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RANDOM FIXED POINT OF MULTIVALUED OPERATERS IN POLISH SPACE

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ABSTRACT

In this paper we introduced some more results of random fixed point for one, two and four mappings in Polish Spaces by using rational contraction.

Keywords: *Polish Space, Random multivalued operator, Random fixed point.*

AMS Subject Classification: *47H10, 54H25.*

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1. INTRODUCTION:

Random fixed point theorem for contraction mappings in polish spaces and random fixed point theorems are of fundamental importance in probabilistic functional analysis. Their study was initiated by the prague school of probabilistic with work of spacek (15) and Hans (5,6).For example survey are refer to Bharucha-Reid (4), Itoh(8) proved several random fixed point theorems and gave their applications to random differential equations in Banach Spaces. Badshah and Gagrani proved existence of common Random fixed points of two Random multivalued operators on Polish spaces

The result of Hardy and Rogers [7] further extended by Wong[16], showing that two self mappings of S and T on a complete metric space satisfying a contractive type condition have a common fixed point. Recently, Beg and Azam [1] further extended it to the case of a pair of multivalued mappings satisfying a more general contractive type condition. In this section we gave a further generalized result of Beg and Shahzad [3] by using fractional inequality

2. PRELIMINARIES

Let (X, d) be a polish space that is a saparable complete metric space and (Ω, $\sum$) be a measurable space with $\sum$ sigma algebra of subsets of Ω. Let 2^X be the family of all non-empty subsets of X and CB(X) the family of all nonempty closed subsets of X. A mapping

T: $\Omega \to 2^X$ is called measurable if, for each open subset C of X,

$$T^{-1}(C) \in \sum, wh\ re\ T^{-1}(C)=\{\omega\in\Omega : T(\omega)\cap C\neq\varphi\}.$$

A mapping $\xi : \Omega \to X$ is called a measurable selector of a measurable mapping T: $\Omega \to 2^X$ if ξ is measurable and $\xi(\omega) \in T(\omega)$ for each $\omega \in \Omega$. A mapping f : $\Omega \times X \to X$ is said to be a random operator if, for each fixed x $\in$ X, f(., x) : $\Omega \to X$ is measurable. A measurable mapping $\xi : \Omega \to X$ is a random fixed point of a random multivalued operator T: $\Omega \times X \to CB(X)$ (f : $\Omega \times X \to X$) if $\xi(\omega) \in T(\omega,\xi(\omega))$ [$\xi(\omega) = f(\omega,\xi(\omega))$]for each $\omega \in \Omega$. Let T : $\Omega \times X \to CB(X)$ be a random operator and $\{\xi_n\}$ a sequence of measurable mappings $\xi_n : \Omega \to X$. The sequence $\{\xi_n\}$ is said to be asymptotically T-regular if $d(\xi_n(\omega), T(\omega,\xi_n(\omega))) \to 0.$

3. MAIN RESULT:

Theorem 3.1: Let X be Polish Space. Let T, S:$\Omega \times X \to CB(X)$ be two continuous random multivalued operators. If there exists measurable mappings a, b, c, d, e: $\Omega \to (0, 1)$, such that

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$$\begin{aligned}
H(S(\omega,x),T(\omega,y)) \le{}& a(\omega)\,d(x,y)+b(\omega)\big[d(x,S(\omega,x))+d(y,T(\omega,y))\big] \\
&+c(\omega)\big[d(x,T(\omega,y))+d(y,S(\omega,x))\big]+d(\omega)\big[d(y,S(\omega,x))+d(x,S(\omega,x))\big] \\
&+e(\omega)\big[d(y,T(\omega,y))+d(x,T(\omega,y))\big]+f(\omega)\left[\frac{d(x,S(\omega,x))\,d(y,T(\omega,y))}{d(x,y)}\right]
\end{aligned}$$

$$for\,each\, x,y\in X,\omega\in\Omega, and\;\; a,b,c,d,e\in R^{+}\; with\, a(\omega)+b(\omega)+c(\omega)+d(\omega)+e(\omega)<1,$$

Then there exists a common fixed point of S and T (Here H represents the hausdroff metric on CB(X) induced by the metric d)

Proof: Let ξ_0: $\Omega\to X$ be an arbitrary measurable mapping and choose a measurable mapping

$$\xi_1:\Omega\to X\;\; such\,that\; \xi_1(\omega)\in S(\omega,\xi_0(\omega))\;\; for\,each\,\omega\in\Omega. Then\;\; for\,each\,\omega\in\Omega$$

$$\begin{aligned}
H(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\le{}& a(\omega)\,d(\xi_0(\omega),\xi_1(\omega)) \\
&+b(\omega)\big[d(\xi_0(\omega),S(\omega,\xi_0(\omega)))+d(\xi_1(\omega),T(\omega,\xi_1(\omega)))\big] \\
&+c(\omega)\big[d(\xi_0(\omega),T(\omega,\xi_1(\omega)))+d(\xi_1(\omega),S(\omega,\xi_0(\omega)))\big] \\
&+d(\omega)\big[d(\xi_1(\omega),S(\omega,\xi_0(\omega)))+d(\xi_0(\omega),S(\omega,\xi_0(\omega)))\big] \\
&+e(\omega)\big[d(\xi_1(\omega),T(\omega,\xi_1(\omega)))+d(\xi_0(\omega),T(\omega,\xi_1(\omega)))\big] \\
&+f(\omega)\left[\frac{d(\xi_0(\omega),S(\omega,\xi_0(\omega)))d(\xi_1(\omega),T(\omega,\xi_1(\omega)))}{d(\xi_0(\omega),\xi_1(\omega))}\right]
\end{aligned}$$

It further implies, than there exists a measurable mapping

$$\xi_2:\Omega\to X\;\; such\,that\; \xi_2(\omega)\in T(\omega,\xi_1(\omega))\;\; for\,each\,\omega\in\Omega\, and$$

$$d(\xi_1(\omega),\xi_2(\omega))=H(S(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))$$

$$\begin{aligned}
d(\xi_1(w),\xi_2(w))\le{}& a(\omega)\,d(\xi_0(\omega),\xi_1(\omega)) \\
&+b(\omega)\big[d(\xi_0(\omega),S(\omega,\xi_0(\omega)))+d(\xi_1(\omega),T(\omega,\xi_1(\omega)))\big] \\
&+c(\omega)\big[d(\xi_0(\omega),T(\omega,\xi_1(\omega)))+d(\xi_1(\omega),S(\omega,\xi_0(\omega)))\big] \\
&+d(\omega)\big[d(\xi_1(\omega),S(\omega,\xi_0(\omega)))+d(\xi_0(\omega),S(\omega,\xi_0(\omega)))\big] \\
&+e(\omega)\big[d(\xi_1(\omega),T(\omega,\xi_1(\omega)))+d(\xi_0(\omega),T(\omega,\xi_1(\omega)))\big] \\
&+f(\omega)\left[\frac{d(\xi_0(\omega),S(\omega,\xi_0(\omega)))d(\xi_1(\omega),T(\omega,\xi_1(\omega)))}{d(\xi_0(\omega),\xi_1(\omega))}\right] \\
={}& a(\omega)\,d(\xi_0(\omega),\xi_1(\omega))+b(\omega)\,[\,d(\xi_0(\omega),\xi_1(\omega))+d(\xi_1(\omega),\xi_2(\omega))] \\
&+c(\omega)\big[d(\xi_0(\omega),\xi_2(\omega))+d(\xi_1(\omega),\xi_1(\omega))\big]+d(\omega)\big[d(\xi_1(\omega),\xi_1(\omega))+d(\xi_0(\omega),\xi_1(\omega))\big] \\
&+e(\omega)\big[d(\xi_1(\omega),\xi_2(\omega))+d(\xi_0(\omega),\xi_2(\omega)\big]+f(\omega)\left[\frac{d(\xi_0(\omega),\xi_1(\omega))d(\xi_1(\omega),\xi_2(\omega))}{d(\xi_0(\omega),\xi_1(\omega))}\right]
\end{aligned}$$

$$
\begin{aligned}
&= a(\omega)\, d(\xi_0(\omega), \xi_1(\omega)) + b(\omega)\, [\, d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega))] \\
&+ c(\omega)\left[d(\xi_0(\omega), \xi_1(\omega)) + d(\xi_1(\omega), \xi_2(\omega))\right] + d(\omega) d(\xi_0(\omega), \xi_1(\omega)) \\
&+ e(\omega)\left[d(\xi_1(\omega), \xi_2(\omega)) + d(\xi_0(\omega), \xi_1(\omega) + d(\xi_1(\omega), \xi_2(\omega))\right] \\
&+ f(\omega) d(\xi_1(\omega), \xi_2(\omega))
\end{aligned}
$$

$$
\begin{aligned}
&= [a(\omega) + b(\omega) + c(\omega) + d(\omega) + e(\omega)] d(\xi_0(\omega), \xi_1(\omega)) \\
&\qquad + [b(\omega) + c(\omega) + 2e(\omega) + f(\omega)] d(\xi_1(\omega), \xi_2(\omega))
\end{aligned}
$$

$$d(\xi_1(\omega), \xi_2(\omega)) \le \frac{a(\omega) + b(\omega) + c(\omega) + d(\omega) + e(\omega)}{1 - b(\omega) - c(\omega) - 2e(\omega) - f(\omega)}\; d(\xi_0(\omega), \xi_1(\omega))$$

$$d(\xi_1(\omega), \xi_2(\omega)) \le k\;\; d(\xi_0(\omega), \xi_1(\omega))$$

$$where\, k = \frac{a(\omega) + b(\omega) + c(\omega) + d(\omega) + e(\omega)}{1 - b(\omega) - c(\omega) - 2e(\omega) - f(\omega)} < 1\; because\;\; a(\omega) + b(\omega) + c(\omega) + d(\omega) + e(\omega) < 1$$

By Beg and Shahjad [3, Lemma 3] in the same manner there exists a measurable mapping $\xi_3 : \Omega \to X$ such that $\xi_3(\omega) \in S(\omega, \xi_2(\omega))$ for each $\omega \in \Omega$ and $d(\xi_2(\omega), \xi_3(\omega)) = H(T(\omega, \xi_1(\omega)), S(\omega, \xi_2(\omega))$

$$
\begin{aligned}
d(\xi_2(\omega), \xi_3(\omega)) \le\; & a(\omega)\, d(\xi_1(\omega), \xi_2(\omega)) + b(\omega)\left[d(\xi_1(\omega), T(\omega, \xi_1(\omega))) + d(\xi_2(\omega), S(\omega, \xi_2(\omega)))\right] \\
&+ c(\omega)\left[d(\xi_1(\omega), S(\omega, \xi_2(\omega))) + d(\xi_2(\omega), T(\omega, \xi_1(\omega)))\right] \\
&+ d(\omega)\left[d(\xi_2(\omega), T(\omega, \xi_1(\omega))) + d(\xi_1(\omega), T(\omega, \xi_1(\omega)))\right] \\
&+ e(\omega)\left[d(\xi_2(\omega), S(\omega, \xi_2(\omega))) + d(\xi_1(\omega), S(w, \xi_2(\omega)))\right] \\
&+ f(\omega)\left[\frac{d(\xi_1(\omega), T(\omega, \xi_1(\omega))) d(\xi_2(\omega), S(\omega, \xi_2(\omega)))}{d(\xi_1(\omega), \xi_2(\omega))}\right]
\end{aligned}
$$

$$
\begin{aligned}
&\le a(\omega)\, d(\xi_1(\omega), \xi_2(\omega)) + b(\omega)[\, d(\xi_1(\omega), \xi_2(\omega)) + d(\xi_2(\omega), \xi_3(\omega))] \\
&+ c(\omega)[d(\xi_1(\omega), \xi_3(\omega)) + d(\xi_2(\omega), \xi_2(\omega))] + d(\omega)[d(\xi_2(\omega), \xi_2(\omega)) + d(\xi_1(\omega), \xi_2(\omega))] \\
&+ e(\omega)[d(\xi_2(\omega), \xi_3(\omega)) + d(\xi_1(\omega), \xi_3(\omega))] + f(\omega)\left[\frac{d(\xi_1(\omega), \xi_2(\omega)) d(\xi_2(\omega), \xi_3(\omega))}{d(\xi_1(\omega), \xi_2(\omega))}\right]
\end{aligned}
$$

$$
\begin{aligned}
d(\xi_2(\omega), \xi_3(\omega)) &\le k\;\; d(\xi_1(\omega), \xi_2(\omega)) \\
&\le k^2 d(\xi_0(\omega), \xi_1(\omega))
\end{aligned}
$$

Similarly, proceeding in the same way, by induction we produce a sequence of measurable mapping $\xi_n : \Omega \to X$ such that for $\gamma > 0$ and any $\omega \in \Omega,\;\; \xi_{2\gamma+1}(\omega) \in S(\omega, \xi_{2\gamma}(\omega)), \xi_{2\gamma+2}(\omega) \in T(\omega, \xi_{2\gamma+1}(\omega))$

and $d(\xi_n(\omega), \xi_{n+2}(\omega)) \le k\;\; d(\xi_{n-1}(\omega), \xi_n(\omega)) .... \le k^n d(\xi_0(\omega), \xi_1(\omega))$

$$
\begin{aligned}
d(\xi_n(\omega), \xi_m(\omega)) &\le\; d(\xi_n(\omega), \xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) + ... + d(\xi_{m-1}(\omega), \xi_m(\omega)) \\
&\le [k^n + k^{n-1} + ... + k^{m-1}] d(\xi_0(\omega), \xi_1(\omega)) \\
&\le k^n [1 + k + k^2 + ... + k^{m-n-1}] d(\xi_0(\omega), \xi_1(\omega))
\end{aligned}
$$

$$d(\xi_n(\omega),\xi_m(\omega)) \le \frac{k^n}{1-k} d(\xi_0(\omega),\xi_1(\omega)) \to 0 \quad as \quad m,n \to \infty.$$

It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping ξ: $\Omega \to X$ such that $\xi_n(\omega) \to \xi(\omega)$ for each $\omega \in \Omega$.

It further implies that $\xi_{2\gamma+1}(\omega) \to \xi(\omega)$ and ξ $\xi_{2\gamma+2}(\omega) \to \xi(\omega)$.

Thus we have for any $\omega \in \Omega$,

$$\begin{aligned} d(\xi(\omega),S(\omega,\xi(\omega))) &\le d(\xi(\omega),\xi_{2\gamma+2}(\omega)) + d(\xi_{2\gamma+2}(\omega),S(\omega,\xi(\omega))) \\ &\le d(\xi(\omega),\xi_{2\gamma+2}(\omega)) + H(T(\omega,\xi_{2\gamma+1}(\omega)),S(\omega,\xi(\omega)) \end{aligned}$$

$$\begin{aligned} d(\xi(\omega),S(\omega,\xi(\omega))) \le{} & d(\xi(\omega),\xi_{2\gamma+2}(\omega)) + a(\omega)\, d(\xi_{2\gamma+1}(\omega),\xi(\omega)) \\ & + b(\omega)\left[d(\xi_{2\gamma+1}(\omega),S(\omega,\xi_{2\gamma+1}(\omega))) + d(\xi(\omega),T(\omega,\xi(\omega)))\right] \\ & + c(\omega)\left[d(\xi_{2\gamma+1}(\omega),T(\omega,\xi(\omega))) + d(\xi(\omega),S(\omega,\xi_{2\gamma+1}(\omega)))\right] \\ & + d(w)\left[d(\xi(\omega),S(\omega,\xi_{2\gamma+1}(\omega))) + d(\xi_{2\gamma+1}(\omega),S(\omega,\xi_{2\gamma+1}(\omega)))\right] \\ & + e(w)\left[d(\xi(\omega),T(\omega,\xi(\omega))) + d(\xi_{2\gamma+1}(\omega),T(\omega,\xi(\omega)))\right] \\ & + f(\omega)\left[\frac{d(\xi_{2\gamma+1}(\omega),S(\omega,\xi_{2\gamma+1}(\omega)))d(\xi(\omega),T(\omega,\xi(\omega)))}{d(\xi_{2\gamma+1}(\omega),\xi(\omega))}\right] \end{aligned}$$

Letting $\gamma \to \infty$, *we have*

$$d(\xi(\omega),S(\omega,\xi(\omega))) \le 0$$

Hence $\xi(\omega) \in S(\omega,\xi(\omega))$ *for* $\omega \in \Omega$.

Similarly, for any $\omega \in \Omega$,

$$d(\xi(\omega),T(\omega,\xi(\omega))) \le d(\xi(\omega),\xi_{2\gamma+1}(\omega)) + H(S(\omega,\xi_{2\gamma+1}(\omega)),T(\omega,\xi(\omega))$$

$$d(\xi(\omega),T(\omega,\xi(\omega))) \le 0$$

Therefore $\xi(\omega) \in T(\omega,\xi(\omega))$ *for* $\omega \in \Omega$.

Theorem 3.2: Let X be Polish Space. Let S_i, T_j: $\Omega \times X \to CB(X)$ be sequence of random multivalued operators. If there exists measurable mappings *a, b, c, d, e:* $\Omega \to (0, 1)$, such that

$$\begin{aligned} H(S_i(\omega,x),T_j(\omega,y)) \le{} & a(\omega)\, d(x,y) + b(\omega)\left[d(x,S_i(\omega,x)) + d(y,T_j(\omega,y))\right] \\ & + c(\omega)\left[d(x,T_j(\omega,y)) + d(y,S_i(\omega,x))\right] + d(\omega)\left[d(y,S_i(\omega,x)) + d(x,S_i(\omega,x))\right] \\ & + e(\omega)\left[d(y,T_j(\omega,y)) + d(x,T_j(\omega,y))\right] + f(\omega)\left[\frac{\left[d(x,S_i(\omega,x))\, d(y,T_j(\omega,y))\right]}{d(x,y)}\right] \end{aligned}$$

for each $x, y \in X$, $\omega \in \Omega$ *and* $a,b,c,d,e \in R^+$ *with* $a(\omega)+b(\omega)+c(\omega)+d(\omega)+e(\omega) < 1, i, j = 1,2...n...\infty$

Then there exists a common fixed point of S and T (Here H represents the hausdroff metric on CB(X) induced by the metric d)

Proof: Similar to the proof of the theorem 4.9 putting S =S_i *and* $T = T_j$.

Theorem 3.3: Let X be Polish Space. Let $T: \Omega \times X \to CB(X)$ be a continuous random multivalued operator. If there exists measurable mappings $a, b, c, d, e: \Omega \to (0, 1)$, such that

$$\begin{aligned} H(T(\omega,x),T(\omega,y)) \le{}& a(\omega)\, d(x,y) + b(\omega)\big[d(x,T(\omega,x)) + d(y,T(\omega,y))\big] \\ &+ c(\omega)\big[d(x,T(\omega,y)) + d(y,T(\omega,x))\big] + d(\omega)\big[d(y,T(\omega,x)) + d(x,T(\omega,x))\big] \\ &+ e(\omega)\big[d(y,T(\omega,y)) + d(x,T(\omega,y))\big] + f(\omega)\left[\frac{d(x,T(\omega,x))\, d(y,T(\omega,y))}{d(x,y)}\right] \end{aligned}$$

$$for\,each\, x, y \in X, \omega \in \Omega \text{ and } a,b,c,d,e \in R^+ \, with\, a(\omega)+b(\omega)+c(\omega)+d(\omega)+e(\omega) < 1,$$

Then there exists a common fixed point of *T* (Here H represents the hausdroff metric on CB(X) induced by the metric d)

Proof: Similar to the proof of the theorem 4.9 putting S = *T*.

Theorem 3.4: Let X be Polish Space. Let $T_j: \Omega \times X \to CB(X)$ be a sequence of continuous random multivalued operator. If there exists measurable mappings $a, b, c, d, e: \Omega \to (0, 1)$, such that

$$\begin{aligned} H(T_i(\omega,x),T_j(\omega,y)) \le{}& a(\omega)\, d(x,y) + b(\omega)\big[d(x,T_i(\omega,x)) + d(y,T_j(\omega,y))\big] \\ &+ c(\omega)\big[d(x,T_j(\omega,y)) + d(y,T_i(\omega,x))\big] + d(\omega)\big[d(y,T_i(\omega,x)) + d(x,S_i(\omega,x))\big] \\ &+ e(\omega)\big[d(y,T_j(\omega,y)) + d(x,T_j(\omega,y))\big] + f(\omega)\left[\frac{\big[d(x,T_i(\omega,x))\, d(y,T_j(\omega,y))\big]}{d(x,y)}\right] \end{aligned}$$

$$for\,each\, x, y \in X, \omega \in \Omega \text{ and } a,b,c,d,e \in R^+ \, with\, a(\omega)+b(\omega)+c(\omega)+d(\omega)+e(\omega) < 1, i, j = 1,2...n...\infty$$

Then there exists a common fixed point of *T* (Here H represents the hausdroff metric on CB(X) induced by the metric d)

Proof: Similar to the proof of the theorem 4.10 putting Si = *Tj*.

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