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ON SQUARING A NUMBER

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ABSTRACT

In this paper we give a method of finding the square of an integer and illustrate it by giving examples.

INTRODUCTION AND STATEMENT OF RESULT

Square of an integer means multiple it by itself in our routine life squaring a number is an important problem. A square number can end only with digits 0, 1,4,6,9, or 25 in base 10, as follows: [1, 2]

1. If the last digit of a number is 0, its square ends in an even number of 0s (so at least 00) and the digits preceding the ending 0s must also form a square.

2. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.

3. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.

4. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.

5. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be odd.

6. If the last digit of a number is 5, its square ends in 25 and the preceding digits must be 0, 2, 06, or 56.

IN THE PRESENT PAPER WE GIVE METHOD OF GETTING THE SQUARE OF ANY INTEGER.

Let $m = \left(a_n a_{n-1} a_{n-2} \ldots a_2 a_1\right)_{10}$ be a number in the decimal system (1)

Then for $n = 1, ie(a_1)$

$$m^2 = a_1^{\,2}$$

For $n = 2, ie(a_2 a_1)$

$$m^2 = (a_2)^2 (2a_1 a_2)^2 (a_1)^2$$

For $n = 3, ie(a_3 a_2 a_1)$

$$m^2 = (a_3)^2 (2a_2 a_3)(2a_1 a_3 + a_2^{\,2})(2a_1 a_2) a_1^{\,2}$$

For $n = 4, ie(a_4 a_3 a_2 a_1)$

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$$m^2 = (a_4)^2(2a_3a_4)(2a_2a_4 + a_3{}^2)\left[2(a_1a_4 + a_2a_3)\right](2a_1a_2)(a_1)^2$$

In general the square of every integer" m" given by equation first can be expressed as

$$\left(a_n\right)^2\left(2a_na_{n-1}\right).....\left[2(a_1a_4 + a_2a_3\right]\left[2a_1a_3 + a_2{}^2\right]\left(2a_1a_2\right)\left(a_1\right)^2$$

At each stage we have to carry on the digits to the next place if it is more than one digit number in calculation

FOR 2 DIGIT NUMBER

Examples

$\left(21\right)^2$

$a_2 = 2, a_1 = 1$

$$\begin{aligned}\left(m\right)^2 &= \left(a_2\right)^2\left(2a_1a_2\right)\left(a_1\right)^2\\ &= (2)^2\left(2\times1\times2\right)(1)^2\\ &= 441\end{aligned}$$

$\left(99\right)^2$

$a_2 = 9, a_1 = 9$

$$\begin{aligned}\left(m\right)^2 &= \left(a_2\right)^2\left(2a_1a_2\right)\left(a_1\right)^2\\ &= (9)^2\left(2\times9\times9\right)(9)^2\left(81\right)\left(162\right)\left(81\right)\\ &= 9801\end{aligned}$$

$\left(76\right)^2$

$a_2 = 7, a_1 = 6$

$$\begin{aligned}\left(m\right)^2 &= \left(a_2\right)^2\left(2a_1a_2\right)\left(a_1\right)^2\\ &= \left(7\right)^2(2\times7\times6)(6)^2\\ &= \left(49\right)\left(84\right)\left(36\right)\\ &= 5776\end{aligned}$$

FOR 3 DIGIT NUMBERS

$$(m)^2 = \left(a_3\right)^2\left(2a_2a_3\right)\left(2a_1a_3 + a_2{}^2\right)\left(2a_1a_2\right)\left(a_1\right)^2$$

$\left(126\right)^2$

$a_1 = 6, a_2 = 2, a_3 = 1$

$$\begin{aligned}&= (1)^2(2\times2\times1)(2\times6\times1+2^2)(2\times6\times2)(6)^2\\ &= (1)(4)(16)(24)(36)\\ &= 15876\end{aligned}$$

$\left(269\right)^2$

$a_1 = 9, a_2 = 6, a_3 = 2$

$$\begin{aligned}&= (2)^2(2\times6\times2)[(2\times9\times2)+6^2)](2\times9\times6)(9)^2\\ &= (4)(24)(72)(108)(81)\\ &= 72361\end{aligned}$$

$(987)^2$

$a_1 = 7, a_2 = 8, a_3 = 9$

$= (9)^2(2\times 8\times 9)[(2\times 7\times 9)+8^2](2\times 7\times 8)(7)^2$

$= (81)(144)(190)(112)(49)$

$= 974169$

FOR FOUR DIGIT NUMBERS

$$(m)^2 = (a_4)^2(2a_3a_4)\left(2a_2a_4+a_3{}^2\right)\left[2\left(a_1a_4+a_2a_3\right)\right]\left(2a_1a_3+a_2{}^2\right)\left(2a_1a_2\right)(a_1)^2$$

$(1376)^2$

$a_1 = 6, a_2 = 7, a_3 = 3, a_4 = 1$

$= (1)^2(2\times 3\times 1)\left[(2\times 7\times 1)+3^2\right]\left[2\left[(6\times 1)+(7\times 3)\right]\right]\left[(2\times 6\times 3)+7^2\right](2\times 6\times 7)(6)^2$

$= (1)^2(2\times 3\times 1)\left[(2\times 7\times 1)+3^2\right]\left[2\left[(6\times 1)+(7\times 3)\right]\right]\left[(2\times 6\times 3)+7^2\right](2\times 6\times 7)(6)^2$

$= 1893376$

$(9876)^2$

$a_1 = 6, a_2 = 7, a_3 = 8, a_4 = 9$

$= (9^2)(2\times 8\times 9\ [(2\times 7\times 9+)8^2]\left[2\left[(6\times 9+)(7\times 8)\right]\right]\left[(2\times 6\times 8)+7^2\right]\left[(2\times 6\times 7)\right](6^2$

$= (81)(144)(190)(220)(145)(84)(36)$

$= 9735376$

FOR 5 DIGIT NUMBER

$$(m)^2 = \left[2\left(a_2a_5+a_3a_4\right)\right]\left[2\left(a_1a_5+a_2a_4\right)+a_3{}^2\right]\left[2\left(a_1a_4+a_2a_3\right)\right]\left(2a_1a_3+a_2{}^2\right)\left(2a_1a_2\right)\left(a_1\right)^2$$

$$\left(a_5\right)^2\left(2a_4a_5\right)\left(2a_3a_5+a_4{}^2\right)$$

$(13546)^2$

$a_1 = 6, a_2 = 4, a_3 = 5, a_4 = 3, a_5 = 1$

$= (1)^2(2\times 3\times 1)\left[(2\times 5\times 1)+3^2\right]\left[2\left[(4\times 1)+(5\times 3)\right]\right]\left[2\left[(6\times 1)+(4\times 3)\right]+5^2\right]\left[2(6\times 3+4\times 5)\right]$

$= \left[2(6\times 5)+4^2\right](2\times 6\times 4)(6)^2$

$= (1)(6)(19)(38)(61)(76)(76)(48)(36)$

$= 183494116$

FOR SIX DIGIT NUMBER

$$(m)^2 = \left[2\left(a_1a_5+a_2a_4\right)+a_3{}^2\right]\left[2\left(a_1a_4+a_2a_3\right)\right]\left[2\left(a_1a_3+a_2{}^2\right)\left(2a_1a_2\right)\left(a_1\right)^2\right]$$
$$\left[2a_3a_6+a_4a_5\right]\left[2\left(a_2a_6+a_4a_5\right)+a_4^2\right]\left[2\left(a_1a_6+a_2a_5+a_3a_4\right)\right]$$
$$\left(a_6\right)^2\left(2a_5a_6\right)\left(2a_4a_6+a_5^2\right)$$

$(376475)^2$

$$a_1=5, a_2=7, a_3=4, a_4=6, a_5=7, a_6=3$$
$$=\left[2(5\times 7)+(7\times 6)+4^2\right]\left[2(5\times 6)+(7\times 4)\right]\left[2(5\times 4)+7^2\right]\left[2(5\times 7)\right](5)^2$$
$$\left[2(6\times 3)+7^2\right]\left[2((4\times 3)+6(7))\right]\left[2\left((7\times 3)+(4\times 7)+6^2\right)\right]\left[2((5\times 3)+(7\times 7)+(4\times 6))\right]$$
$$(3)^2[2(7\times 3)]$$
$$=(9)(42)(85)(108)(134)(176)(170)(116)(89)(70)(25)$$
$$=141733425625$$

FOR SEVEN DIGIT NUMBER

$$(m)^2=\left[2(a_1a_5+a_2a_4)+a_3^2\right]\left[2(a_1a_4+a_2a_3)\right]\left(2a_1a_3+a_2^2\right)\left(2a_1a_2\right)\left(a_1\right)^2$$
$$\left[2\left(a_2a_7+a_3a_6+a_4a_5\right)\right]\left[2\left(a_1a_7+a_2a_6+a_3a_5\right)+a_4^2\right]\left[2\left(a_1a_6+a_2a_5+a_3a_4\right)\right]$$
$$\left(a_7\right)^2 2\left(a_6a_7\right)\left(2a_5a_7+a_6^2\right)\left[2\left(a_4a_2+a_5a_6\right)\right]\left[2\left(a_3a_7+a_4a_6\right)+a_5^2\right]$$

Examples

$(1111111)^2$

$$a_1=1, a_2=1, a_3=1, a_4=1, a_5=1, a_6=1, a_7=1$$
$$=\left[2\left((1\times 1)+(1\times 1)\right)+1^2\right]\left[2\left((1\times 1)+(1\times 1)\right)\right]\left[2(1\times 1)+1^2\right]\left(2(1\times 1)\right)(1)^2$$
$$=\left[2\left((1\times 1)+(1\times 1)+(1\times 1)\right)\right]\left(2\left((1\times 1)+(1\times 1)+(1\times 1)\right)+1^2\right)\left(2(1\times 1)+(1\times 1)+(1\times 1)\right)$$
$$=\left(2\left((1\times 1)+1^2\right)\right)\left(2\left((1\times 1)+(1\times 1)+1^2\right)\right)(1)^2\left(2(1\times 1)\right)$$
$$=1234567654321$$

$(2132143)^2$

$$a_1=3, a_2=4, a_3=1, a_4=2, a_5=3, a_6=1, a_7=2$$
$$\left[2\left((3\times 3)+(4\times 2)\right)+1^2\right]\left[2((3\times 2)+(4\times 1))\right]\left[2(3\times 1)+4^2\right]\left(2(3\times 4)\right)(3)^2$$
$$\left[2((4\times 2)+(1\times 1)+(2\times 3))\right]\left(2\left((3\times 2)+(4\times 1)+(1\times 3)\right)+2^2\right)\left(2(3\times 1)+(4\times 3)+(1\times 2)\right.$$
$$(2)^2\left(2((1\times 2))\right)\left(2\left((3\times 2)+1^2\right)\right)\left(2(2\times 2)+(3\times 1)\right)\left(2((1\times 2)+(2\times 1))+3^2\right)$$
$$=(4)(4)(13)(14)(17)(30)(30)(34)(35)(20)(22)(24)(9)$$
$$=4546033772449$$

CONCLUSION

From the above examples we conclude that the square of every integer can be expressed as

$$\left(a_n\right)^2\left(2a_na_{n-1}\right)\ .....\left[2(a_1a_4+a_2a_3\right]\left[2a_1a_3+a_2^2\right]\left(2a_1a_2\right)\left(a_1\right)^2$$

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