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Fuzzy Anti n-Normed Linear Spaces

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ABSTRACT

The main purpose of this paper is to introduce the concept of Fuzzy Anti- n Normed Linear Space. Standard results in Fuzzy n-normed linear spaces are extended to fuzzy anti n- normed spaces.

Key words: *Fuzzy normed space, Fuzzy 2 normed space, Fuzzy anti 2-normed space, Fuzzy n- normed space and Fuzzy anti n-normed space*

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in 1965 and thereafter several authors applied it different branches of pure and applied mathematics. The concept of fuzzy norms was introduced by katsaras in 1984. In 1992 Felbin introduced the concept of Fuzzy normed linear space. A satisfactory theory of 2 norms of a linear space has been introduced and developed by the Gahler. The concept of Fuzzy 2 normed linear space introduced by A.R. Meenakshi and R. Gokilavani in 2001 and Fuzzy 2 linear operators by R.M. Somasundaram and Thangaraj Beaula. Jebril and Samanta gave the definition of Fuzzy anti normed linear space in 2011 .B. Sundander reddy introduced the idea of Fuzzy anti 2-normed linear spaces. Parijat Sinha, Divya Mishra, Ghanshyam lal have introduced the concept of Fuzzy anti 2-continuous linear operator. AL. Narayanan and S. Vijaya Balaji have introduced the notion of Fuzzy n- normed linear space. In this paper we introduce the concept of Fuzzy anti n- normed linear spaces.

2. PRELIMINARIES

For the sack of completeness, we reproduce the following definitions

Definition 2.1: Fuzzy normed space.

Let X be a vector space over the field K (K = Real or Complex). Let $\|.\|: X \to (0, \infty)$ be a function which assigns to each point x in X , $x \in (0,1)$, a non negative real number $\|x\|$ such that

(FN1) $\|x\| = 0$ if and only if $x = 0$

(FN2) $\|\lambda x\| = |\lambda| \, \|x\|$ for all $\lambda \varepsilon K$

(FN3) $\|x + y\| \le \|x\| + \|y\|$

(FN4) If $\|x_\alpha\| < r$ where $r > 0$ then there exists $0 < \alpha < 1$ such that $\|x_\alpha\| < r$

Then $\|.\|$ is called Fuzzy norm and (x, $\|.\|$) is called Fuzzy normed space.

Definition 2.2: 2-normed linear space.

Let X be a real vector space of dimension greater than 1 and let $\|.,.\|$ be a real valued function on X x X satisfying the following conditions.

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(2N1) $\|x, y\| = 0$ if and only if x and y are linearly dependent.

(2N2) $\|x, y\| = \|y, x\|$

(2N3) $\|x, \alpha y\| = |\alpha| \|x, y\|$ where α is real

(2N4) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$

Then $\|.,.\|$ is called norms on X and the pair $(X, \|.,.\|)$ is called 2-normed linear space

Definition 2.3: Fuzzy normed linear space.

Let X be a linear space over the field F. A fuzzy subset N of X×R (R is the set of all real numbers) is called a fuzzy norm on X if and only if for all x , u ∈ X and c ∈ K

(FN1) for all $t \in R$ with $t \leq 0$, $N(x, t) = 0$

(FN2) for all $t \in R$ with $t > 0$, $N(x, t) = 1$ if and only if $x = 0$

(FN3) for all $t \in R$ with $t > 0$, $N(c x, t) = N(x, \frac{t}{|c|})$ if $c \neq 0$

(FN4) for all $s, t \in R$, $x, u \in X$, $N(x + u, s + t) \geq \min\{N(x, s), N(x, t)\}$

(FN5) $N(x, *)$ is non decreasing function of R and $\lim_{t \to \infty} N(x, t) = 1$

The pair (X, N) will be referred as a fuzzy normed linear space

Definition 2.4: Fuzzy 2 - normed linear space.

Let X be a linear space over the field F. A fuzzy subset N of X×X×R (R is the set of real numbers) is called a fuzzy 2-norm on X if and only if

(F2N1) for all $t \in R$ with $t \leq 0$, $N(x_1, x_2, t) = 0$

(F2N2) for all $t \in R$ with $t > 0$, $N(x_1, x_2, t) = 1$ if and only if x_1 and x_2 are linearly dependent

(F2N3) $N(x_1, x_2, t)$ is invariant under any permutation of x_1, x_2

(F2N4) for all $t \in R$, with $t > 0$, $N(x_1, c x_2, t) = N(x_1, x_2, \frac{t}{|c|})$ is $c \neq 0$, $c \in F$

(F2N5) for all $s, t \in R$, $N(x_1, x_2 + x_2', s + t) \geq \min\{N(x_1, x_2, s), N(x_1, x_2', t)\}$

(F2N6) $N(x_1, x_2, *)$ is non decreasing function of R and $\lim_{t \to \infty} N(x_1, x_2, t) = 1$

Then (X, N) is called fuzzy 2-normed linear space

Definition 2.5: Fuzzy anti 2-normed linear space.

Let X be a linear space over the field F. A fuzzy subset N^* of X×X×R (R is the set of real numbers) is called a fuzzy anti 2- norm on X if and only if

(FA2N1) for all $t \in R$ with $t \leq 0$, $N^*(x_1, x_2, t) = 1$

(FA2N2) for all $t \in R$ with $t > 0$, $N^*(x_1, x_2, t) = 0$ if and only if x_1 and x_2 are linearly dependent

(FA2N3) $N^*(x_1, x_2, t)$ is invariant under any permutation of x_1, x_2

(FA2N4) for all $t \in R$, with $t > 0$, $N^*(x_1, c\,x_2, t) = N^*(x_1, x_2, \frac{t}{|c|})$ is $c \neq 0$, $c \in F$

(FA2N5) for all s, $t \in R$ with $t > 0$,

$$N^*(x_1, x_2 + x_2', s + t) \leq \max\{N^*(x_1, x_2, s), N^*(x_1, x_2, t)\}$$

(FA2N6) $N^*(x_1, x_2, *)$ is non increasing function of $t \in R$ and $\lim_{t \to \infty} N^*(x_1, x_2, t) = 0$

Then (X, N) is called fuzzy anti 2-normed linear space

3. FUZZY N-NORMED LINEAR SPACE AND FUZZY ANTI N-NORMED LINEAR SPACE

Definition 3.1: Fuzzy n-normed linear space.

Let X be a linear space over the field F. A fuzzy subset N of X×X ,…, X×X×R

(R is the set of real numbers) is called a fuzzy n-norm on X if and only if

(FnN1) for all $t \in R$ with $t \leq 0$, $N(x_1, x_2, \ldots, x_n, t) = 0$

(FnN2) for all $t \in R$ with $t > 0$, $N(x_1, x_2, \ldots, x_n, t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly independent

(FnN3) $N(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$

(FnN4) for all $t \in R$, with $t > 0$,

$$N(x_1, x_2, \ldots, c\,x_n, t) = N(x_1, x_2, \ldots, x_n, \frac{t}{|c|}) \quad \text{is} \quad c \neq 0, c \in F$$

(FnN5) for all s, $t \in R$,

$$N(x_1, x_2, \ldots, x_n + x_n', s + t) \geq \min\{N(x_1, x_2, \ldots, x_n, s), N(x_1, x_2, \ldots, x_n', t)\}$$

(FnN6) $N(x_1, x_2, \ldots, x_n, *)$ is non decreasing function of R and

$$\lim_{t \to \infty} N(x_1, x_2, \ldots, x_n, t) = 1$$

Then (X, N) is called fuzzy n-normed linear space

Definition 3.2: Fuzzy anti n-normed linear space

Let X be a linear space over the field F. A fuzzy subset N^* of X×X,…, X×X×R

(R is the set of real numbers) is called a fuzzy anti n – norm on X if and only if

(FAnN1) for all $t \in R$ with $t \leq 0$, $N^*(x_1, x_2, .., x_n, t) = 1$

(FAnN2) for all $t \in R$ with $t > 0$, $N^*(x_1, x_2, \ldots, x_n, t) = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly independent

(FAnN3) $N^*(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$

(FAnN4) for all $t \in R$, with $t > 0$,

$$N^*(x_1, x_2, \ldots, c\,x_n, t) = N^*(x_1, x_2, \ldots, x_n, \frac{t}{|c|}) \quad \text{is} \quad c \neq 0, c \in F$$

(FAnN5) for all s, $t \in R$, with $t > 0$

$$N^*(x_1, x_2, \ldots, x_n + x_n', s + t) \leq \max\{N^*(x_1, x_2, \ldots, x_n, s), N^*(x_1, x_2, \ldots, x_n', t)\}$$

(FAnN6) $N^*(x_1, x_2, \dots, x_n, *)$ is non increasing function of $t \in R$ and

$$\lim_{t\to\infty} N^*(x_1, x_2, \dots, x_n, t) = 0$$

Then (X, N^*) is called fuzzy anti n-normed linear space

Example: Let $(X, \|*,*, \dots, *\|)$ be an n- normed linear space . define

$$N^*(x_1, x_2, \dots, x_n, t) = \frac{\|x_1, x_2, \dots, x_n\|}{t + \|x_1, x_2, \dots, x_n\|} \text{ when } t > 0, t \in R$$

$$= 1, \text{ when } t \le 0$$

Proof:

(N1) for all $t \in R$ with $t \le 0$, we have

$$N^*(x_1, x_2, \dots, x_n, t) = \frac{\|x_1, x_2, \dots, x_n\|}{t + \|x_1, x_2, \dots, x_n\|} = 1$$

If and only if, $\|x_1, x_2, \dots, x_n\| = t + \|x_1, x_2, \dots, x_n\|$

If and only if, $\|x_1, x_2, \dots, x_n\| = 0$

If and only if $x_1, x_2, \dots, x_n$ are linearly dependent

(N2) If $t > 0$, we have $N^*(x_1, x_2, \dots, x_n, t) = \dfrac{\|x_1, x_2, \dots, x_n\|}{t + \|x_1, x_2, \dots, x_n\|} = 0$

This implies $\|x_1, x_2, \dots, x_n\| = 0$

(N3) for all $t \in R$ with $t > 0$

$$N^*(x_1, x_2, \dots, x_n, t) = \frac{\|x_1, x_2, \dots, x_n\|}{t + \|x_1, x_2, \dots, x_n\|} = \frac{\|x_1, x_2, \dots, x_n, x_{n-1}\|}{t + \|x_1, x_2, \dots, x_n, x_{n-1}\|}$$

$$= N^*(x_1, x_2, \dots, x_n, x_{n-1}, t)$$

(N4) for all $t \in R$ with $t > 0$ and $c \in F$, $c \neq 0$

$$N^*\left(x_1, x_2, \dots, x_n, \frac{t}{|c|}\right) = \frac{\|x_1, x_2, \dots, x_n\|}{(t/|c|) + \|x_1, x_2, \dots, x_n\|}$$

$$= \frac{(|c|\,\|x_1, x_2, \dots, x_n\|)/(|c|)}{(t + \|x_1, x_2, \dots, x_n\|\;|c|)/(|c|)}$$

$$= \frac{\|x_1, x_2, \dots, cx_n\|}{t + \|x_1, x_2, \dots, cx_n\|} = N^*(x_1, x_2, \dots, cx_n, t)$$

(N5) we have to prove

$$N^*(x_1, x_2, \dots, x_n + x_n', s + t) \le \max\{N^*(x_1, x_2, \dots, x_n, s), N^*(x_1, x_2, \dots, x_n', t)\}$$

If

(a). $s + t < 0$

(b). $s = t = 0$

(c). $s + t > 0$, $s > 0$, $t < 0$; $s < 0$, $t > 0$ then the above relation is obvious

(d). $S + t > 0$, $s > 0$, $t > 0$, then

$$N^*(x_1, x_2, \ldots, x_{n}+x'_n, s+t) = \frac{\|x_1, x_2, \ldots, x_n + x'_n\|}{s+t+\|x_1, x_2, \ldots, x_n, x'_n\|}$$

$$\leq \frac{\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|}{s+t+\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|}$$

If
$$\frac{\|x_1, x_2, \ldots, x_n\|}{s+\|x_1, x_2, \ldots, x_n\|} \leq \frac{\|x_1, x_2, \ldots, +x'_n\|}{t+\|x_1, x_2, \ldots, +x'_n\|}$$

$$\frac{\|x_1, x_2, \ldots, x_n\|}{s+\|x_1, x_2, \ldots, x_n\|} - \frac{\|x_1, x_2, \ldots, x'_n\|}{t+\|x_1, x_2, \ldots, x_n\|} \leq 0$$

Which implies

$$\left(\|x_1, x_2, \ldots, x_n\|\right)\left(t+\|x_1, x_2, \ldots, x_n\|\right) - \left(\|x_1, x_2, \ldots, x_n\|\right)\left(s+\|x_1, x_2, \ldots, x_n\|\right) \leq 0$$

$$t\left(\|x_1, x_2, \ldots, x_n\|\right) - s\left(\|x_1, x_2, \ldots, x_n\|\right) \leq 0 \qquad (1)$$

So, $\frac{\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|}{s+t+\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|} - \frac{\|x_1, x_2, \ldots, + x'_n\|}{t+\|x_1, x_2, \ldots, + x'_n\|} \leq 0$,

Using (1),
$$\frac{\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|}{s+t+\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|} \leq \frac{\|x_1, x_2, \ldots, + x'_n\|}{t+\|x_1, x_2, \ldots, + x'_n\|}$$

Similarly,
$$\frac{\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|}{s+t+\|x_1, x_2, \ldots, x_n\| + \|x_1, x_2, \ldots, + x'_n\|} \leq \frac{\|x_1, x_2, \ldots, +x_n\|}{s+\|x_1, x_2, \ldots, +x_n\|}$$

Then,

$N^*(x_1, x_2, \ldots, x_{n}+x'_n, s+t) \leq \max\{N^*(x_1, x_2, \ldots, x_n, s), N^*(x_1, x_2, \ldots, x'_n, t)\}$

(N6) for all $t_1, t_2 \in R$, if $t_1, t_2 > 0$, then by definition

$$N^*(x_1, x_2, \ldots, x_n, t_1) = N^*(x_1, x_2, \ldots, x_n, t_2)$$

Suppose $t_1 < t_2 \leq 0$, this implies

$$\frac{\|x_1, x_2, \ldots, +x_n\|}{t_2+\|x_1, x_2, \ldots, +x_n\|} - \frac{\|x_1, x_2, \ldots, +x_n\|}{t_1+\|x_1, x_2, \ldots, +x_n\|}$$
$$= \frac{\|x_1, x_2, \ldots, +x_n\|(t1-t2)}{(t_2+\|x_1, x_2, \ldots, +x_n\|)(t_1+\|x_1, x_2, \ldots, +x_n\|)} \leq 0$$

For all $(x_1, x_2, \ldots, +x_n) \in X \times X \times \ldots \times X$, implies
$$\frac{\|x_1, x_2, \ldots, +x_n\|}{t_2+\|x_1, x_2, \ldots, +x_n\|} \leq \frac{\|x_1, x_2, \ldots, +x_n\|}{t_1+\|x_1, x_2, \ldots, +x_n\|}$$

Which is implies

$$N^*(x_1, x_2, \dots, x_n, t_2) \leq N^*(x_1, x_2, \dots, x_n, t_1)$$

Thus $N^*(x_1, x_2, \dots, x_n, t)$ is a non increasing function

Also $\lim_{t\to\infty} N^*(x_1, x_2, \dots, x_n, t) = \lim_{t\to\infty} \frac{\| x_1, x_2, \dots, x_n \|}{t + \| x_1, x_2, \dots, + x_n \|} = 0$

Thus (X, N^*) is an fuzzy anti n – normed linear space

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