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## EQUILIBRIUM STRUCTURE OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED PRASAD MODEL INCLUDING MASS VARIATION INSIDE THE STAR

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#### Abstract

In this paper we propose suitable modifications in the concept of Roche equipotentials to account for the effect of mass variation inside the star on its equipotential surfaces and use this in conjunction with Kippenhahn and Thomas ${ }^{4}$ approach, in a manner earlier used by Mohan et. al.[12], to incorporate the effects of differentially rotating and tidally distorted stellar models of stars using the law of the type $\omega=b_{1}+b_{2} s^{2}$ where $b_{1}, b_{2}$ are numerical constants and s is the distance of rotating fluid element from the axis of rotation. The proposed method has been used to compute the structure parameters of differentially rotating and tidally distorted Prasad model of the star.


Keywords: Roche equipotentials, equilibrium structure of stars, Prasad model, rotating stars, stars in binary systems

## 1. INTRODUCTION

Observations show that most of the binary stars are rotating as well as revolving around their common centers of mass. Rotational forces as well as tidal distortion of the companion star have their effect on the equilibrium structure, shape and other observable physical parameters of binary stars. Prasad[13] introduced a model (called Prasad model thereafter) in which density $\rho$ inside the star varies according to the law $\rho=\rho_{c}\left(1-x^{2}\right), \rho_{c}$ being the density of fluid at the centre and x a non-dimensional measure of the distance of a fluid element from its centre. Since then several authors such as Gurm[3], Prasad and Mohan[14] , Agarwal[1], Sharma[15] etc. have addressed themselves to these types of problems.

The mathematical problem of determining the effects of rotation and tidal forces on the equilibrium structure of a star is quite complex. Approximate methods have therefore often been used in literature to study such problems. In some such approximations Kopal [5,6] ; Mohan and Singh[7] ; Mohan and Saxena[9]; Mohan et al[10.11], the actual equipotential surfaces of a rotationally and tidally distorted star are approximated by equivalent Roche equipotentials, assuming both stars in the binary system to be point masses. This approximation is valid for highly centrally condensed types of stars. However in the case of stars in which the central condensation is not too large, this approximation is not very justified. Therefore, it will be of interest to analyze the effect of incorporating suitable modifications in the Roche equipotentials to account for the mass distribution inside the star. Baur [2] recently modified the Roche potentials to account for mechanical effects of the mutual irradiation of the binary components. In a somewhat similar manner we have suitably modified the Roche equipotentials to account for the mass distribution inside the primary component (e.g. Mohan et al.[12].

The paper is organized as follows: expressions for the modified Roche equipotentials of a rotationally and tidally distorted star are obtained in Section 2. This modified concept of Roche equipotentials is next used in Section 3 to obtain the equilibrium structure of a differentially and tidally distorted Prasad model of a star. Computational results for the inner structure, shapes and certain other physical parameters of differentially and tidally distorted Prasad model including mass variation inside the star is next obtained in Section 4 and compared with corresponding results earlier obtained by some other authors.

## 2. MODIFIED ROCHE EQUIPOTENTIALS

In order to investigate the equilibrium structures of binary stars, the concept of Roche equipotentials has been frequently used in literature Kopal[5]. However, while computing the Roche equipotential surfaces, the whole mass of the stars (the primary as well as the secondary) is assumed to be concentrated at their centers. This approximation,

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though reasonably valid for highly centrally condensed stars, is not reasonably true for stars, such as main sequence and pre-main sequence stars which are not very highly centrally condensed. The concept of Roche equipotentials, therefore, needs to be suitably modified to incorporate the effects of mass distribution inside the star so that it can provide a better approximation for the structure of a differentially and tidally distorted star which is not very highly centrally condensed.

In this section we suitably modify the mathematical expression for Roche equipotentials to reasonably account for the mass distribution inside the primary star (in whose inner structure we are primarily interested) assuming as earlier that the secondary star is still a point mass.

Let $M_{0}$ and $M_{1}$ be the masses of the primary and the secondary components of a binary system of stars in which the primary is assumed to be much massive than the secondary $\left(M_{0} \gg M_{1}\right)$. Let $M_{0}(r)$ represent the mass interior to a sphere of radius $r$ inside the primary component. Let $D$ be the mutual separation between the centers of the two stars. Further suppose that the position of the two components of this binary system is referred to a rectangular system of Cartesian coordinates having the origin at the center of gravity of the primary of mass $M_{0}$, the $x$-axis along the line joining the centers of the two components of the binary, and $Z$ - axis perpendicular to the plane of the orbit of the two components (Fig 1).


FIG. 1 AXES OF REFERENCE OF ROTATING BINARY SYSTEM

Then the total potential $\psi$ due to the gravitational, rotational and other disturbing forces acting at an arbitrary point $P(x, y, z)$ distant $r$ from the centre of the primary and $r_{1}$ from the centre of the secondary may be expressed as

$$
\begin{equation*}
\psi=G \frac{M^{*}}{r}+G \frac{M_{1}}{r_{1}}+\frac{1}{2} \Omega^{2}\left(\left(x-\frac{M_{1} D}{M_{0}+M_{1}}\right)^{2}+y^{2}\right) \tag{1}
\end{equation*}
$$

where $M^{*}=\left\{\begin{array}{l}M_{0} \text { if } \quad r>R \\ M_{0}(r) \text { if } r<R\end{array}\right.$. Here $\Omega$ is the angular velocity of rotation and $G$ the gravitational constant. The three terms on the right hand side of (1) are, respectively, the potential arising from the mass of the primary, the disturbing potential of its companion of mass $M_{1}$ and the potential arising from the centrifugal force. In the first term on the right hand side of (1) $M_{0}$ normally used in the definition of Roche potential has been replaced by $M^{*}$ to account for the mass distribution inside the primary. This has been done keeping in view the fact that where as in the case of a sphere the gravitational potential at a point outside the sphere is $\frac{G M}{r}$, the gravitational potential at a point inside the sphere is $\frac{G M(r)}{r}, M(r)$ being the mass contained inside a sphere of radius $r$ concentric with the centre of stars of mass $M$.

Equation (1) in nondimensional form can be expressed as

$$
\begin{aligned}
& \psi=\frac{z}{r}+\frac{q}{r_{1}}+\frac{1}{2} b_{1}^{2}\left\{r^{2}\left(1-v^{2}\right)-\frac{2 q r \lambda}{1+q}+\frac{q^{2}}{(1+q)^{2}}\right\}+\frac{b_{1} b_{2}}{2}\left\{r^{2}\left(1-v^{2}\right)-\frac{2 q r \lambda}{1+q}+\frac{q^{2}}{(1+q)^{2}}\right\}^{2} \\
& +\frac{b_{2}^{2}}{6}\left\{r^{2}\left(1-v^{2}\right)-\frac{2 q r \lambda}{1+q}+\frac{q^{2}}{(1+q)^{2}}\right\}^{3} \\
& \text { where } \quad \psi^{*}=\frac{D \psi}{G M_{0}}-\frac{M_{1}^{2}}{2 M_{0}\left(M_{0}+M_{1}\right)}, \text { and } \\
& \qquad z=\left\{\begin{array}{ll}
\frac{M_{0}(r)}{M_{0}}, & \text { if } r<R \\
1, & \text { if } r>R
\end{array} \quad \text { Also } r^{*}=\frac{r}{D}\right. \text { is a }
\end{aligned}
$$

nondimensional measure of the distance and $\lambda=\sin \theta \cos \phi, \mu=\sin \theta \sin \phi, v=\cos \theta, r, \theta, \phi$ being the spherical polar coordinates of the point $P$. Obviously, $z$ is a nondimensional parameter, which becomes zero at the center of the primary $M_{0}$ and one at the points on and outside the surface of the primary. It will have a value between 0 and 1 at points inside the primary. Also $q=\frac{M_{1}}{M_{0}}$ is a nondimensional parameter representing the ratio of the mass of the secondary over the mass of the primary ( $q<1$ ), and $2 n$ represents the square of the normalized angular velocity $\Omega$. In equation (2) if $q=0$ it reduces to the potential of a rotating spherical model rotating with angular velocity $\Omega$ and if $n=0$, then it reduces to the potential of a spherical model distorted only by the tidal effects of the companion. For a binary system in synchronous rotation, $\Omega^{2}=\frac{G\left(M_{0}+M_{1}\right)}{D^{3}}$ this in terms of the nondimensional variables as defined above, becomes $n=\frac{q+1}{2}$.

Equipotential surfaces represented by $\psi^{*}=$ constant as given in (2) are the modified Roche equipotential surfaces of the primary component of a rotationally and tidally distorted binary system which reasonably account for the mass distribution in the interior of the primary. This, however, modifies potential at the points inside the primary only. On substituting $z=1$ in (2) or $M_{0}(r)=M_{0}$ in (1), it reduces to the expression for the Roche equipotential which has been earlier used by Kopal ${ }^{5}$ and other authors.

Following Kopal ${ }^{5}$ it can be shown that the values of $r, \theta, \phi$ on the surfaces of the Roche equipotentials as given by (2) are connected through the relation

$$
\begin{align*}
r= & r_{0} D\left[1+\left(\frac{q P_{2}}{z^{2}}+\frac{b_{1}^{2}}{2 z} x\right) r_{0}^{3}+\frac{q P_{3}}{z^{2}} r_{0}^{4}+\left(\frac{q P_{4}}{z^{2}}+\frac{b_{1} b_{2}}{2 z} x^{2}+\frac{5}{2 z} \lambda b_{1}^{2} q x\right) r_{0}^{5}+\right. \\
& +\left(\frac{q P_{5}}{z^{2}}+\frac{3 q^{2} P_{2}^{2}}{z^{2}}+\frac{3 q P_{2} b_{1}^{2}}{z^{2}} x\right) r_{0}^{6}+\left(\frac{q P_{6}}{z^{2}}+\frac{b_{2}^{2}}{6 z} x^{3}+\frac{7}{4 z^{2}} \lambda b_{1}^{2} q x^{2}+\frac{7 q^{2} P_{2} P_{3}}{z^{2}}\right) r_{0}^{7}+ \\
& +\left(\frac{q P_{7}}{z^{2}}+\frac{8 q^{2} P_{2} P_{4}}{z^{2}}+\frac{4 q P_{4} b_{1}^{2}}{z^{2}} x+\frac{4 q^{2} P_{3}^{2}}{z^{2}}\right) r_{0}^{8}+  \tag{3}\\
& +\left(\frac{q P_{8}}{z^{2}}+\frac{3 \lambda b_{2}^{2} q}{2 z^{2}} x^{3}+\frac{9 q^{2} P_{3} P_{4}}{z^{2}}+\frac{9 q 3_{3} B b_{1}^{2}}{4 z^{2}} x^{2}+\frac{9 q^{2} P_{2} P_{5}}{z^{2}}\right) r_{0}^{9}+ \\
& \left.+\left(\frac{q P_{9}}{z^{2}}+\frac{10 q^{2} P_{2} P_{6}}{z^{2}}+\frac{10 q^{2} P_{3} P_{5}}{z^{2}}+\frac{5 q b_{2}^{2} P_{2}}{3 z^{2}} x^{3}+\frac{5 q^{2} P_{4}^{2}}{z^{2}}\right) r_{0}^{10}+\ldots\right]
\end{align*}
$$

where $r_{0}=\frac{z}{\psi-q}, \quad a_{0}=\frac{q P_{2}}{z}+\frac{n\left(1-v^{2}\right)}{z}, \quad P_{j}=P_{j}(\lambda)$ Legendre polynomials and terms up to second order of smallness in $n$ and $q$ are retained. This relation can be used to obtain the shapes of Roche equipotential $\psi=$ constant. Whereas on account of inclusion of mass distribution inside the primary, these will get modified at points inside the primary due to the presence of $Z$, outside the primary (where $Z=1$ ), these will be same as earlier obtained by Kopal[5]. Following Kopal[5] and Mohan et. al.[12], modified expressions for the volume enclosed $V_{\psi}$, and the surface area $S_{\psi}$ of an equipotential surface can be obtained if desired in series form.

## 3. EQUILIBRIUM STRUCTURES OF DIFFERENTIALLY AND TIDALLY DISTORTED PRASAD MODEL

If we assume that the primary component of binary system behaves as Prasad model and rotating about its axis then its equilibrium structure will be distorted by rotation as well as the tidal effects of the companion. In order to determine the equilibrium structure of this rotationally and tidally distorted stellar model we may follow the approach of Mohan and Saxena[9],it is assumed that the rotational velocity and the mass of the secondary as compared to the primary are suitably small.

Let $r_{\psi}$ denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface $\psi=$ constant of this differentially and tidally distorted Prasad model. Also, let $R_{\psi}$ be the value of $r_{\psi}$ on the equipotentials surface $\psi=$ constant of this differentially and tidally distorted model. Following the approach as discussed in Seema[16] $r_{\psi}$ and $R_{\psi}$ are given as.

$$
\begin{gather*}
r_{\psi}=D r_{0}\left[1+\frac{2 b_{1}^{2} r_{0}^{3}}{3 z}+\frac{4 b_{1} b_{2} r_{0}^{5}}{15 z}+\frac{4 q^{2} r_{0}^{6}}{4 z^{2}} \cdot+\frac{8 b_{2}^{2} r_{0}^{7}}{15 z}+\frac{5 q^{2} r_{0}^{8}}{7 z^{2}}+\frac{2 q^{2} r_{0}^{10}}{3 z^{2}}\right]  \tag{4}\\
R_{\psi}=D r_{0 s}\left[1+\frac{4 n r_{0 s}^{3}}{3 z}+\left(\frac{4 q^{2} r_{0}^{6}}{4 z^{2}}+\frac{8 n q}{15 z^{2}}+\frac{76 n^{2}}{45 z^{2}}\right) r_{0 s}^{6}+\frac{5 q^{2} r_{0 s}^{8}}{7 z^{2}}+\frac{2 q^{2} r_{0 s}^{10}}{3 z^{2}}+\ldots\right] \tag{5}
\end{gather*}
$$

and

$$
\mathrm{Z}=\frac{5}{2}\left(\frac{r_{\psi}}{R_{\psi}}\right)^{3}-\frac{3}{2}\left(\frac{r_{\psi}}{R_{\psi}}\right)^{5}
$$

where $\quad r_{0}=\frac{Z}{\psi-q}$

Further let $\rho_{\psi}$ denote the value of density on an equipotentials $\psi=$ constant. The density distribution law of differentially and tidally distorted Prasad model is given as

$$
\begin{equation*}
\rho_{\psi}=\rho_{c}\left(1-\frac{r_{\psi}^{2}}{R_{\psi}^{2}}\right) \tag{7}
\end{equation*}
$$

On substituting the value of $r_{\psi}$ and $R_{\psi}$ from equation (4) and (5) in equation (7) we get

$$
\begin{equation*}
\rho_{\psi}=\rho_{c}\left[1-\frac{D^{2} r_{0}^{2}}{R_{\psi}{ }^{2}}\left\{1+\frac{2 b_{1}^{2} r_{0}^{3}}{3 z}+\frac{8 b_{1} b_{2} r_{0}^{5}}{15 z}+\frac{8 q^{2} r_{0}^{6}}{5 z^{2}}+\frac{6 b_{2}^{2} r_{0}^{7}}{105 z}+\frac{10 q^{2} r_{0}^{8}}{7 z^{2}}+\frac{4 q^{2} r_{0}^{10}}{3 z^{2}}+\ldots\right\}\right] \tag{8}
\end{equation*}
$$

On substituting value of $\rho_{\psi}$ from (8) and using the approach used by Saxena[9] integrating w.r.t. $r_{0}$ and using the fact that $M_{\psi}=0$ at center $r_{0}=0$ we get

$$
\begin{align*}
& M_{\psi}=\frac{4 \pi \rho_{c} D^{3} r_{0}{ }^{3}}{3}\left[1-\frac{3 D^{2}}{5 R_{\psi}{ }^{2}} r_{0}{ }^{2}+\frac{b_{1}{ }^{2} r_{0}{ }^{3}}{z}+\frac{4 b_{1} b_{2} r_{0}{ }^{5}}{5 z^{2}}-\frac{b_{1}{ }^{2} R^{2} r_{0}{ }^{5}}{z R_{\psi}{ }^{2}}+\frac{12 q^{2} r_{0}{ }^{6}}{5 z^{2}}\right. \\
& \left.+\frac{16 b_{2}{ }^{2} r_{0}{ }^{7}}{70 z}-\frac{4 b_{1} b_{2} R^{2} r_{0}{ }^{7}}{5 z R_{\psi}{ }^{2}}+\left\{\frac{15 q^{2}}{7 z^{2}}-\frac{12 q^{2} R^{2}}{5 z^{2} R_{\psi}{ }^{2}}\right\} r_{0}{ }^{8}+\left(\frac{2 q^{2}}{5 z^{2}}-\frac{15 q^{2} R^{2}}{7 R_{\psi}{ }^{2}}\right) r_{0}{ }^{10}+\ldots\right] \tag{9}
\end{align*}
$$

Similarly on substituting $\rho_{\psi}$ from (8) and $M_{\psi}$ from (9) and using the approach used by Saxena[9] and integrating with respect to $r_{0}$ we get

$$
\begin{align*}
P_{\psi}=\frac{2 \pi z G \rho_{c}{ }^{2} D^{2}}{3} & {\left[K-r_{0}{ }^{2}+\frac{4 D^{2} r_{0}{ }^{4}}{5 R_{\psi}{ }^{2}}-\frac{2 b_{1}{ }^{2} r_{0}{ }^{5}}{5 z}-\frac{D^{4} r_{0}{ }^{6}}{5 R_{\psi}{ }^{2}}-\frac{8 b_{1} b_{2} r_{0}{ }^{7}}{35 z}+\frac{16 D^{2} b_{1}{ }^{2} r_{0}{ }^{7}}{21 z R_{\psi}{ }^{2}}+\right.} \\
& +\frac{q^{2} r_{0}{ }^{8}}{2 z^{2}}-\frac{16 b_{2}{ }^{2} r_{0}{ }^{9}}{315 z}+\frac{64 b_{1} b_{2} D^{2} r_{0}{ }^{9}}{135 z R_{\psi}{ }^{2}}-\frac{14 D^{4} b_{1}{ }^{2} r_{0}{ }^{9}}{45 z R_{\psi}{ }^{4}}  \tag{10}\\
& \left.-\frac{3 q^{2} r_{0}{ }^{10}}{10 z^{2}}+\frac{144 D^{2} q^{2} r_{0}{ }^{10}}{125 z^{2} R_{\psi}{ }^{2}}+. .\right]
\end{align*}
$$

where $K$ is a constant of integration whose value may be calculated by using boundary condition say $P_{\psi /}=0$ $r_{0}=r_{0 \mathrm{~s}}$. This yield

$$
\begin{align*}
K=r_{0 s}{ }^{2} & -\frac{4 D^{2} r_{0 s}{ }^{4}}{5 R_{\psi}{ }^{2}}+\frac{2 b_{1}{ }^{2} r_{0 s}{ }^{5}}{5 z}+\frac{D^{4} r_{0 s}{ }^{6}}{5 R_{\psi}{ }^{2}}+\frac{8 b_{1} b_{2} r_{0 s}{ }^{7}}{35 z}-\frac{16 D^{2} b_{1}{ }^{2} r_{0 s}{ }^{7}}{21 z R_{\psi}{ }^{2}}+\frac{q^{2} r_{0 s}{ }^{8}}{2 z^{2}} \\
& +\frac{16 b_{2}{ }^{2} r_{0 s}{ }^{9}}{315 z}-\frac{64 b_{1} b_{2} D^{2} r_{0 s}{ }^{9}}{135 z R_{\psi}{ }^{2}}+\frac{14 D^{4} b_{1}{ }^{2} r_{0 s}{ }^{9}}{45 z R_{\psi}{ }^{4}}+\frac{3 q^{2} r_{0 s}{ }^{10}}{10 z^{2}}-\frac{144 D^{2} q^{2} r_{0 s}{ }^{10}}{125 z^{2} R_{\psi}{ }^{2}}+\ldots \tag{11}
\end{align*}
$$

Similarly the volume $V_{\psi}$, surface area $S_{\psi}, g^{-}$and $g^{-1}$ of rotationally and tidally distorted Prasad model are obtained as

$$
\begin{align*}
& V_{\psi}=\frac{4 \pi r_{0}^{3}}{3}\left[1+\frac{b_{1}^{2} r_{0}^{3}}{z}+\frac{4 b_{1} b_{2} r_{0}^{5}}{5 z}+\frac{12 q^{2}}{5 z^{2}}+\frac{8 b_{2}^{2} r_{0}^{7}}{35 z}+\frac{15 q^{2} r_{0}^{8}}{7 z^{2}}+\frac{2 q^{2} r_{0}^{10}}{z^{2}}+\ldots\right]  \tag{12}\\
& S_{\psi}=4 \pi r_{0}^{2}\left[1+\frac{2 b_{1}^{2} r_{0}^{3}}{3 z}+\frac{8 b_{1} b_{2} r_{0}^{5}}{15 z}+\frac{7 q^{2} r_{0}^{6}}{5 z^{2}}+\frac{16 b_{2}^{2} r_{0}^{7}}{105 z}+\frac{9 q^{2} r_{0}^{8}}{7 z^{2}}+\frac{11 q^{2} r_{0}^{10}}{9 z^{2}}+\ldots\right]  \tag{13}\\
& \bar{g}=\frac{z G M_{\psi}}{r_{0}^{2}}\left[1-\frac{4 b_{1}^{2} r_{0}^{3}}{3 z}-\frac{8 b_{1} b_{2} r_{0}^{5}}{5 z}-\frac{3 q^{2} r_{0}^{6}}{z^{2}}-\frac{64 b_{2}^{2} r_{0}^{7}}{105 z}-\frac{51 q^{2} r_{0}^{8}}{14 z^{2}}-\frac{13 q^{2} r_{0}^{10}}{3 z^{2}}+\ldots\right]  \tag{14}\\
& \bar{g}^{-1}=\frac{r_{0}^{2}}{z G M_{\psi}}\left[1+\frac{4 b_{1}^{2} r_{0}^{3}}{3 z}+\frac{8 b_{1} b_{2} r_{0}^{5}}{5 z}+\frac{31 q^{2} r_{0}^{6}}{5 z^{2}}+\frac{64 b_{2}^{2} r_{0}^{7}}{105 z}+\frac{101 q^{2} r_{0}^{8}}{14 z^{2}}+\frac{75 q^{2} r_{0}^{10}}{9 z^{2}}+\ldots\right] \tag{15}
\end{align*}
$$

## 4. NUMERICAL EVALUATION OF STRUCTURE FOR PRASAD MODEL AND ANALYSIS OF RESULTS

For a better appreciation of the effects of differentially and tidally distorted stars the values of density, mass and pressure at various points inside the star we have used equations (8), (9) and (10) to numerically compute the values of $\rho_{\psi}, M_{\psi}$ and $P_{\psi}$ at various points inside Prasad model.

The results presented in Tables 1(A-D) give the values of certain structures parameters and related observable quantities of undistorted, rotationally distorted, tidally distorted and rotationally and tidally distorted Prasad model for
$\psi=5.0$. Results show that with the modification of expression for potential to account for mass variation inside the star on its equipotential surfaces, our results show marginal changes but no significant trend is observed.

Table 1 A:
Structure Parameters of Uniformly Distorted Prasad Model For $\psi_{s}^{*}=5, \mathrm{~b}_{1}=0, \mathrm{~b}_{2}=0, q=0.1$

| $\mathbf{x}$ | $V_{\psi}$ | $s_{\psi}$ | $\rho_{\psi}$ | $M_{\psi}$ | $P_{\psi}$ | $\sigma$ | $\varepsilon$ | $T_{e} / T_{P}$ | $L_{e} / L_{P}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.00001 | 0.00042 | 0.99000 | 0.00248 | 0.01313 | 0.00035 | 0.00035 | 0.14283 | 0.99825 |
| 0.2 | 0.00006 | 0.00166 | 0.96000 | 0.01952 | 0.01499 | 0.00036 | 0.00036 | 0.20199 | 0.99817 |
| 0.3 | 0.00023 | 0.00374 | 0.91000 | 0.06385 | 0.01317 | 0.00038 | 0.00038 | 0.24738 | 0.99806 |
| 0.4 | 0.00054 | 0.00666 | 0.84000 | 0.14464 | 0.01081 | 0.00041 | 0.00041 | 0.28565 | 0.99791 |
| 0.5 | 0.00106 | 0.01041 | 0.75000 | 0.26563 | 0.00819 | 0.00044 | 0.00044 | 0.31936 | 0.99772 |
| 0.6 | 0.00183 | 0.01499 | 0.64000 | 0.42336 | 0.00559 | 0.00049 | 0.00049 | 0.34983 | 0.99745 |
| 0.7 | 0.00291 | 0.20408 | 0.51000 | 0.60536 | 0.00327 | 0.00056 | 0.00056 | 0.37784 | 0.99709 |
| 0.8 | 0.00435 | 0.02665 | 0.36000 | 0.78848 | 0.00149 | 0.00066 | 0.00066 | 0.40391 | 0.99657 |
| 0.9 | 0.00622 | 0.03373 | 0.18999 | 0.93676 | 0.00036 | 0.00081 | 0.00081 | 0.42837 | 0.99576 |
| 1.0 | 0.00800 | 0.41692 | 0.0000 | 1.0000 | 0.00000 | 0.00100 | 0.00100 | 0.45148 | 0.99438 |

Table 1 B
Structure Parameters of Rotationally and Tidally Distorted Prasad Model $\psi_{s}^{*}=5, \mathrm{~b}_{1}=0.3162, \mathrm{~b}_{2}=0, \mathrm{q}=0.1$

| x | $V_{\psi}$ | $s_{\psi}$ | $\rho_{\psi}$ | $M_{\psi}$ | $P_{\psi}$ | $\sigma$ | $\varepsilon$ | $T_{e} T_{p}$ | $L_{e} L_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.00001 | 0.00041 | 0.99000 | 0.00284 | 0.01305 | 0.00052 | 0.00052 | 0.14282 | 0.99733 |
| 0.2 | 0.00006 | 0.00166 | 0.96001 | 0.01951 | 0.01499 | 0.00053 | 0.00053 | 0.20197 | 0.99729 |
| 0.3 | 0.00023 | 0.00374 | 0.91002 | 0.06382 | 0.01317 | 0.00056 | 0.00056 | 0.24736 | 0.99715 |
| 0.4 | 0.00054 | 0.00666 | 0.84005 | 0.14457 | 0.01081 | 0.00059 | 0.00059 | 0.28562 | 0.99697 |
| 0.5 | 0.00106 | 0.01041 | 0.75007 | 0.26551 | 0.00820 | 0.00064 | 0.00064 | 0.31933 | 0.99671 |
| 0.6 | 0.00183 | 0.01499 | 0.64010 | 0.42321 | 0.00559 | 0.00071 | 0.00071 | 0.34979 | 0.99637 |
| 0.7 | 0.00291 | 0.02041 | 0.51012 | 0.60523 | 0.00327 | 0.00080 | 0.00080 | 0.37780 | 0.99589 |
| 0.8 | 0.00435 | 0.02666 | 0.36012 | 0.78834 | 0.00146 | 0.00093 | 0.00093 | 0.40385 | 0.99519 |
| 0.9 | 0.00620 | 0.03375 | 0.19010 | 0.93670 | 0.00036 | 0.00114 | 0.00114 | 0.42830 | 0.99411 |
| 1.0 | 0.00850 | 0.41673 | 0.00000 | 1.00000 | 0.00000 | 0.001499 | 0.001497 | 0.451384 | 0.99226 |

Table C
Structures Parameters of Rotationally and Tidally Distorted Prasad Model $\psi_{s}^{*}=5, \mathrm{~b}_{1}=0, \mathrm{~b}_{2}=0.3162, \mathrm{q}=0.1$

| X | $V_{\psi}$ | $s_{\psi}$ | $\rho_{\psi}$ | $M_{\psi}$ | $P_{\psi}$ | $\sigma$ | $\varepsilon$ | $T_{e} / T_{P}$ | $L_{e} / L_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.00001 | 0.00041 | 0.99000 | 0.00249 | 0.01313 | 0.00035 | 0.00035 | 0.14832 | 0.998245 |
| 0.2 | 0.00006 | 0.00166 | 0.96000 | 0.01952 | 0.01499 | 0.00036 | 0.00036 | 0.20199 | 0.998168 |
| 0.3 | 0.00022 | 0.00374 | 0.91000 | 0.06386 | 0.01317 | 0.00038 | 0.00038 | 0.24738 | 0.998060 |
| 0.4 | 0.00054 | 0.00666 | 0.84000 | 0.14464 | 0.01081 | 0.00041 | 0.00041 | 0.28565 | 0.99915 |
| 0.5 | 0.00106 | 0.01041 | 0.75000 | 0.26562 | 0.00819 | 0.00044 | 0.00044 | 0.31936 | 0.997721 |
| 0.6 | 0.00183 | 0.01499 | 0.64000 | 0.42333 | 0.00559 | 0.00049 | 0.00049 | 0.34983 | 0.997459 |
| 0.7 | 0.00291 | 0.02040 | 0.51000 | 0.60539 | 0.00327 | 0.00056 | 0.00056 | 0.37784 | 0.997096 |
| 0.8 | 0.00435 | 0.02665 | 0.36000 | 0.78848 | 0.00146 | 0.00066 | 0.00066 | 0.40391 | 0.996572 |
| 0.9 | 0.00619 | 0.03373 | 0.18999 | 0.93676 | 0.00035 | 0.00081 | 0.00081 | 0.42837 | 0.995764 |
| 1.0 | 0.00850 | 0.41649 | 0.00000 | 1.00000 | 0.00000 | 0.00107 | 0.00107 | 0.45148 | 0.994383 |

Table: D
Structure Parameters of Differentially Rotating and Tidally Distorted Prasad Model

$$
\psi_{s}^{*}=5, \mathrm{~b}_{1}=.3162, \mathrm{~b}_{2}=.32, \mathrm{q}=0.1
$$

| X | $V_{\psi}$ | $s_{\psi}$ | $\rho_{\psi}$ | $M_{\psi}$ | $P_{\psi}$ | $\sigma$ | $\varepsilon$ | $T_{e} / T_{P}$ | $L_{e} L_{p}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.00001 | 0.00042 | 0.99004 | 0.00248 | 0.01304 | 0.00053 | 0.00053 | 0.14282 | 0.99731 |
| 0.2 | 0.00006 | 0.00166 | 0.96001 | 0.01951 | 0.01489 | 0.00054 | 0.00054 | 0.20198 | 0.99726 |
| 0.3 | 0.00023 | 0.00375 | 0.91003 | 0.06385 | 0.01314 | 0.00056 | 0.00056 | 0.24736 | 0.99713 |
| 0.4 | 0.00054 | 0.00666 | 0.84005 | 0.14457 | 0.01087 | 0.00060 | 0.00060 | 0.28562 | 0.99694 |
| 0.5 | 0.00106 | 0.01041 | 0.75008 | 0.26551 | 0.00819 | 0.00065 | 0.00065 | 0.31933 | 0.99668 |
| 0.6 | 0.00184 | 0.01499 | 0.64001 | 0.42320 | 0.00559 | 0.00072 | 0.00072 | 0.34983 | 0.99632 |


| 0.7 | 0.00292 | 0.02041 | 0.51013 | 0.60522 | 0.00327 | 0.00082 | 0.00081 | 0.37782 | 0.99582 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.8 | 0.00435 | 0.02665 | 0.36014 | 0.78833 | 0.00146 | 0.00095 | 0.00095 | 0.40386 | 0.99512 |
| 0.9 | 0.00619 | 0.03378 | 0.19010 | 0.93669 | 0.00035 | 0.00116 | 0.00116 | 0.42830 | 0.99399 |
| 1.0 | 0.00850 | 0.04165 | 0.00000 | 1.00000 | 0.00000 | 0.00153 | 0.00153 | 0.45147 | 0.99203 |

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