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ON TOTAL EDGE FIXED GEODOMINATING SETS AND POLYNOMIALS OF GRAPHS

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ABSTRACT

We introduce total edge fixed geodomination sets and polynomial of a graph G. The total edge fixed geodomination polynomial of G is defined as $G_t(G, x) = \sum_{e_k \in E(G)} G_{e_k}(G,x)$ *where* $G_{e_k}(G,x) = \sum_{i=g_{e_k}(G)}^{n-2} g_{e_k}(G,i)x^i$, $g_{e_k}(G, i)$ *is the number of edge fixed geodominating sets of graph G with cardinality i, and* e_k *is a fixed edge of G and* $g_{e_k}(G)$ *is the* e_k *-geodomination number of G. we obtain some properties of* $G_t(G, x)$ *and its coefficients. Also, we compute the polynomial for some specific graphs.*

Keywords: *Edge fixed geodominating set, Total edge fixed geodomination polynomial, unimodal.*

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1. INTRODUCTION

Let G = (u, v) be a simple graph of order n .An edge of a graph is said to be pendant if one of its vertices is a pendant vertex. A vertex of a graph is said to be pendant if its neighbouhood contains exactly one vertex. The distance d(u, v) between two vertices u and v in a connected graph G is the length of the shortest u – v path in G. A u –v path of length d(u, v) is called u – v geodesic. The closed interval I [u, v] consists of all vertices lying on some u – v geodesic of G, while for $S \subseteq V$, $I[S] = \bigcup_{u,v \in s} I[u,v]$. A set S of vertices is a geodetic set if I [S] = V, and the minimum cardinality of a geodetic set is the geodetic number, g(G). The concept of edge fixed domination was introduced in [8]. Let e = *x*y be any edge of a connected graph G of order atleast 3. A set S of vertices of G is an e – geodominating set if every vertex of G is lies on either an *x*-u geodesic or a y- u geodesic in G for some element u in S. The minimum cardinality of an e-geodominating set of G is defined as the e-geodomination number of G and is denoted by $g_e(G)$ or $g_{xy}(G)$.

The corona of two graphs G_1 and G_2, as defined by Frucht and Harary in [6] is the graph G = $G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. The corona $G \circ K_1$, in particular, is the graph constructed from a copy of G, where for each vertex v $\in$ V(G), a new vertex v' and a pendant edge vv' are added.

A finite sequence of real numbers $(a_0, a_1, a_2 \dots a_n)$ is said to be unimodal if there is some $k \in \{0,1,\dots,n\}$ called the mode of a sequence, such that $a_0 \le \dots \le a_{k-1} \le a_k \ge a_{k+1} \ge \dots \ge a_n$; the mode is unique if $a_{k-1} < a_k > a_{k+1}$. A polynomial is called unimodal if the sequence of its coefficients is unimodal.

In the next section, we introduce the total edge fixed geodomination polynomial. In section 3 we study the coefficients of the total edge fixed geodomination polynomial. In the last section, we study the total edge fixed geodetic polynomial of the graph $G \circ K_1$, where $G \circ K_1$, is the corona of two graphs G and K_1.

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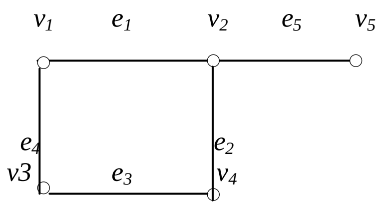
2. TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL OF A GRAPH

Definition 2.1: The edge fixed geodomination polynomial is defined as $G_{e_k}(G,x) = \sum_{i=g_{e_k}(G)}^{n-2} g_{e_k}(G,i)x^i$, where g_{e_k} (G, i) is the number of edge fixed geodominating sets of graph G with cardinality i,and e_k is a fixed edge of G.The total edge fixed geodomination polynomial of G is defined as G_t (G, x) = $\sum_{e_k \in E(G)} G_{e_k}(G,x)$ clearly, $1 \leq g_{e_k}(G) \leq n-2$.

If $g_t(G) = \min_{e_k \in E(G)} \{ g_{e_k}(G)\}$, then we can write $G_t(G, x) = \sum_{i=g_t(G)}^{n-2} g_t(G, i)x^i$, where g_t (G, i) is the number of total edge fixed geodominating sets of cardinality i.

Example 2.2: Consider a graph G.



Clearly $g_{e_1}(G) = 2$
$g_{e_1}(G, 2) = 2$. Here S = {{v_5, v_3}, {v_5, v_4}} are the edge fixed geodomination set of cardinality 2.
$g_{e_1}(G, 3) = 1$. Here S = {v_3, v_4, v_5} is the edge fixed geodominating set.

Therefore $G_{e_1}(G, x) = 2x^2 + x^3$.

Similarly $G_{e_2}(G, x) = 2x^2 + x^3$
$G_{e_3}(G, x) = 2x^2 + x^3$

$g_{e_4}(G) = 1$
$g_{e_4}(G, 1) = 1$. Here S = {v_5}
$g_{e_4}(G, 2) = 2$. Here S = {{v_5, v_3}, {v_5, v_4}}
$g_{e_4}(G, 3) = 1$. Here S = {v_3, v_4, v_5 }

Therefore $G_{e_4}(G, x) = x + 2x^2 + x^3$
$g_{e_5}(G) = 1$.
$g_{e_5}(G, 1) = 1$. Here S = {v_3}
$g_{e_5}(G, 2) = 2$ Here S = {{v_3, v_1}, {v_3, v_4}}
$g_{e_5}(G, 3) = 1$ Here S = {(v_1, v_3, v_4) }

Therefore $G_{e_5}(G, x) = x + 2x^2 + x^3$

Hence $G_t(G, x) = 2x^2 + x^3 + 2x^2 + x^3 + 2x^2 + x^3 + x + 2x^2 + x^3 + x + 2x^2 + x^3$

$= 2x + 10x^2 + 5x^3$.

Theorem 2.3: The total edge fixed geodomination polynomial of a complete graph K_n ($n \geq 3$) is $G_t(K_n, x) = n\,C_2\, x^{n-2}$.

Proof: Let $v_1, v_2, \ldots\ldots, v_n$ be the vertices of K_n. Let $e_1, e_2, \ldots.. e_{nC_2}$ be the edges of K_n. By theorem (2.8) in [8], $g_{e_k}(K_n) = n - 2$, for any edge e_k in K_n. Fix a edge e_1 in K_n. Then $g_{e_1}(K_n, n - 2) = 1$.

$$g_{e_{nC_2}}(K_n, n-2) = 1.$$

Since $1 \leq g_{e_k}(G) \leq n-2$ for any edge e_k in G we have $G_{e_1}(K_n, x) = x^{n-2}$, $G_{e_2}(K_n, x) = x^{n-2}, \ldots, G_{e_{nC_2}} = x^{n-2}$.

Therefore $G_t(K_n, x) = (x^{n-2}+x^{n-2}+\ldots\ldots+x^{n-2})$ nC_2 times
$= nC_2x^{n-2}$.

Theorem 2.4: The total edge fixed geodomination polynomial of a stargraph $K_{1,n}$ $(n \geq 2)$ is $G_t(K_{1,n}, x) = n\,x^{n-1}$.

Proof: Let u, $v_1, v_2, \ldots, v_n$ be the vertices of $K_{1,n}$.

By a theorem 2.7 in [8],

$g_{e_k}(K_{1,n}) = n-1$ for any edge e_k in $K_{1,n}$ .Fix a edge e_1 in $k_{1,n}$

Then $g_{e_1}(K_{1,n}, n-1) = 1$

Similarly, $g_{e_2}(K_{1,n}, n-1) = 1, \ldots, g_{e_n}(K_{1,n}, n-1) = 1$

Then, we have $G_{e_1}(K_{1,n}, x) = x^{n-1}$, $G_{e_2}(K_{1,n}, x) = x^{n-1}, \ldots, G_{e_n}(K_{1,n}, x) = x^{n-1}$.

Therefore, $G_t(K_{1,n}, x) = (x^{n-1}+x^{n-1}+\ldots\ldots x^{n-1})$ n times

$= nx^{n-1}$.

Theoram 2.5: The total edge fixed geodomination polynomial of a complete bipartite graph $K_{m,n}$ $(m \geq n)$ is
(i) $G_t(K_{m,n}, x) = 2nx((1+x)^{n-1})$, m =2.
(ii) $G_t(K_{m,n}, x) = mn\{[(1+x)^{n-1}-1][(1+x)^{m-1}-1] + x^{m-1} + x^{n-1}\}$, $m \geq 3$

Proof: Let $K_{m,n}$ be a complete bipartite graph with bipartition (X, Y).Let $X = \{u_1, u_2, \ldots, u_m\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$.

Then $|X| = m$ and $|Y| = n$

Case (i): If m = 2.

Then $X = \{u_1, u_2\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$.

Let $e_1, e_2, \ldots, e_{2n}$ be the edges of $K_{m,n}$.

By a theorem (2.13) in [8], $g_{e_k}(K_{m,n}) = 1$, for any edge e_k in $K_{m,n}$. Fix e_k
$g_{e_k}(K_{m,n}, 1) = 1$
$g_{e_k}(K_{m,n}, 2) = (n-1)C_1$
$g_{e_k}(K_{m,n}, 3) = (n-1)C_2$..............................
$g_{e_k}(K_{m,n}, n) = 1 = (n-1)C_{n-1}$

Therefore, $G_{e_k}(K_{m,n}, x) = x + (n-1)C_1x^2 + (n-1)C_2x^3 + \ldots\ldots + (n-1)C_{n-1}x^n$.

$= x(1 + (n-1)C_1x + (n-1)C_2x^2 + \ldots\ldots + (n-1)C_{n-1}x^{n-1})$.

$= x(1+x)^{n-1}$,for $k = 1,2,\ldots\ldots 2n$.

Therefore, $G_t(K_{m,n}, x) = [x(1+x)^{n-1} + x(1+x)^{n-1} + \ldots\ldots + x(1+x)^{n-1}]$ (2n times)

$= 2nx[(1+x)^{n-1}]$

Case (ii): If $m \geq 3$.

Let $e_1, e_2, \ldots, e_{mn}$ be the edges of $K_{m,n}$.

By a theorem (2.13) in [8] $g_{e_k}(K_{m,n}) = 2$ for any edge e_k in $K_{m,n}$. Fix e_k in G.

$g_{e_k}(K_{m,n},2) = (m-1)C_1 \times (n-1)C_1$.

$g_{e_k}(K_{m,n},3) = (m-1)C_2 \times (n-1)C_1 + (m-1)C_1 \times (n-1)C_2$

$g_{e_k}(K_{m,n},4) = (m-1)C_3 \times (n-1)C_1 + (m-1)C_2 \times (n-1)C_2 + (m-1)C_1 \times (n-1)C_3$

$g_{e_k}(K_{m,n},m\text{-}2) = (m-1)C_{m-3} \times (n-1)C_1 + (m-1)C_{m-4} \times (n-1)C_2 + (m-1)C_{m-5} \times (n-1)C_3 \ldots\ldots + (m-1)C_1 \times (n-1)C_{m-3}$

$g_{e_k}(K_{m,n}, m) = (m-1)C_{m-1} \times (n-1)C_1 + (m-1)C_{m-2} \times (n-1)C_{2} + \ldots\ldots + (m-1)C_1 \times (n-1)C_{m-1}$

$g_{e_k}(K_{m,n}, n\text{-}2) = (m-1)C_{m-2} \times (n-1)C_{n-m} + (m-1)C_{m-3} \times (n-1)C_{n-m+1} + \ldots\ldots + (m-1)C_1 \times (n-1)C_{n-3}$

$g_{e_k}(K_{m,n}, n\text{-}1) = (m-1)C_{m-1} \times (n-1)C_{n-m} + (m-1)C_{m-2} \times (n-1)C_{n-m+1} + \ldots\ldots + (m-1)C_1 \times (n-1)C_{n-2} + (n\text{-}1)C_{n-1}$

$g_{e_k}(K_{m,n}, m+n\text{-}3) = (m-1)C_{m-1} \times (n-1)C_{n-2} + (n-1)C_{n-1} \times (m-1)C_{m-2}$

$g_{e_k}(K_{m,n}, m+n\text{-}2) = (m-1)C_{m-1} \times (n-1)C_{n-1}$

Therefore,

$$G_{ek}(K_{m,n}, x) = [(m-1)C_1 \times (n-1)C_1]\, x^2 + [(m-1)C_2 \times (n-1)C_1 + (m-1)C_1 \times (n-1)C_2]\, x^3 + [(m-1)C_3 \times (n-1)C_1 + (m-1)C_2 \times (n-1)C_2 + (m-1)C_1 \times (n-1)C_3]\, x^4 + \ldots + [(m-1)C_{m-1} \times (n-1)C_{n-2} + (n-1)C_{n-1} \times (m-1)C_{m-2}]\, x^{m+n-3} + (m-1)C_{m-1} \times (n-1)C_{n-1}\, x^{m+n-2} + (m-1)C_{m-1}\, x^{m-1} + (n-1)C_{n-1}\, x^{n-1.}$$

$$= (m-1)C_1 [(n-1)C_1 x^2 + (n-1)C_2 x^3 + \ldots\ldots + (n-1)C_{n-1} x^n] + (m-1)C_2 [(n-1)C_1 x^3 + (n-1)C_2 x^4 + \ldots\ldots + (n-1)C_{n-1} x^{n+1}] + (m-1)C_3 [(n-1)C_1 x^4 + (n-1)C_2 x^5 + \ldots\ldots + (n-1)C_{n-1} x^{n+2})] + \ldots\ldots + (m-1)C_{m-2}[(n-1)C_1 x^{m-1} + (n-1)C_2 x^m + \ldots.. + (n-1)C_{n-1} x^{m+n-3}] + (m-1)C_{m-1}[(n-1)C_1 x^m + (n-1)C_2 x^{m+1} + \ldots. + (n-1)C_{n-1} x^{m+n-2}] + x^{m-1} + x^{n-1}$$

$$= (m-1)C_1\, x[(1+x)^{n-1}-1] + (m-1)C_2\, x^2 [(1+x)^{n-1}-1] + (m-1)C_3\, x^3[(1+x)^{n-1}-1] + \ldots. + (m-1)C_{m-2}\, x^{m-2}[(1+x)^{n-1}-1] + (m-1)C_{m-1}\, x^{m-1}[(1+x)^{n-1}-1] + x^{m-1} + x^{n-1}$$

$$= [(1+x)^{n-1}-1] + [(m-1)C_1\, x + (m-1)C_2\, x^2 + (m-1)C_3\, x^3 + \ldots\ldots + (m-1)C_{m-2}\, x^{m-2} + (m-1)C_{m-1}\, x^{m-1}] + x^{m-1} + x^{n-1}$$

$$= [(1+x)^{n-1}-1] \times [(1+x)^{m-1}-1] + x^{m-1} + x^{n-1}$$

Since $K_{m,n}$ has mn edges

$G_t(K_{m,n},x) = mn\, \{[(1+x)^{n-1}-1]\,[(1+x)^{m-1}-1] + x^{m-1} + x^{n-1}\}$.

3. COEFFICIENTS OF TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL

In this section we obtain some properties of total edge fixed geodomination polynomial of G.

Theorem 3.1: Let G be a graph with $|V(G)| = n$, $n \geq 3$.

Then (i) If G is connected, then $g_{e_k}(G, n\text{-}2) = 1$ for any edge e_k in G.
(ii) $g_{e_k}(G, i) = 0$ if and only if $I < g_{e_k}(G)$ or $i > n\text{-}2$.
(iii) $G_t(G, x)$ has no constant term.
(iv) $G_t(G, x)$ is strictly increasing function in $[0, \infty)$
(v) Let G be a graph and H be any induced subgraph of G, then $\deg(G_t(G,x)) \geq \deg(G_t(H, x))$
(iv) zero is a root of $(G_t(G,x))$ with multiplicity $g_t(G)$.

Proof: Proof is obvious.

4. TOTAL EDGE FIXED GEODOMINATION POLYNOMIAL OF G O K_1

Let G be any graph with vertex set { $v_1, v_2,\ldots,v_n$}. Add a new set of vertices {$u_1,u_2\ldots u_n$} and join u_i to v_i for $1\leq i \leq n$. By the definition of corona of two graphs. We shall denote this graph by $G\circ K_1$.

Lemma 4.1: For any graph G of order n,

$$g_{e_k}(G\circ K_1) = \begin{cases} n-1 & \text{if } e_k \text{ is a pendant edge} \\ n & \text{if } e_k \text{ is a non-pendant edge} \end{cases}$$

Proof:

Case (i): e_k is a pendant edge. If G_1 is a edge fixed geodominating set of G then $G_1 = \{u_1,u_2,\ldots\ldots u_{i-1},u_{i+1},\ldots,u_n\}$ or $G_1 = \{u_1, u_2,\ldots\ldots u_{i-1},u_{i+1},\ldots,u_n\} \bigcup A$ where $A\subset \{v_1, v_2,\ldots\ldots v_n\}$.

Therfore $|G_1| \geq$ n-1. since$G_1 = \{u_1,u_2,\ldots\ldots u_i,u_{i+1},\ldots,u_n\}$ is a edge fixed geodominating set of $G\circ K_1$, we have $g_{e_k}(G\circ K_1)$ = n-1.

Case (ii): e_k is a non pendant edge. If G_1 is a edge fixed geodominating set of G, then $G_1 = \{u_1,u_2,\ldots,u_n\}$ or $G_1 = \{u_1,u_2,\ldots,u_n\} \bigcup A$ where $A\subset\{v_1,v_2,\ldots\ldots v_n\}$.

Therefore $|G_1| \geq$ n. since$G_1 = \{u_1, u_2,\ldots,u_n\}$ is a edge fixed geodominating set of $G\circ K_1$, we have $g_{e_k}(G\circ K_1)$ = n.

By lemma 4.1 $g_{e_k}(G\circ K_1$, m) = 0 if m < n-1 for pendant edge e_k and if m< n for non pendant edge e_k. so we shall compute g_e ($G\circ K_1$,m) for $n\text{-}1\leq m \leq 2$ (n-1) if e_k is a pendant edge and for $n \leq m\leq 2$ (n-1) if e_k is a non pendant edge.

Theorem 4.2: For any graph G of order n and p be the number of edges. Then

$$g_{e_k}(G\circ K_1, m) = \begin{cases} (n-1)C_{m-(n-1)} & \text{if } e_k \text{ is a pendant edge and } n - 1\leq m\leq 2(n-1) \\ (n-2)C_{m-n} & \text{if } e_k \text{ is a non-pendant edge and } n\leq m\leq 2(n-1) \end{cases}$$

Hence $G_t(G\circ K_1, x) = (1 + x)^{n-2}\,[\,nx^{n-1} + (n+p)x^n]$.

Proof:

Case (i): e_k is a pendant edge.

Suppose that G_1 is a edge fixed geodominating set of size m. When m = n-1, the edge fixed geodminating set with cardinality n-1 is $\{u_1, u_2,\ldots\ldots u_{i-1},u_{i+1},\ldots,u_n\}$. Therfore, $g_{e_k}(G\circ K_1$, n-1) = (n-1) C_o . When m = n, the edge fixed geodominating set with cardinality n is $\{u_1, u_2,\ldots\ldots u_{i-1}, u_{i+1},\ldots,u_n\} \bigcup \{v_j\}$, $1\leq j\leq n\text{-}1$.

Therefore $g_{e_k}(G\circ K_1$, n) = (n-1) C_1. Similarly, $g_{e_k}(G\circ K_1$, n+1) = (n-1) C_2,….., $g_{e_k}(G\circ K_1$, 2(n-1)) = (n-1) C_{n-1} .

Therefore $G_{e_k}(G\circ K_1, x) = (n-1)\, C_0 x^{n-1} + (n-1)\, C_1x^n +(n-1)\, C_2\, x^{n+1} +\ldots\ldots+ (n-1)\, C_{n-2}x^{2n-3} + (n-1)\, C_{n-1}x^{2(n-1)}$

$$= x^{n-1} (1+ (n-1)\, C_1x + (n-1)\, C_2\, x^2 +\ldots\ldots+ (n-1)\, C_{n-1}x^{n-1})$$

$$= x^{n-1} (1+x)^{n-1}$$

Case (ii): e_k is a non pendant edge. Suppose that G_1 is a edge fixed geodominating set of size m. When m = n, the edge fixed geodominating set with cardinality n is $\{u_1, u_2,\ldots u_n\}$. Therefore $g_{e_k}(G \circ K_1, n) = 1 = (n-2)\,C_o$. when m = n+1, the edge fixed geodominating set with cardinality n+1 is $\{u_1,u_2,\ldots u_n\} \bigcup \{v_j\}$, $1 \le j \le n-2$.

Therefore, $g_{e_k}(G \circ K_1, n+1) = (n-2)\,C_1$.

Similarly $g_{e_k}(G \circ K_1, n+2) = (n-2)\,C_2, \ldots, g_{e_k}(G \circ K_1, 2n-3) = (n-2)\,C_{(n-3)}$.

$$g_{e_k}(G \circ K_1, 2n-2) = (n-2)\,C_{(n-2)}.$$

Therefore, $G_{e_k}(G \circ K_1, x) = (n-2)\,C_0\, x^{n} + (n-2)\,C_1 x^{n+1} + (n-2)\,C_2\, x^{n+2} + \ldots + (n-2)\,C_{n-3} x^{2n-3} + (n-2)\,C_{n-2} x^{2n-2}$

$$= x^{n}\left(1 + (n-2)\,C_1 x + (n-2)\,C_2\, x^2 + \ldots + (n-2)\,C_{n-3} x^{n-3} + (n-2)\,C_{n-2} x^{n-2}\right)$$

$$= x^{n}(1+x)^{n-2}$$

Since $G \circ K_1$ has n pendant edges and p non pendant edges, we have,

$$G_t(G \circ K_1, x) = n x^{n-1}(1+x)^{n-1} + p x^{n}(1+x)^{n-2}$$

$$= x^{n-1}(1+x)^{n-2}[n(1+x) + px]$$

$$= (1+x)^{n-2}[n x^{n-1} + (n+p)\,x^{n}].$$

Here, we discuss about unimodality of the geodetic polynomial of $G_n \circ K_1$, where G_n denote a graph with *n* vertices.

Let us denotes $G \circ K_1$ simply by G^*

Theorem 4.3: For every $n \in N$,

$$\begin{cases} g_{e_k}(G_n*,\ n-1) = g_{e_k}(G_n*,\ 2(n-1)) = 1, \text{ if } e_k \text{ is a pendant edge} \\ g_{e_k}(G_n*,\ n) = g_{e_k}(G_n*,\ 2(n-1)) = 1 \text{ if } e_k \text{ is a non-pendant edge} \end{cases}$$

Proof: By lemma 4.2, If e_k is a pendant edge, then $g_{e_k}(G_n^*, n-1) = (n-1)\,C_0 = 1$, $g_{e_k}(G_n^*, 2(n-1)) = (n-1)\,C_{n-1} = 1$.

If e_k is a non pendant edge, then $g_{e_k}(G_n^*, n) = (n-2)\,C_0 = 1$, $g_{e_k}(G_n^*, 2(n-1)) = (n-2)\,C_{n-2} = 1$.

Theorem 4.4: (Unimodal theorem for $G \circ K_1$)

For every $n \in N$

(a) If e_k is a pendant edge, then

(i) $1 = g_{e_k}(G_{3n}^*, 3n-1) < g_{e_k}(G_{3n}^*, 3n) < \ldots < g_{e_k}(G_{3n}^*, 4n-2) > g_{e_k}(G_{3n}^*, 4n-1) > g_{e_k}(G_{3n}^*, 4n) \ldots > g_{e_k}(G_{3n}^*, 6n-3) > g_{e_k}(G_{3n}^*, 6n-1) = 1.$

(ii) $1 = g_{e_k}(G_{3n+1}^*, 3n) < g_{e_k}(G_{3n+1}^*, 3n+1) < \ldots < g_{e_k}(G_{3n+1}^*, 4n-1) > g_{e_k}(G_{3n+1}^*, 4n) > g_{e_k}(G_{3n+1}^*, 4n+1) > \ldots > g_{e_k}(G_{3n+1}^*, 6n-1) > g_{e_k}(G_{3n+1}^*, 6n) = 1.$

(iii) $1 = g_{e_k}(G_{3n+2}^*, 3n+1) < g_{e_k}(G_{3n+2}^*, 3n+2) < \ldots < g_{e_k}(G_{3n+2}^*, 4n) > g_{e_k}(G_{3n+2}^*, 4n+1) > g_{e_k}(G_{3n+2}^*, 4n+2) > \ldots > g_{e_k}(G_{3n+2}^*, 6n+1) > g_{e_k}(G_{3n+2}^*, 6n+2) = 1.$

(b) If e_k is a non pendant edge, then

(i) $1 = g_{e_k}(G_{3n}^*, 3n) < g_{e_k}(G_{3n}^*, 3n+1) < \ldots < g_{e_k}(G_{3n}^*, 4n-1) > g_{e_k}(G_{3n}^*, 4n+1) > \ldots > g_{e_k}(G_{3n}^*, 6n-3) > g_{e_k}(G_{3n}^*, 6n-2) = 1.$

(ii) $1 = g_{e_k}(G_{3n+1}^*, 3n+1) < g_{e_k}(G_{3n+1}^*, 3n+2) < \ldots < g_{e_k}(G_{3n+1}^*, 4n) > g_{e_k}(G_{3n+1}^*, 4n+1) > g_{e_k}(G_{3n+1}^*, 4n+2)$
$> \ldots > g_{e_k}(G_{3n+1}^*, 6n-1) > g_{e_k}(G_{3n+1}^*, 6n) = 1.$

(iii) $1 = g_{e_k}(G_{3n+2}^*, 3n+2) < g_{e_k}(G_{3n+2}^*, 3n+3) < \ldots < g_{e_k}(G_{3n+2}^*, 4n+2) > g_{e_k}(G_{3n+2}^*, 4n+3) \square > g_{e_k}(G_{3n+2}^*, 4n+4)$
$> \ldots > g_{e_k}(G_{3n+2}^*, 6n+1) > g_{e_k}(G_{3n+2}^*, 6n+2) = 1.$

Proof: Since the proofs of all parts are similar, we only prove the part (a) (i).

(i) Clearly $g_{e_k}(G_{3n}^*, 3n-1) = 1$ and $g_{e_k}(G_{3n}^*, 6n-2) = 1$.

we shall prove that $g_{e_k}(G_{3n}^*, i) < g_{e_k}(G_{3n}^*, i+1)$ for $3n-1 \le i \le 4n-2$ and $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ for $4n-1 \le i \le 6n-2$.

Suppose that $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ by theorem 4.2, we have

$$\binom{3n-1}{i-(3n-1)} < \binom{3n-1}{i+1-(3n-1)}$$

So we have $i \le 4n-2$. But $i \ge 3n-1$. Together we have $3n-1 \le i \le 4n-2$. Similarly, we have $g_{e_k}(G_{3n}^*, i) > g_{e_k}(G_{3n}^*, i+1)$ for $4n-1 \le i \le 6n-2$.

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