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g**- closed sets in topological spaces

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ABSTRACT

***In** this paper, we introduce a new class of sets namely, g**-closed sets, which is settled in between the class of closed sets and the class of g-closed sets. Applying these sets, we introduce the new class of spaces, namely $T_{½}$** spaces, ** $T_{½}$ spaces, * $T_{½}$ * spaces, T_c*spaces and $_{\alpha}T_c$* spaces. Further we introduce g** continuous maps and g** irresolute maps.*

***Key words**: g**-closed sets; g** continuous maps; g** irresolute maps; $T_{½}$** spaces; $_{\alpha}T_c$* spaces, *$T_{½}$* spaces, T_c*spaces and ** $T_{½}$ spaces.*

1. Introduction

Levine **[13]** introduced the class of g-closed sets in 1970. Maki.et.al **[15]** defined αg-closed sets and α**g-closed sets in 1994. S.P. Arya and N. Tour **[3]** defined gs-closed sets in 1990. Dontchev **[10],** Gnanambal**[12]** and Palaniappan and Rao**[22]** introduced gsp-closed sets, gpr-closed sets and rg-closed sets respectively. M.K.R.S. Veerakumar**[24]** introduced g*-closed sets in 1991. We introduce a new class of sets called g**-closed sets, which is properly placed in between the class of closed sets and the class of g-closed sets.

Levine **[13]** Devi.et.al **[7]** and Devi et.al **[6]** introduced $T_{½}$ spaces, T_b spaces and $_{\alpha}T_b$ spaces respectively. M.K.R.S. Veerakumar**[24]** introduced T_c, $T_{½}$* and $_{\alpha}T_c$ spaces. Applying g**-closed sets, five new spaces namely, $T_{½}$** spaces, ** $T_{½}$ spaces, *T_c*, $T_{½}$* and $_{\alpha}T_c$* spaces are introduced.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively.

The class of all closed subsets of a space (X, τ) is denoted by C(X, τ). The smallest semi-closed (resp. pre-closed and α-closed) set containing a subset A of (X, τ) is called the semi-closure(resp. pre-closure and α-closure) of A and is denoted by scl(A)(resp. pcl(A) and αcl(A)).

Definition 2-1: A subset A of a toplogical space(X, τ) is called
1) a *pre-open* set **[18]** if A ⊆ int(cl(A)) and a *preclosed* set if cl(int(A)) ⊆ A.
2) a *semi-open* set [**14**] if A ⊆ cl(int(A)) and and *semi-closed* set if int (cl(A)) ⊆ A.
3) a *semi-preopen* set **[1]** if A ⊆ cl(int(cl(A))) and a *semi preclosed* set [1] if int(cl(int(A))) ⊆A
4) an α-*open* set [**20**] if A ⊆ int(cl(int(A))) and an α-*closed* set **[19]** if cl(int(cl(A))) ⊆ A.

Definition 2-2: A subset A of a toplogical space(X, τ) is called
1)a *generalized closed* set (brfly *g-closed*)[**13**] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
2) a *semi-generalized closed* set (briefly *sg-closed*) **[5]** if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in (X, τ).
3) a *generalized semi-closed* set (briefly *gs-closed*) **[3]** if scl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).

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4) an α-*generalized closed* set (briefly α*g-closed*) [**15**] if αcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
5) a *generalized α-closed* set (briefly *gα-closed*) [**16**] if αcl(A) ⊆U whenever A ⊆ U and U is α-open in (X, τ).
7) an α**-*generalized closed* set (briefly α***g-closed*) [**15**] if αcl(A)⊆int(cl(U)) whenever A ⊆ U and U is open in (X, τ).
8) a *generalized semi-preclosed* set (briefly *gsp-closed* [**10**] if spcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
9) a *regular generalized closed* set (briefly *rg-closed*) [**22**] if cl(A) ⊆ Uwhenever A ⊆ U and U is regular open in (X, τ).
10) a *generalized preclosed* set (briefly *gp-closed*) [**17**] if pcl(A) ⊆ U whenever A ⊆ U and U is open in (X, τ).
11) a *generalized preregular closed* set (briefly *gpr-closed*) [**12**] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in (X, τ).
12) a *gα*-closed* set [**16**] if αcl(A) ⊆ int(U) whenever A ⊆U and U is α-open in (X, τ).
13) *a g*-closed* set**[24]** if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X, τ).
14) a wg – closed set **[21]** if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).
14) a wg – closed set **[21]** if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in (X, τ).

Definition 2.3: A function f **:** (X, τ) → (Y, σ) is called
1)a *g-continuous* **[4]** if $f^{-1}(V)$ is a g-closed set of (X, τ) for every closed set V of (Y, σ).
2) an α*g-continuous* [**12**] if $f^{-1}(V)$ is an αg-closed set of (X, τ) for every closed set V of (Y, σ).
3) a *gs-continuous* [**8**] if $f^{-1}(V)$ is a gs-closed set of (X, τ) for every closed set V of (Y, σ).
4) a *gsp-continuous* [**10**] if $f^{-1}(V)$ is a gsp-closed set of (X, τ) for every closed set V of (Y, σ).
5) a *rg-continuous* [**22**] if $f^{-1}(V)$ is a rg-closed set of (X, τ) for every closed set V of (Y, σ).
6) a *gp-continuous* [**2**] if $f^{-1}(V)$ is a gp-closed set of (X, τ) for every closed set V of (Y, σ).
7) a *gpr-continuous* [**12**] if $f^{-1}(V)$ is a gpr-closed set of (X, τ) for every closed set V of (Y, σ).
8) a *g*-continuous* [**24**] if $f^{-1}(V)$ is a g*-closed set of ((X, τ) for every closed set V of (Y, σ).
9) a *g*-irresolute* [**24**] if $f^{-1}(V)$ is a g*-closed set of (X, τ) for every g*-closed set V of (Y, σ).

Further we call a function f **:** (Y, σ)→ (X, τ) → as α**g-continuous **[15]** if $f^{-1}(V)$ is an α**g-closed set of (X, τ) whenever V is a closed set of (Y, σ) and and wg-continuous **[21]** if $f^{-1}(V)$ is an wg -closed set of (X, τ) whenever V is a closed set of (Y, σ).

Definition 2.4: A topological space (X, τ) is said to be
1)a $T_{1/2}$ space [**13**] if every g-closed set in it is closed.
2) a T_b space [**7**] if every gs-closed set in it is closed.
3) a T_d space [**7**] if every gs-closed set in it is g-closed.
4) an $_{\alpha}T_d$ space **[4]** if every αg-closed set in it is g-closed.
5) an $_{\alpha}T_b$ space **[4]** if every αg-closed set in it is closed.
6) a $T_{1/2}$***[24]** *space* if every g*-closed set in it is closed.
7) *$T_{1/2}$ **[24]** space if every g-closed set in it is g*-closed set.

3. Basic properties of $g^{**}-closed$ sets

We now introduce the following definitions.

Definition 3.1: A subset A of (X,τ) is said to be a $g^{**}-closed$ set if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $g^{*}-open$ in X .

The class of $g^{**}-closed$ subset of (X,τ) is denoted by $G^{**}C(X,\tau)$.

Proposition 3.2: Every closed set is $g^{**}-closed$.

Proof follows from the definition.

The following example supports that a $g^{**}-closed$ set need not be closed in general.

Example 3.3: Let $X = \{a,b,c\}$, $\tau = \{\phi, \{a\}, X\}$. Let $A = \{b\}$. A is a $g^{**}-closed$ set but not a closed set of (X,τ).

So, the class of $g^{**}-closed$ sets properly contains the class of closed sets.

Next we show that the class of $g^{**}-closed$ sets is properly contained in the class of $g-closed$, $rg-closed$, $gpr-closed$ and $wg-closed$ sets.

Proposition 3.4: Every $g^{**}-closed$ set is
(i) $g-closed$
(ii) $rg-closed$
(iii) $gpr-closed$
(iv) $wg-closed$.

Proof: Let A be a $g^{**}-closed$ set
(i) Let $A \subseteq U$ and U be open then U is g^*-open. Since A is $g^{**}-closed$, $cl(A) \subseteq U$ therefore A is $g-closed$.

(ii) Let $A \subseteq U$ and U be regular open. Then U is open and hence U is g^*-open. Since A is $g^{**}-closed$, $cl(A) \subseteq U$, therefore A is $rg-closed$.

(iii) Let $A \subseteq U$ where U is regular open. Then U is g^*-open. Since A is $g^{**}-closed$, $cl(A) \subseteq U$ which implies $pcl(A) \subseteq cl(A) \subseteq U$. Therefore A is $gpr-closed$.

(iv) Let $A \subseteq U$ where U is open. Then U is g^*-open. $Int(A) \subseteq A$ implies $cl(\text{int}(A)) \subseteq cl(A) \subseteq U$. Therefore A is $wg-closed$.

The converse of the above proposition need not be true in general as it can be seen from the following example.

Example 3.5: In example (3.3), $A = \{a\}$ is $gpr-closed$ but not $g^{**}-closed$.

Example 3.6: Let $X = \{a,b,c\}$, $\tau = \{\phi, \{a\}, \{a,c\}, X\}$. $A = \{c\}$ is $g-closed$ and hence it is $rg-closed$ and $\alpha g-closed$. That is A is $gsp-closed$, $rg-closed$ and $wg-closed$ but it is not $g^{**}-closed$.

Proposition 3.7: Every $g^*-closed$ set is $gpr-continuous.$

Proof: Let A be a $g^*-closed$ set. Let $A \subseteq U$ and U be g^*-open. Then $A \subseteq U$ and U is $g-open$. Hence $cl(A) \subseteq U$, and U is g^*-open. Therefore A is $g^{**}-closed$.

The converse of the above proposition need not be true.

Example 3.8: In example (3.3), $\{a,c\}$ is $g^{**}-closed$ but not $gsp-continuous.$ So the class of $g^{**}-closed$ sets properly contains the class of $g^*-closed$ sets.

Proposition 3.9: Every $g^{**}-closed$ set is $\alpha g-closed$ set and hence $gs-closed$, $gsp-closed$, $gp-closed$ and also $\alpha^{**}g-closed$ but not conversely.

Proof: Let A be a $g^{**}-closed$ set of (X,τ). By proposition (3.2), A is $g-closed$. By the implications (2.4) in Maki.et.al. **[18]**, A is $\alpha g-closed$ and $\alpha^{**}g-closed$. Therefore every $g^{**}-closed$ set is $\alpha g-closed$ and $\alpha^{**}g-closed$. From the investigations of Dontchev **[11]** and Gnanambal **[15],** we know that every

$g-closed$ set is $gs-closed$, $gsp-closed$ and $gp-closed$. Therefore every $g**-closed$ set is $gs-closed$, $gsp-closed$ and $gp-closed$.

Example 3.10: In example (3.6), $\{c\}$ is $g-closed$ and hence it is $gs-closed$, $gsp-closed$ and $gp-closed$, $\alpha g-closed$ and $\alpha **g-closed$ but it is not $g**-closed$.

So the class of $g**-closed$ sets is properly contained in the class of $\alpha g-closed$ sets, the class of $gs-closed$ sets, the class of $gsp-closed$ sets, the class of $gp-closed$ sets, the class of $\alpha g-closed$ sets and the class of $\alpha **g-closed$ sets.

Proposition 3.11: If A and B are $g**-closed$ sets, then $A\cup B$ is also a $g**-closed$ set.

Proof follows from the fact that $cl(A\cup B)=cl(A)\cup cl(B)$.

Proposition 3.12: If A is both $g*-open$ and $g**-closed$, then A is closed.

Proof follows from the definition of $g**-closed$ sets.

Proposition 3.13: A is a $g**-closed$ set of (X,τ) if $cl(A)\setminus A$ does not contain any non-empty $g*-closed$ set.

Proof: Let F be a $g*-closed$ set of (X,τ) such that $F\subseteq cl(A)\setminus A$. Then $A\subseteq X\setminus F$. Since A is $g**-closed$ and $X\setminus F$ is $g*-open$, $cl(A)\subseteq X\setminus F$. This implies $F\subseteq X\setminus cl(A)$. So $F\subseteq (X\setminus cl(A))\cap(cl(A)\setminus A)\subseteq (X\setminus cl(A))\cap cl(A)=\phi$. Therefore $F=\phi$.

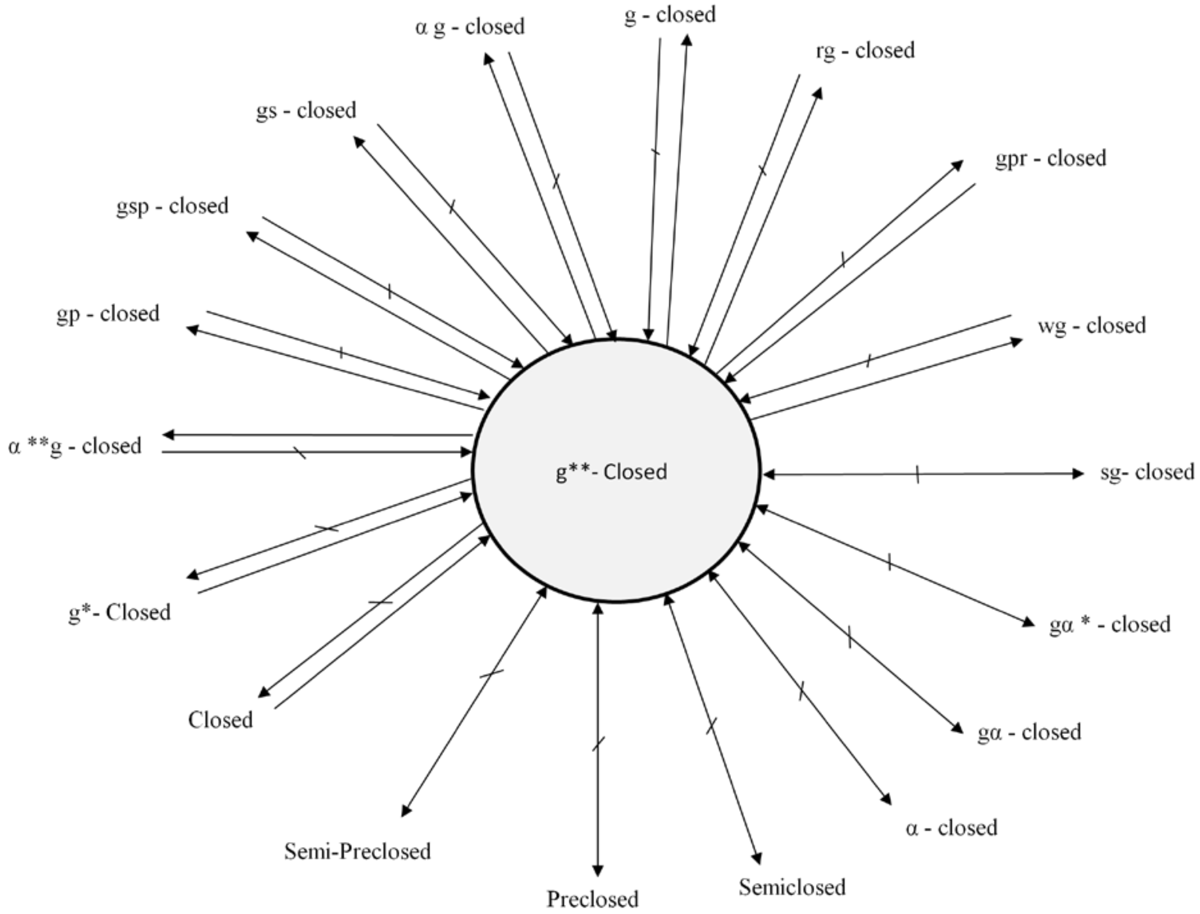
Remark 3.14: $g**-closedness$ is independent from $\alpha-closedness$, $semi-closedness$, $g:(Y,\sigma)\to(Z,\eta).,g_o f$, $g**-continuous$, $pre-closedness$ and $semi-preclosedness$.

Proof: Let $X=\{a,b,c\}$, $\tau=\{\phi,X,\{a\},\{a,c\}\}$. Let $A=\{a,b\}$ and $B=\{c\}$. A is $g**-closed$. A is neither $\alpha-closed$ nor $semi-closed$, in fact, it is not even a $semi-preclosed$ set. Also it is not $preclosed$. B is $\alpha-closed$ and hence $semi-closed$, $preclosed$, $semi-preclosed$, $sg-closed$, $g\alpha-closed$ and $g\alpha*-closed$ but it is not $g**-closed$.

Proposition 3.15: If A is a $g**-closed$ set of (X,τ) such that $A\subseteq B\subseteq cl(A)$, then B is also a $g**-closed$ set of (X,τ).

Proof: Let U be a $g*-open$ set of (X,τ) such that $B\subseteq U$. Then $A\subseteq U$, since A is $g**-closed$, then $cl(A)\subseteq U$. Now $cl(B)\subseteq cl(cl(A))=cl(A)\subseteq U$. Therefore B is also a $g**-closed$ set of (X,τ).

The above results can be represented in the following figure.



Where A ⟶ B (resp. A ⟷̸ B) represents A implies B (resp. A and B are independent).

4. $g^{**}-continuous$ **and** $g^{**}-irresolute$ **maps.**

We introduce the following definitions.

Definition 4.1: A function $f:(X,\tau)\to(Y,\sigma)$ is called $g^{**}-continuous$ if $f^{-1}(V)$ is a $g^{**}-closed$ set of (X,τ) for every closed set of (Y,σ).

Theorem 4.2: Every continuous map is $g^{**}-continuous$.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be continuous and let F be any closed set of Y. Then $f^{-1}(F)$ is closed in X. Since every closed set is $g^{**}-closed$, $f^{-1}(F)$ is $g^{**}-closed$. Therefore f is $g^{**}-continuous$.

The following example shows that the converse of the above theorem need not be true in general.

Example 4.3: Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,X,\{a\}\}$, $\sigma=\{\phi,Y,\{b\}\}$, $f:(X,\tau)\to(Y,\sigma)$ is defined as the identity map. The inverse image of all the closed sets of (Y,σ) are $g^{**}-closed$ in (X,τ). Therefore f is $g^{**}-continuous$ but not continuous.

Thus the class of all $g^{**}-continuous$ maps properly contains the class of continuous maps.

Theorem 4.4: Every $g^{**}-continuous$ map is $g-continuous$ and hence an $\alpha g-continuous$, $\alpha^{**}g-continuous$, $gs-continuous$, $gsp-continuous$, $gp-continuous$, $rg-continuous$, $gpr-continuous$ and $wg-continuous$ maps.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a $g^{**}-continuous$ map. Let V be a closed set of (Y,σ).

Since f is $g^{**}-continuous$, then $f^{-1}(V)$ is a $g^{**}-closed$ set in (X,τ).

By proposition (3.4) and (3.9), $f^{-1}(V)$ is $g-closed$, $rg-closed$, $gpr-closed$, $wg-closed$, $\alpha g-closed$, $gs-closed$, $gsp-closed$, $gp-closed$ and $\alpha^{**}g-closed$ set of (X,τ).

The converse of the above theorem need not be true as seen in the following example.

Example 4.5: Let $X=Y=\{a,b,c\}, \tau=\{\phi,X,\{a\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a,b\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. Then $f^{-1}(\{c\})=\{c\}$ is not $g^{**}-closed$ in X. But $\{c\}$ is $g-closed$ and hence $rg-closed$, $gpr-closed$, $wg-closed$, $\alpha g-closed$, $gs-closed$, $gsp-closed$, $gp-closed$ and $\alpha^{**}g-closed$. Therefore f is $g-continuous$ and hence $rg-continuous$, $gpr-continuous$, $wg-continuous$, $\alpha g-continuous$, $gs-continuous$, $gsp-continuous$, $gp-continuous$ and $\alpha^{**}g-continuous$ but f is not $g^{**}-continuous$.

Thus the class of all $g^{**}-continuous$ maps is properly contained in the classes of $g-continuous$, $rg-continuous$, $gpr-continuous$, $wg-continuous$, $\alpha g-continuous$, $gs-continuous$, $gsp-continuous$, $gp-continuous$ and $\alpha^{**}g-continuous$ maps.

The following example shows that the compositions of two $g^{**}-continuous$ maps need not be a $g^{**}-continuous$ map.

Example 4.6: Let $X=Y=Z=\{a,b,c\}$ and let $f:(X,\tau)\to(Y,\sigma)$, $g:(Y,\sigma)\to(Z,\eta)$, be the identity maps. $\tau=\{\phi,X,\{a\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a\}\}$, $\eta=\{\phi,Z,\{b\}\}$.
$(f_o g)^{-1}(\{a,c\})=f^{-1}(g^{-1}\{a,c\})=f^{-1}(\{a,c\})=\{a,c\}$ is not $g^{**}-closed$ in (X,τ). But f and g are $g^{**}-continuous$ maps.

Theorem 4.7: Every $g^{*}-continuous$ map is $g^{**}-continuous$.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be $g^{*}-continuous$ and let V be a closed set of Y. Then $f^{-1}(V)$ is $g^{*}-closed$ and hence by proposition (3.7), it is $g^{**}-closed$. Hence f is $g^{**}-continuous$.

The following example shows that the converse of the above theorem is not true in general.

Example 4.8: Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,X,\{a\}\}$, $\sigma=\{\phi,X,\{b\}\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. $A=\{a,c\}$ is closed in (Y,σ) and is $g^{**}-closed$ in (X,τ) but not $g^{*}-closed$ in (X,τ). Therefore f is $g^{**}-continuous$ but not $g^{*}-continuous$.

Definition 4.9: A function $f:(X,\tau)\to(Y,\sigma)$ is called $g^{**}-irresolute$ if $f^{-1}(V)$ is a $g^{**}-closed$ set of (X,τ) for every $g^{**}-closed$ set V of (Y,σ).

Theorem 4.10: Every $g^{**}-irresolute$ function is $g^{**}-continuous$.

Proof follows from the definition.

Theorem 4.11: Every $g^{*}-irresolute$ function is $g^{**}-continuous$.

Proof follows from the definition.

Converse of theorems (4.10) and (4.11) need not be true in general as seen in the following example.

Example 4.12: Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$, $\sigma=\{\phi,Y,\{a,b\}\}$.

Define $h:(X,\tau)\to(Y,\sigma)$ by $h(a)=b$, $h(b)=c$ and $h(c)=a$. $\{c\}$ is the only closed set of Y. $h^{-1}(\{c\})=\{b\}$ is $g^{**}-closed$ in X. Therefore h is $g^{**}-continuous$. $\{b,c\}$ is a $g^{*}-closed$ set of Y but $h^{-1}\{b,c\}=\{a,b\}$ is not $g^{*}-closed$ in X. Therefore h is not $g^{*}-irresolute$. Therefore h is $g^{**}-continuous$ but not $g^{*}-irresolute$. $\{b,c\}$ is a $g^{**}-closed$ set in Y but $h^{-1}\{b,c\}=\{a,b\}$ is not $g^{**}-closed$ in X.

Therefore h is not $g^{**}-irresolute$. Therefore h is $g^{**}-continuous$ but not $g^{**}-irresolute$.

Theorem 4.13: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\eta)$ be any two functions then
(i) $g_o f$ is $g^{**}-continuous$ if g is continuous and f is $g^{**}-continuous$.
(ii) $g_o f$ is $g^{**}-irresolute$ if both f and g are $g^{**}-irresolute$.
(iii) $g_o f$ is $g^{**}-continuous$ if g is $g^{**}-continuous$ and f is $g^{**}-irresolute$.

5. Applications of $g^{**}-closed$ sets

As applications of $g^{**}-closed$ sets, new spaces, namely, $T_{1/2}^{**}\ space$, ${}_{\alpha}T_c^{*}\ spaces$ and ${}^{**}T_{1/2}\ spaces$, $*T_{1/2}*$ and $T_c^{*}\ spaces$ are introduced.

We introduce the following definition.

Definition 5.1: A space (X,τ) is called a $T_{1/2}^{**}space$ if every $g^{**}-closed$ set is closed.

Theorem 5.2: Every $T_{1/2}\ space$ is $T_{1/2}^{**}\ space$.

A $T_{1/2}^{**}\ space$ need not be $T_{1/2}\ space$, which can be seen from the following example.

Example 5.3: Let X = {a, b, c, d}, τ = {φ, X,{a}}. (X,τ) is a $T_{½}$ ** space but not a $T_{½}$ space since A = {b} is g-closed but not closed.

Therefore the class of $T_{1/2}^{**}\ space$ properly contains the class of $T_{1/2}-spaces$.

Theorem 5.4: Every $T_{1/2}^{**}\ space$ is $T_{1/2}^{*}\ space$

The converse need not be true in general as seen in the following example.

Example 5.5: Let $X=\{a,b,c\}$, $\tau=\{\phi,X,\{a\}\}$, $G^{*}C(X,\tau)=\{\phi,X,\{b,c\}\}=C(X,\tau)$. Therefore (X,τ) is a $T_{1/2}^{*}-space$ but not $T_{1/2}^{**}\ space$ since $\{a,b\}$ is a $g^{**}-closed$ set but not a closed set of (X,τ).

Theorem 5.6: Every $T_b-space$ is a $T_{1/2}^{**}-space$.

The converse need not be true in general as seen in the following example.

Example 5.7: Let, X = {a, b, c,}, τ = {φ, X,{a},{b},{a, b}}. (X, τ) is a $T_{½}$ ** space but not a T_b space since A = {a} is gs-closed but not closed.

Remark 5.8: T_d-ness is independent of $T_{1/2}^{**}-ness$ as it can be seen from the following example.

Example 5.9: In example (5.7), (X, τ) is a $T_{½}$ ** space but not a ${}_{\alpha}T_d$ space since A = {a} is gs-closed but not g-closed.

Example 5.10: X = {a, b, c,}, τ = {φ, X,{a},{b, c}}. (X, τ) is a T_d space but not a $T_{½}$** space since A = {b} is g**-closed but not closed.

Theorem 5.11: Every ${}_{\alpha}T_b - space$ is a $T_{1/2}^{**} - space$.

Example 5.12: In example (5.3). (X, τ) is a $T_{½}$ ** space but not a ${}_{\alpha}T_b$ space since A = {b} is αg-closed but not closed.

Theorem 5.13: The following conditions are equivalent in a topological space (X,τ).
(i) (X,τ) is a $T_{1/2}^{**} - space$.
(ii) Every singleton of X is either $g^*-closed$ or open.

Proof:
(i) = (ii): Let (X,τ) be a $T_{1/2}^{**} - space$. Let $x \in X$ and suppose $\{x\}$ is not $g^*-closed$. Then $X \setminus \{x\}$ is not g^*-open. This implies that X is the only g^*-open set containing $X \setminus \{x\}$. Therefore $X \setminus \{x\}$ is a $g^{**}-closed$ set of (X,τ). Therefore $X \setminus \{x\}$ is closed since (X,τ) is a $T_{1/2}^{**} - space$. Therefore $\{x\}$ is open in (X,τ).

(ii) = (i): Let A be a $g^{**}-closed$ set of (X,τ) $A \subseteq cl(A)$ and let $x \in cl(A)$. By (ii) $\{x\}$ is $g^*-closed$ or open.

Case (i): Let $\{x\}$ be $g^*-closed$. If $x \notin A$, then $cl(A) \setminus A$ contains a non-empty $g^*-closed$ set $\{x\}$. But it is not possible by proposition (3.13). Therefore $x \in A$.

Case (ii): Let $\{x\}$ be open. Now $x \in cl(A)$, then $\{x\} \cap A \neq \phi$. Therefore $x \in A$ and so $cl(A) \subseteq A$ and hence $A = cl(A)$ or A is closed. Therefore (X,τ) is a $T_{1/2}^{**} - space$.

We introduce the following definition.

Definition 5.14: A space (X,τ) is called an ${}_{\alpha}T_c^{*} - space$ if every $\alpha g - closed$ set of (X,τ) is $g^{**} - closed$
We show that the class of ${}_{\alpha}T_c^{*} - spaces$ properly contains the class of ${}_{\alpha}T_b - spaces$ and is properly contained in the class of ${}_{\alpha}T_d - spaces$. Moreover we prove that ${}_{\alpha}T_c^{*} - ness$ and $T_{1/2}^{**} - ness$ are independent from each other.

Theorem 5.15: Every ${}_{\alpha}T_b - space$ is an ${}_{\alpha}T_c^{*} - space$ but not conversely.

Example 5.16: Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a,b\}\}$, (X,τ) is an ${}_{\alpha}T_c^{*} - space$ but not ${}_{\alpha}T_b - space$ since $\{a,c\}$ is $\alpha g - closed$ but not closed.

Theorem 5.17: Every ${}_{\alpha}T_c^{*} - space$ is an ${}_{\alpha}T_d - space$ but not conversely.

Example 5.18: In example (5.3). (X, τ) is a ${}_{\alpha}T_d$ space but not a ${}_{\alpha}T_c$* space since A = {b} is αg-closed but not g**-closed.

Theorem 5.19: A space (X,τ) is an ${}_{\alpha}T_b - space$ if and only if it is a ${}_{\alpha}T_c^{*} - space$ and a $T_{1/2}^{**} - space$.

Proof: Necessity – Follows from theorem 5.4 and 5.10

Sufficiency: Suppose (X,τ) is ${}_{\alpha}T_c^{*} - space$ and $T_{1/2}^{**} - space$. Let A be $\alpha g - closed$. Since (X,τ) is an ${}_{\alpha}T_c^{*} - space$, A is $g^{**} - closed$ and since (X,τ) is an $T_{1/2}^{**} - space$, A is closed. Therefore (X,τ) is an ${}_{\alpha}T_b - space$.

Remark 5.20: ${}_{\alpha}T_c^{*} - ness$ is independent from $T_{1/2}^{**} - ness$, as it is clear from the following examples.

Example 5.21: In example (5.16), (X,τ) is an ${}_{\alpha}T_c^* - space$ but not $T_{1/2}^{**}$ since, $\{a,c\}$ is g**-closed but not closed.

Example 5.22: In example (5.3). (X,τ) is a $T_{½}$** space but not a ${}_{\alpha}T_c$* space since A = {b} is αg-closed but not g**-closed.

Definition 5.23: A subset A of a space (X,τ) is called a $g^{**}-open$ set if its complement is a $g^{**}-closed$ set of (X,τ).

Theorem 5.24: If (X,τ) is an ${}_{\alpha}T_c^* - space$ for each $x \in X$, $\{x\}$ is either $\alpha g - closed$ or $g^{**}-open$.

Proof: Let $x \in X$ suppose that $\{x\}$ is not an $\alpha g - closed$ set of (X,τ). Then $\{x\}$ is not a closed set since every closed set is an $\alpha g - closed$ set. Therefore $X \setminus \{x\}$ is not open. Therefore $X \setminus \{x\}$ is an $\alpha g - closed$ set since X is the only open set which contains $X \setminus \{x\}$. Since (X,τ) is an ${}_{\alpha}T_c^* - space$, $X \setminus \{x\}$ is a $g^{**}-closed$ set or $\{x\}$ is $g^{**}-open$.

Remark 5.25: The converse of the above theorem is not true as it can be seen from the following example.

Example 5.26: X = {a, b, c}, τ = {φ, X, {a},{a, b}}. (X, τ) is not a ${}_{\alpha}T_c$* space but {c} and {b} are αg-closed and {a} is g**-open.

We introduce the following definition.

Definition 5.27: A space (X,τ) is called an ${}^{**}T_{1/2} - space$ if every $g^{**}-closed$ set of (X,τ) is a $g^{*}-closed$ set.

Remark 5.28: $T_{1/2}^{*} - ness$ is independent from ${}^{**}T_{1/2} - ness$ as it is clear from the following example.

Example 5.29: In example (5.26), (X,τ) is a ${}^{**}T_{1/2} - space$ but not a $T_{1/2}^{*} - space$ since A = {a, c}is g*-closed but not closed.

Example 5.30: Let X = {a, b, c}, τ = {φ, X, {a}}. (X, τ) is a $T_{½}$* space but not a **$T_{½}$ since A = {a, c} is g**-closed but not g*-closed.

Theorem 5.31: Every $T_{1/2}^{**} - space$ is ${}^{**}T_{1/2} - space$.

Proof: Let (X,τ) be an $T_{1/2}^{**} - space$. Let A be a $g^{**}-closed$ set of (X,τ). Since (X,τ) is a $T_{1/2}^{**} - space$, A is closed. By theorem (3.2) of **[24],** A is $g^{*}-closed$. Therefore (X,τ) is a ${}^{**}T_{1/2} - space$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.32: In example (5.26), (X,τ) is a ${}^{**}T_{1/2} - space$ but not a $T_{1/2}^{**} - space$ since A = {a, c} is g**-closed but not closed.

Theorem 5.33: Every $T_b - space$ is a ${}^{**}T_{1/2} - space$.

Proof: Let (X,τ) be a $T_b - space$. Then by theorem 5.6, it is a $T_{1/2}^{**} - space$. Therefore by theorem 5.31, it is a ${}^{**}T_{1/2} - space$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.34: In example (5.26), (X,τ) is a ${}^{**}T_{1/2} - space$ but not a $T_b - space$ since A = {a, c} is gs-closed but not closed.

Theorem 5.35: Every ${}_{\alpha}T_b - space$ is a ${}^{**}T_{1/2} - space$.

Proof: Let (X,τ) be a ${}_{\alpha}T_b - space$. Then by theorem (5.11), it is a $T_{1/2}^{**} - space$. Therefore by theorem (5.31), (X,τ) is a ${}^{**}T_{1/2} - space$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.36: Let $X = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{a,b\},X\}$, (X,τ) is an ${}^{**}T_{1/2} - space$ but not a ${}_{\alpha}T_b - space$ since $A = \{a,c\}$ is a $\alpha g - closed$ set but not a closed set of (X,τ).

Theorem 5.37: Every ${}_{\alpha}T_c - space$ is a ${}^{**}T_{1/2} - space$.

Proof: Let (X,τ) be a ${}_{\alpha}T_c - space$. Let A be a $g** - closed$ set. Then by proposition (3.9), A is $\alpha g - closed$. Since (X,τ) is an ${}_{\alpha}T_c - space$, A is $g* - closed$. Therefore it is a ${}^{**}T_{1/2} - space$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.38: In example (5.36), (X,τ) is an ${}^{**}T_{1/2} - space$ but not a ${}_{\alpha}T_c - space$ since $A = \{b\}$ is a $\alpha g - closed$ set but not a $g* - closed$.

Theorem 5.39: A space (X,τ) is a ${}_{\alpha}T_c - space$ if and only if it is ${}_{\alpha}T_c^* - space$ and ${}^{**}T_{1/2}\ space$.

Proof: Necessity follows from theorem (5.37) and theorem (5.15).

Sufficiency: Let A be $\alpha g - closed$. Then it is $g** - closed$, since (X,τ) is an ${}_{\alpha}T_c^* - space$. Since (X,τ) is ${}^{**}T_{1/2} - space$, A is $g* - closed$. Therefore (X,τ) is an ${}_{\alpha}T_c - space$.

Definitions 5.40: A space (X,τ) is called a ${}^{*}T_{1/2}^{*} - space$ if every $g - closed$ set of (X,τ) is $g** - closed$.

Theorem 5.41: Every $T_{1/2} - space$ is a ${}^{*}T_{1/2}^{*} - space$.

Proof: Let (X,τ) be a $T_{1/2} - space$. Let A be a $g - closed$ set of (X,τ). Then A is closed since (X,τ) is a $T_{1/2} - space$. But by proposition (3.2), A is $g** - closed$. Therefore (X,τ) is a ${}^{*}T_{1/2}^{*} - space$.

The converse of the above theorem need not be true as seen in the following example.

Example 5.42: In example (5.36), (X,τ) is a ${}^{*}T_{1/2}^{*} - space$ but not a $T_{1/2} - space$ since A = {a, c} is g-closed but not closed.

Remark 5.43: ${}^{*}T_{1/2}^{*} - ness$ and $T_{1/2}^{*} - ness$ are independent as it is shown in the following examples.

Example 5.44: In example (5, 36), (X,τ) is a ${}^{*}T_{1/2}^{*} - space$ but not a $T_{1/2}^{*} - space$ since A = {a, c} is a g*-closed set but not a closed set. $A = \{a,b\}$ is a $g* - closed$ set but not a closed set.

Example 5.45: In example (5.3), (X,τ) is a $T_{½}$*space but not a *$T_{½}$* since A = { c} is a g-closed set but not a g**-closed set.

Theorem 5.46: Every ${}_{\alpha}T_c - space$ is a ${}^{*}T_{1/2}^{*} - space$.

Proof: Let (X,τ) be an ${}_{\alpha}T_c - space$. Let A be a $g-closed$ set of (X,τ). Then A is also an $\alpha g-closed$ set. Since (X,τ) is an ${}_{\alpha}T_c - space$, A is $g*-closed$. Then by proposition (3.8), A is $g**-closed$. Therefore (X,τ) is a ${}^{*}T_{1/2}^{*} - space$.

The converse need not be true in general as seen in the following example.

Example 5.47: In example (5.36), (X,τ) is a $*T_{½}*$space but not a ${}_{\alpha}T_c$ since A = {b} is a αg-closed set but not a g*-closed set.

Theorem 5.48: Every ${}^{*}T_{1/2} - space$ is a ${}^{*}T_{1/2}^{*} - space$.

Proof: Let (X,τ) be an ${}^{*}T_{1/2} - space$. Let A be a $g-closed$ set of (X,τ). Then A is $g*-closed$. (X,τ) is a ${}^{*}T_{1/2} - space$. Then by proposition (3.8), A is $g**-closed$. Therefore (X,τ) is a ${}^{*}T_{1/2}^{*} - space$.

The converse need not be true in general as seen in the following example.

Example 5.49: In example (3.3), (X,τ) is a ${}^{*}T_{1/2}^{*} - space$ but not a ${}^{*}T_{1/2} - space$ since $A=\{b\}$ is a $g-closed$ but not a $g*-closed$.

Theorem 5.50: The space (X,τ) is a $T_{1/2}^{**} - space$ and a ${}^{*}T_{1/2}^{*} - space$ if and only if it is a $T_{1/2} - space$.

Proof Necessity: Let (X,τ) be a $T_{1/2}^{**} - space$ and a ${}^{*}T_{1/2}^{*} - space$. Let A be a $g-closed$ set of (X,τ). Then A is $g**-closed$ since (X,τ) is a ${}^{*}T_{1/2}^{*} - space$. Also since (X,τ) is a $T_{1/2}^{**} - space$, A is a closed set. Therefore (X,τ) is a $T_{1/2} - space$.

Sufficiency: Let (X,τ) be a $T_{1/2} - space$. Then by theorem (5.2) and (5.41), (X,τ) is a ${}^{*}T_{1/2}^{*} - space$ and a $T_{1/2}^{**} - space$.

Theorem 5.51: If (X,τ) is a ${}^{*}T_{1/2}^{*} - space$, then for each $x \in X$, $\{x\}$ is either closed or $g**-open$.

Proof: Suppose (X,τ) is a ${}^{*}T_{1/2}^{*} - space$. Let $x \in X$ and let $\{x\}$ be not closed. Then $X/\{x\}$ is not open set. Therefore $X/\{x\}$ is a $g-closed$ set since X is the only open set which contains $X/\{x\}$. Since (X,τ) is a ${}^{*}T_{1/2}^{*} - space$, $X/\{x\}$ is $g**-closed$. Therefore $\{x\}$ is $g**-open$.

We introduce the following definition.

Definition 5.52: A space (X,τ) is called a $T_c^{*} - space$ if every $gs-closed$ set of (X,τ) is $g**-closed$.

Theorem 5.53: Every $T_c - space$ is a $T_c^{*} - space$.

Proof: Let (X,τ) be a $T_c - space$. Let A be a $gs-closed$ set of (X,τ). Then A is $g*-closed$ since (X,τ) is a $T_c - space$. But by proposition (3.8), A is $g**-closed$. Therefore (X,τ) is a $T_c^{*} - space$.

The converse need not be true in general as seen in the following example.

Example 5.54: Let X = {a,b,c} τ = {φ, X,{a},{b, c}. (X,τ) is a $T_c^{*} space$ but not a $T_c - space$ since $A=\{b\}$ is a $gs-closed$ but not $g*-closed$.

Theorem 5.55: Every $T_b - space$ is a $T_c^* - space$.

Proof: Let (X,τ) be a $T_b - space$. Let A be a $gs - closed$ set of (X,τ). Then A is also closed. But by proposition (3.3), A is $g ** - closed$. Therefore (X,τ) is a $T_c^* - space$.

The converse need not be true in general as seen in the following example.

Example 5.56: In example (5.54), (X,τ) is a $T_c^* - space$ but not a $T_b - space$ since $A = \{a,b\}$ is a $gs - closed$ set but not a closed set.

Theorem 5.57: Every $T_c^* - space$ is a $T_d - space$.

Proof: Let (X,τ) be a $T_c^* - space$. Let A be a $gs - closed$ set of (X,τ). Since (X,τ) is a $T_c^* - space$, A is $g ** - closed$. But by proposition (3.4), A is $g - closed$. Therefore (X,τ) is a $T_d - space$.

The converse need not be true in general as seen in the following example.

Example 5.58: In example (5.3), (X, τ) is a T_d space but not a T_c* space since A = {a, b} is gs-closed but not g**-closed.

Theorem 5.59: Every $T_c^* - space$ is a ${}_\alpha T_d - space$.

Proof: Let (X,τ) be a $T_c^* - space$. Let A be a $\alpha g - closed$ set of (X,τ). Then A is also $gs - closed$ a set. Since (X,τ) is a $T_c^* - space$, A is $g ** - closed$. But by proposition (3.2), A is $g - closed$. Therefore (X,τ) is a ${}_\alpha T_d - space$.

The converse need not be true in general as seen in the following example.

Example 5.60: In example (3.3), (X,τ) is a ${}_\alpha T_d - space$ but not a $T_c^* - space$ since $A = \{a\}$ is a $gs - closed$ set but not a $g ** - closed$ set.

Theorem 5.61: If (X,τ) is a $T_c^* - space$ and a $T_{1/2}^{**} - space$, then it is a ${}_\alpha T_b - space$.

Proof: Let (X,τ) be a $T_c^* - space$ and a $T_{1/2}^{**} - space$. Let A be a $\alpha g - closed$ set of (X,τ). Then it is also a $gs - closed$ set. Hence A is $g ** - closed$ since (X,τ) is a $T_c^* - space$. But every $g ** - closed$ set is closed since (X,τ) is a $T_{1/2}^{**} - space$. Therefore A is closed and hence (X,τ) is a ${}_\alpha T_b - space$.

Theorem 5.62: Every $T_c^* - space$ is a ${}^*T_{1/2}^* - space$.

Proof: Let (X,τ) be a $T_c^* - space$. Let A be a $g - closed$ set of (X,τ). Then A is also $gs - closed$ a set. Then A is $g ** - closed$, since (X,τ) is a $T_c^* - space$. Therefore (X,τ) is a ${}^*T_{1/2}^* - space$.

The converse need not be true in general as seen in the following example.

Example 5.63: Let $X = \{a,b,c\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},X\}$. (X,τ) is a ${}^*T_{1/2}^*\ space$ but not a $T_c^*\ space$ since $A = \{a\}$ is $gs - closed$ but not $g ** - closed$.

Theorem 5.64: If (X,τ) is a $T_c^* - space$, then for each $x \in X$, $\{x\}$ is either semi-closed or $g ** - open$ in (X,τ).

Proof: Let (X,τ) be a $T_c^* - space$ and Let $x \in X$ suppose $\{x\}$ is not semi-closed, then by the proposition 6.4 (ii) of **[7],** $X \setminus \{x\}$ is $sg-closed$. Also $X \setminus \{x\}$ is $gs-closed$. Since (X,τ) is a $T_c^* - space$, then $X \setminus \{x\}$ is $g^{**}-closed$. Therefore $\{x\}$ is $g^{**}-open$.

Theorem 5.65: Let $f:(X,\tau)\to(Y,\sigma)$ be a $g^{**}-continuous$ map. If (X,τ) is $T_{1/2}^{**}$ then f is continuous.

Theorem 5.66: Let $f:(X,\tau)\to(Y,\sigma)$ be an $\alpha g-continuous$ map. If (X,τ) is ${}_{\alpha}T_c{}^*$, then f is $g^{**}-continuous$.

Theorem 5.67: Let $f:(X,\tau)\to(Y,\sigma)$ be a $g-continuous$ map. If (X,τ) is ${}^*T_{1/2}^*$, then f is $g^{**}-continuous$.

Theorem 5.68: Let $f:(X,\tau)\to(Y,\sigma)$ be a $gs-continuous$ map. If (X,τ) is $T_c{}^*$, then f is $g^{**}-continuous$.

Theorem 5.69: Let $f:(X,\tau)\to(Y,\sigma)$ be a $g^*-irresolute$ map and a closed map. Then $f(A)$ is a $g^{**}-closed$ set of (Y,σ) for every $g^{**}-closed$ set A of (X,τ).

Proof: Let A be a $g^{**}-closed$ set of (X,τ). Let U be a g^*-open set of (Y,σ) such that $f(A)\subseteq U$. Since f is $g^*-irresolute$, $f^{-1}(U)$ is g^*-open in (X,τ). Now $f^{-1}(U)$ and A is $g^{**}-closed$ set of (X,τ), then $cl(A)\subseteq f^{-1}(U)$. Then $f(cl(A)) = cl[f(cl(A))]$.

Therefore
$cl[f(A)] \subseteq cl[f(cl(A))] = f(cl(A)) \subseteq U$. Therefore $f(A)$ is a $g^{**}-closed$ set of (Y,σ).

Theorem 5.70: Let $f:(X,\tau)\to(Y,\sigma)$ be onto, $g^{**}-irresolute$ and closed. If (X,τ) is $T_{1/2}^{**}$, then (Y,σ) is also a $T_{1/2}^{**} - space$.

We introduce the following definition.

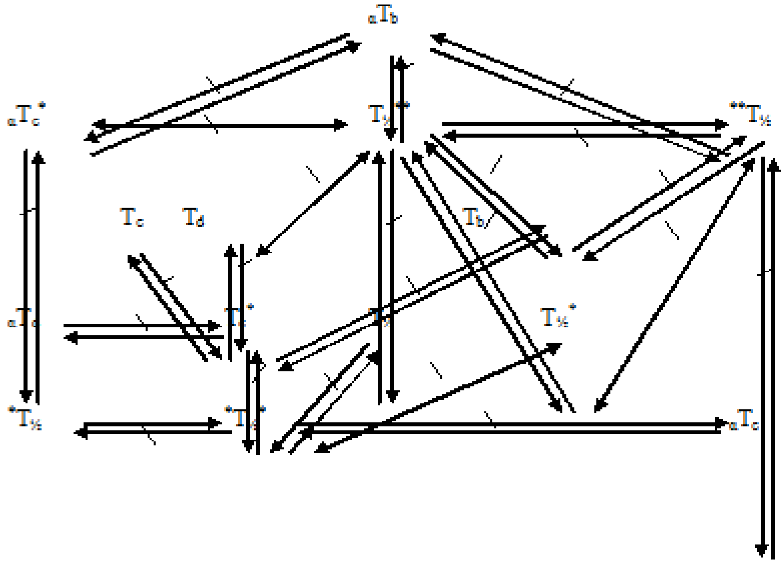
Definition 5.71: A function $f:(X,\tau)\to(Y,\sigma)$ is called a $pre-g^{**}-closed$ set if $f(A)$ is a $g^{**}-closed$ set of (Y,σ) for every $g^{**}-closed$ set of (X,τ).

Theorem 5.72: Let $f:(X,\tau)\to(Y,\sigma)$ be onto, $\alpha g-irresolute$ and $pre-g^{**}-closed$. If (X,τ) is a ${}_{\alpha}T_c{}^*-space$, then (Y,σ) is also an ${}_{\alpha}T_c{}^*-space$.

Theorem 5.73: Let $f:(X,\tau)\to(Y,\sigma)$ be onto, $g^{**}-irresolute$ and $pre-g^*-closed$. If (X,τ) is ${}^{**}T_{1/2}$, then (Y,σ) is also a ${}^{**}T_{1/2} - space$.

Theorem 5.74: Let $f:(X,\tau)\to(Y,\sigma)$ be onto, $gs-irresolute$ and $pre-g^{**}-closed$. If (X,τ) is $T_c^* - space$, then (Y,σ) is also a $T_c^* - space$.

The above results can be represented in the following figure.



Where A → B (resp. A ↮ B) represents A implies B (resp. A and B are independent).

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