$. It is found that in the case of generative reaction ($k \leq 1.2$) the temperature depreciates in the left half and enhances in the right half and for higher $k > 2$ it depreciates with $k=2$, while in the case of destructive reaction, we notice an enhancement in the temperature (Fig-15).

The concentration distribution ($C$) is shown in figs.16-24. We follow the convention that the concentration is positive and negative according as the actual concentration is greater or lesser than the ambient concentration. The concentration gradually rises from value to attain its prescribed value $y=1$. The variation of $C$ with Grashoff number $G$ shows that the concentration experience a marginal increment with increase in the entire flow region. With respect to heat source parameter $\alpha$ we notice that for $\alpha \leq 4$, $C$ is positive and is negative for higher $\alpha \geq 6$. The actual concentration enhances with $\alpha$ in the entire flow region (Fig-17). From fig-18 we observe that higher the Lorentz force smaller the actual concentration in the flow region (Fig-18). The variation of $C$ with $N$ shows that $C$ is positive for $N>0$ and negative in the left half for $N<0$. When the molecular buoyancy force dominates over the thermal buoyancy force, the actual concentration experiences a depreciation in the entire flow region, while for the forces acting in opposite direction the actual concentration increases in the flow region (Fig-19). An increase in $N_1$ enhances the actual concentration in the flow region (Fig-20). Also an increase in the suction parameter $S$ leads to an increment in the concentration (Fig-21). A variation of $C$ with Schmidt number $S_c$ reveals that the concentration is positive for all values of $S_c$ except for $S_c=0.6$ where it is negative. It is found that the concentration enhances with $S_c \geq 0.6$ and depreciates with higher $S_c \geq 0.3$ (Fig-22). From fig-23 we find that an increase in $S_0$ results in an enhancements in concentration while in increase in $|S_0| \geq 0$ leads to a depreciation in the concentration. The variation of $C$ with chemical reaction parameter $k$ in the case of generating reaction, $C$ is negative in the vicinity of $y=1$ and positive everywhere while in the case of destructive reaction it is negative in the region $-0.8 \leq y \leq -0.6$. It is found that the concentration experiences an enhancement with $k \leq 1.2$ and depreciates with higher $k \geq 2$ while an increase in $k(<0)$ the concentration depreciates in the entire flow region.

The Nusselt number which represents the rate of heat transfer at $y=\pm 1$ is shown in table 1-6. It is found that the rate of heat transfer experiences an enhancement in magnitude with increase in $G(\leq 0)$. An increase in the suction parameter $S$, depreciates $[Nu]$ at $y=1$ and enhances it at $y=-1$ (tables 3 & 6). The variation of $[Nu]$ with heat source parameter $\alpha$ indicates that $[Nu]$ experiences an enhancement with $\alpha$ at $y=1$ and depreciates at $y=-1$. The variation of $Nu$ with $Sc$ reveals that lesser the molecular diffusivity larger $[Nu]$ and for further lowering of the molecular diffusivity smaller $[Nu]$ and for still lowering of the diffusivity larger $[Nu]$ at $y=1$, while at $y=-1$ larger $|N|$. With respect to variation in $So$, we notice that $[Nu]$ depreciates with increase in $|S_0|\geq 0$ and enhances with $|S_0|(<0)$ (tables 2 & 5). An increase in the radiation parameter $N_1$ enhances the rate of heat transfer at $y=1$ and at $y=-1$ it enhances with $N_1 \geq 1$ and depreciates with $N_1 \geq 1.5$. The variation of $Nu$ with chemical reaction parameter $k$ shows that the rate of heat transfer enhances in the generating reaction and depreciates in the destructive reaction (tables 1 & 4).

The Sherwood number ($Sh$) which represents the rate of mass transfer at $y=\pm 1$ is shown in table 7-12 for different values of the governing parameter. It is found that the rate of mass transfer enhances with increasing $G=0$ at both the walls while an increase in $G(<0)$ enhance $|Sh|$ at $y=1$ and depreciates at $y=-1$. Also an increase in the suction parameter $S$ leads to a reduction $|Sh|$ (tables 9 & 12). The variation of $Sh$ with $Sc$ shows that lesser the molecular...
diffusivity smaller $|\text{Sh}|$ and for further lowering of diffusivity larger $|\text{Sh}|$ at $y=1$ while larger the rate of mass transfer at $y=-1$. An increase in the soret parameter $S_0>0$ enhances the Sherwood number at $y=1$ and depreciates at $y=-1$, while an increase in $|S_0|(<0)$ enhances it at $y=\pm 1$ (tables 8 & 11). From tables 7 & 10, we notice that $|\text{Sh}|$ decreases in the generating reaction and enhances in the destructive reaction, while a reversed effect is observed at $y=-1$.

7. REFERENCES


Figures:
Fig. 5: Variation of \( u \) with \( S \)

- \( M = 2 \times 10^3 \), \( K = 2 \)
- \( S \): 0.1, 0.3, 0.5, 1.3

Fig. 6: \( u \) with \( S_c \) & \( S_p \)

- \( M = 2 \times 10^3 \), \( K = 2 \)
- \( S_c \): 0.24, 0.6, 1.5, 2.01, 1.3, 1.3, 1.3
- \( S_p \): 0.5, 0.5, 0.5, 0.5, 1, -0.5, -1

Fig. 7: Variation of \( U \) with \( K \)

- \( M = 2 \times 10^3 \), \( N = 1 \)
- \( K \): -0.2, -0.02, 0, 0.2, 1.2, 2

Fig. 8: Variation of \( \theta \) with \( G \) & \( \alpha \)

- \( G \): \( 1 \times 10^3 \), \( 3 \times 10^3 \), \( 5 \times 10^3 \), \( 3 \times 10^3 \), \( 3 \times 10^3 \)
- \( \alpha \): 2, 2, 2, 2, 4

Fig. 9: Variation of \( \theta \) with \( M \)

- \( K = 2 \times 10^3 \), \( N = 1 \)
- \( M \): 0, 2, 3, 4

Fig. 10: Variation of \( \theta \) with \( N \)

- \( M = 2 \times 10^3 \), \( K = 2 \)
- \( N \): 1, 2, -0.5, -0.8
Fig. 17 Variation of C with α
M = 2, G = 10^5, N = 1
I II III
α 2 4 6

Fig. 18 Variation of C with M
K = 2, G = 10^5, N = 1
I II III IV
M 0 2 3 4

Fig. 19 Variation of C with N
M = 2, G = 10^5, K = 2
I II III IV
N 1 2 -0.5 -0.8

Fig. 20 Variation of C with N₁
M = 2, G = 10^5, N = 1
I II III IV
N₁ 0 5 10 100

Fig. 21 Variation of C with S
M = 2, G = 10^5, N = 1
I II III IV
S 0.1 0.3 0.5 1.3

Fig. 22 Variation of C with Sc
M = 2, G = 10^5, N = 1
I II III IV
Sc 0.24 0.26 1.3 2.01
### Table 1

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Table 5
Nusselt Number \((Nu)\) at \(y=1\)
\[ G = 10^9, M = 2.00, N = 1.00, S_c = 1.30, S_a = 0.5, S = 0.30 \]
<table>
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<td>0.23153</td>
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<td>-0.25373</td>
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<td>2.01</td>
<td>0.24</td>
<td>0.60</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
</tr>
<tr>
<td>S_a</td>
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<td>0.5</td>
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<td>0.5</td>
<td>1</td>
<td>-0.5</td>
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Table 6
Nusselt Number \((Nu)\) at \(y=1\)
\[ G = 10^9, M = 2.00, N = 1.00, S_c = 1.30, S_a = 0.5, S = 0.30 \]
<table>
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<td>(3 \times 10^3)</td>
<td>(5 \times 10^3)</td>
<td>(10^5)</td>
<td>(10^6)</td>
<td>(10^8)</td>
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Table 7
Sherwood Number \((Sh)\) at \(y=1\)
\[ G = 10^9, M = 2.00, N = 1.00, S_c = 1.30, S_a = 0.5, S = 0.30 \]
<table>
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Table 8
Sherwood Number \((Sh)\) at \(y=1\)
\[ G = 10^9, M = 2.00, N = 1.00, S_c = 1.30, S_a = 0.5, S = 0.30 \]
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### Table 9
**Sherwood Number (Sh) at y=1**

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### Table 10
**Sherwood Number (Sh) at y=-1**

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### Table 11
**Sherwood Number (Sh) at y=-1**

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### Table 12
**Sherwood Number (Sh) at y=-1**

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<td>$5 \times 10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
<td>$10^3$</td>
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