REGULAR WEAKLY GENERALIZED CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy regular weakly generalized continuous mappings in intuitionistic fuzzy topological space. We investigate some of their properties.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy regular weakly generalized closed set, intuitionistic fuzzy regular weakly generalized open set, intuitionistic fuzzy regular weakly generalized continuous mappings, intuitionistic fuzzy rwT_{1/2} space and rwgT_{1/2} space.

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1. INTRODUCTION


2. PRELIMINARIES

Definition 2.1: [1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = \{x, \mu_A(x), \nu_A(x)\} / x \in X where the functions \mu_A(x): X \rightarrow [0, 1] and \nu_A(x): X \rightarrow [0, 1] denote the degree of membership (namely \mu_A(x)) and the degree of non-membership (namely \nu_A(x)) of each element x \in X to the set A, respectively, and 0 \leq \mu_A(x) + \nu_A(x) \leq 1 for each x \in X. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFS’s of the forms A = \{x, \mu_A(x), \nu_A(x)\} / x \in X and B = \{x, \mu_B(x), \nu_B(x)\} / x \in X. Then

(a) A \subseteq B if and only if \mu_A(x) \leq \mu_B(x) and \nu_A(x) \geq \nu_B(x) for all x \in X,
(b) A = B if and only if A \subseteq B and B \subseteq A,
(c) A^\mathcal{A} = \{x, \nu_A(x), \mu_A(x)\} / x \in X,
(d) A \cap B = \{x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\} / x \in X,
(e) A \cup B = \{x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\} / x \in X.

For the sake of simplicity, we shall use the notation A = \langle x, \mu_A, \nu_A \rangle instead of A = \{x, \mu_A(x), \nu_A(x)\} / x \in X. Also for the sake of simplicity, we shall use the notation A = \langle x, (A/\mu_A, B/\nu_A) \rangle instead of A = \langle x, (A/\mu_A, B/\nu_B) \rangle. The intuitionistic fuzzy sets 0. = \{x, 0, 1\} / x \in X and 1. = \{x, 1, 0\} / x \in X are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on a non empty X is a family \tau of IFS in X satisfying the following axioms:

(a) 0. \subset \tau.

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In this case the pair \((X, \tau)\) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in \(\tau\) is known as an intuitionistic fuzzy open set (IFOS for short) in \(X\).

The complement \(A^c\) of an IFOS \(A\) in an IFTS \((X, \tau)\) is called an intuitionistic fuzzy closed set (IFCS for short) in \(X\).

**Definition 2.4:** \([3]\) Let \((X, \tau)\) be an IFTS and \(A = \langle x, \mu_A, \nu_A \rangle\) be an IFS in \(X\). Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

\[
\text{int}(A) = \bigcup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\},
\]

\[
\text{cl}(A) = \bigcap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.
\]

Note that for any IFS \(A\) in \((X, \tau)\), we have \(\text{cl}(A^c) = [\text{int}(A)]^c\) and \(\text{int}(A^c) = [\text{cl}(A)]^c\).

**Definition 2.5:** \([6]\) An IFS \(A = \langle x, \mu_A, \nu_A \rangle\) in an IFTS \((X, \tau)\) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if \(A \subseteq \text{cl}(\text{int}(A))\),

(ii) intuitionistic fuzzy \(\alpha\)-open set (IF\(\alpha\)OS in short) if \(A \subseteq \text{int}(\text{cl}(\text{int}(A)))\),

(iii) intuitionistic fuzzy regular open set (IFROS in short) if \(A = \text{int}(\text{cl}(A))\).

The family of all IFOS (respectively IFSOS, IF\(\alpha\)OS, IFROS) of an IFTS \((X, \tau)\) is denoted by \(\text{IFO}(X)\) (respectively \(\text{IFS}(X), \text{IF}\(\alpha\)S(X), \text{IFRO}(X))

**Definition 2.6:** \([6]\) An IFS \(A = \langle x, \mu_A, \nu_A \rangle\) in an IFTS \((X, \tau)\) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if \(\text{int}(\text{cl}(A)) \subseteq A\),

(ii) intuitionistic fuzzy \(\alpha\)-closed set (IF\(\alpha\)CS in short) if \(\text{cl}(\text{int}(\text{cl}(A))) \subseteq A\),

(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if \(A = \text{cl}(\text{int}(A))\).

The family of all IFCS (respectively IFSCS, IF\(\alpha\)S, IFRCS) of an IFTS \((X, \tau)\) is denoted by \(\text{IFC}(X)\) (respectively \(\text{IFSC}(X), \text{IF}\(\alpha\)C(X), \text{IFRC}(X))

**Definition 2.7:** \([8]\) Let \(A\) be an IFS in an IFTS \((X, \tau)\). Then

\[
\text{sint}(A) = \bigcup \{G / G \text{ is an IFSOS in } X \text{ and } G \subseteq A\},
\]

\[
\text{scl}(A) = \bigcap \{K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K\}.
\]

Note that for any IFS \(A\) in \((X, \tau)\), we have \(\text{scl}(A^c) = (\text{sint}(A))^c\) and \(\text{sint}(A^c) = (\text{scl}(A))^c\).

**Definition 2.8:** \([7]\) An IFS \(A = \langle x, \mu_A, \nu_A \rangle\) in an IFTS \((X, \tau)\) is said to be an

(i) intuitionistic fuzzy regular weakly generalized closed set (IFRWGCS in short) if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X\).

**Result 2.9:** \([7]\) Every IFCS, IFRCS, IFGCS, IFPCS, IF\(\alpha\)CS, IF\(\alpha\)GCS is an IFRWGCS but the converses may not be true in general.

**Definition 2.10:** \([10]\) An IFS \(A\) in an IFTS \((X, \tau)\) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X\).

(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFROS in \(X\).

**Definition 2.11:** \([8]\) An IFS \(A\) in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X\).

**Result 2.12:** \([8]\) Every IFCS, IFSCS, IFGCS, IFRCS, IF\(\alpha\)CS, IF\(\alpha\)GCS is an IFRGCS but the converses may not be true in general.

**Definition 2.13:** \([8]\) An IFS \(A\) is said to be an intuitionistic fuzzy generalized semi open set (IFGSO in short) if the complement \(A^c\) is an IFGCS in \(X\).

The family of all IFGCSs (IFGSOs) of an IFTS \((X, \tau)\) is denoted by IFGCS(X) (IFGSO(X)).

**Definition 2.14:** \([6]\) An IFS \(A\) in an IFTS \((X, \tau)\) is said to be an intuitionistic fuzzy alpha generalized closed set (IF\(\alpha\)GCS in short) if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is an IFOS in \(X, \tau\).
Result 2.15: [6] Every IFCS, IFGCS, IF RCS, IFαCS is an IFαGCS but the converses may not be true in general. Every IFαGCS is IFGCS but the converse is need not be true.

Definition 2.16: [6] An IFS A is said to be an intuitionistic fuzzy alpha generalized open set (IFαGOS in short) in X if the complement A^c is an IFαGCS in X.

The family of all IFαGCSs (IFαGOSs) of an IFTS (X, τ) is denoted by IFαGC(X) (IFGSO(X)).

Definition 2.17: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if f^(-1)(B) ∈ IFO(X) for every B ∈ σ.

Definition 2.18: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) intuitionistic fuzzy semi continuous (IFS continuous in short) if f^(-1)(B) ∈ IFO(X) for every B ∈ σ.
(ii) intuitionistic fuzzy α continuous (IFα continuous in short) if f^(-1)(B) ∈ IFαO(X) for every B ∈ σ.
(iii) intuitionistic fuzzy pre continuous (IFP continuous in short) if f^(-1)(B) ∈ IFPO(X) for every B ∈ σ.

Definition 2.19: [5] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ continuous (IFγ continuous in short) if f^(-1)(B) is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.20: [2] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if f^(-1)(B) ∈ IFGCS(X) for every IFCS B in Y.

Definition 2.21: [8] A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if f^(-1)(B) is an IFGS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.22: [7] An IFTS (X, τ) is said to be an intuitionistic fuzzy rwT₁/₂ (IFrwT₁/₂ in short) space if every IFRWGCS in X is an IFCS in X.

Definition 2.23: [7] An IFTS (X, τ) is said to be an intuitionistic fuzzy rwgT₁/₂ (IFrwgT₁/₂ in short) space if every IFRWGCS in X is an IFPCS in X.

3. INTUITIONISTIC FUZZY REGULAR WEAKLY GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy regular weakly generalized continuous mapping and studied some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy regular weakly generalized continuous (IFRWG continuous in short) if f^(-1)(A) is an IFRWGCS in (X, τ) for every IFCS A of (Y, σ).

Example 3.2: Let X = {a, b}, Y = {u, v} and G₁ = ⟨x, (0.4, 0.4), (0.6, 0.6)⟩, G₂ = ⟨y, (0.8, 0.8), (0.2, 0.2)⟩. Then τ = {0, G₁, 1} and σ = {0, G₂, 1} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFRWG continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFRWG continuous mapping but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is an IF continuous mapping, f^(-1)(A) is an IFCS in X. Since every IFCS is an IFRWGCS, f^(-1)(A) is an IFRWGCS in X. Hence f is an IFRWG continuous mapping.

Example 3.4: Let X = {a, b}, Y = {u, v} and G₁ = ⟨x, (0.1, 0.2), (0.8, 0.8)⟩, G₂ = ⟨y, (0.5, 0.7), (0.5, 0.3)⟩. Then τ = {0, G₁, 1} and σ = {0, G₂, 1} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFRWG continuous mapping.

Therefore f is an IFRWG continuous mapping but not an IF continuous mapping.

Remark 3.5: The converse of the above theorem is true if X is an IFrwT₁/₂ space.

Proof: Let A be an IFCS in Y. Then f^(-1)(A) is an IFRWGCS in X, by hypothesis. Since X is an IFrwT₁/₂ space, f^(-1)(A) is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.6: Every IFP continuous mapping is an IFRWG continuous mapping but not conversely.
Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFP continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IFPCS in \( X \). Since every IFPCS is an IFRWGCS, \( f^{-1}(A) \) is an IFRWGCS in \( X \). Hence \( f \) is an IFRWG continuous mapping.

Example 3.7: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.1, 0.2), (0.9, 0.8) \rangle \), and \( G_2 = \langle y, (0.4, 0.4), (0.5, 0.6) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.5, 0.6), (0.4, 0.4) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is IFRWGCS in \( X \) but not an IFPCS in \( X \). Therefore \( f \) is an IFRWG continuous mapping but not an IFP continuous mapping.

Remark 3.8: The converse of the above theorem is true if \( X \) is an IFrwgT1/2 space.

Proof: Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRWGCS in \( X \), by hypothesis. Since \( X \) is an IFrwgT1/2 space, \( f^{-1}(A) \) is an IFCS in \( X \). Hence \( f \) is an IFP continuous mapping.

Theorem 3.9: Every IF\( \alpha \) continuous mapping is an IFRWG continuous mapping but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \alpha \)G continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF\( \alpha \)CS in \( X \). Since every IF\( \alpha \)CS is an IFRWGCS, \( f^{-1}(A) \) is an IFRWGCS in \( X \). Hence \( f \) is an IFRWG continuous mapping.

Example 3.10: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle \), and \( G_2 = \langle y, (0.7, 0.8), (0.3, 0.1) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.3, 0.1), (0.7, 0.8) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRWGCS in \( X \) but not an IFCS in \( X \).

Theorem 3.11: Every IF\( \alpha \)G continuous mapping is an IFRWG continuous mapping but not conversely.

Proof: Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IF\( \alpha \)G continuous mapping. Let \( A \) be an IFCS in \( Y \). Then by hypothesis \( f^{-1}(A) \) is an IF\( \alpha \)CS in \( X \). Since every IF\( \alpha \)CS is an IFRWGCS, \( f^{-1}(A) \) is an IFRWGCS in \( X \). Hence \( f \) is an IFRWG continuous mapping.

Example 3.12: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle \), and \( G_2 = \langle y, (0.6, 0.5), (0.4, 0.5) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.4, 0.5), (0.6, 0.5) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRWGCS in \( X \) but not an IF\( \alpha \)CS in \( X \).

Theorem 3.13: Every IF continuous mapping is an IFRWG continuous mapping but not conversely.

Proof: Let \( A \) be an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFCS in \( X \). Since every IFCS is an IFRWGCS, \( f^{-1}(A) \) is an IFRWGCS in \( X \). Hence \( f \) is an IFRWG continuous mapping.

Example 3.14: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.7, 0.7), (0.3, 0.2) \rangle \), and \( G_2 = \langle y, (0.8, 0.8), (0.2, 0.2) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.2, 0.2), (0.8, 0.8) \rangle \) is IFCS in \( Y \). Then \( f^{-1}(A) \) is IFRWGCS in \( X \) but not IFCS in \( X \).

Proposition 3.15: IFRWG continuous mapping and IFCS continuous mapping are independent to each other.

Example 3.16: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.8, 0.8), (0.2, 0.2) \rangle \), \( G_2 = \langle y, (0.2, 0.2), (0.8, 0.7) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.8, 0.7), (0.2, 0.2) \rangle \) is an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRWGCS in \( X \) but not an IFCS in \( X \).

Example 3.17: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.5, 0.2), (0.5, 0.6) \rangle \), \( G_2 = \langle y, (0.5, 0.6), (0.5, 0.2) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.5, 0.2), (0.5, 0.6) \rangle \) is an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFSC in \( X \) but not an IFRWGCS in \( X \).

Proposition 3.18: IFRWG continuous mapping and IFGS continuous mapping are independent to each other.

Example 3.19: Let \( X = \{a, b\}, Y = \{u, v\} \) and \( G_1 = \langle x, (0.7, 0.9), (0.3, 0.1) \rangle \), \( G_2 = \langle y, (0.4, 0.3), (0.6, 0.7) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \)
and \( f(b) = v \). The IFS \( A = \langle y, (0.6, 0.7), (0.4, 0.3) \rangle \) is an IFCS in \( Y \). Then \( f^{-1}(A) \) is an IFRWGCS in \( X \) but not an IFGSC in \( X \).

**Example 3.20:** Let \( X = \{a, b\} \), \( Y = \{u, v\} \) and \( G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle \), \( G_2 = \langle y, (0.5, 0.6), (0.5, 0.4) \rangle \). Then \( \tau = \{0, G_1, 1\} \) and \( \sigma = \{0, G_2, 1\} \) are IFTs on \( X \) and \( Y \) respectively. Define a mapping \( f: (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = u \) and \( f(b) = v \). The IFS \( A = \langle y, (0.5, 0.4), (0.5, 0.6) \rangle \) is an IFCS in \( Y \). Then \( f^{-1}(A) \) is IFGSC in \( X \) but not an IFRWGCS in \( X \).

**Theorem 3.21:** If the mapping \( f: X \rightarrow Y \) is an IFRWG continuous then the inverse image of each IFOS in \( Y \) is an IFRWGOS in \( X \).

**Proof:** Let \( A \) be an IFOS in \( Y \). This implies \( A^c \) is IFCS in \( Y \). Since \( f \) is IFRWG continuous, \( f^{-1}(A^c) \) is IFRWGCS in \( X \). Since \( f^{-1}(A^c) = (f^{-1}(A))^c \), \( f^{-1}(A) \) is an IFRWGOS in \( X \).

**Theorem 3.22:** Let \( f: (X, \tau) \rightarrow (Y, \sigma) \) be an IFRWG continuous mapping and \( g: (Y, \sigma) \rightarrow (Z, \delta) \) be IF continuous, then \( g \circ f: (X, \tau) \rightarrow (Z, \delta) \) is an IFRWG continuous.

**Proof:** Let \( A \) be an IFCS in \( Z \). Then \( g^{-1}(A) \) is an IFCS in \( Y \), by hypothesis. Since \( f \) is an IFRWG continuous mapping, \( f^{-1}(g^{-1}(A)) \) is an IFRWGCS in \( X \). Hence \( g \circ f \) is an IFRWG continuous mapping.

**4. CONCLUSION**

In this paper we have introduced intuitionistic fuzzy regular weakly generalized continuous mappings and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy regular weakly generalized continuous mappings and some of the intuitionistic fuzzy continuous mappings already exists.

**REFERENCES**


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