International Journal of Mathematical Archive-3(6), 2012, 2194-2203
IJMA Available online through www.ijma.info ISSN 2229 – 5046

Slightly *sg*-continuous; Somewhat sg-continuous and Somewhat sg-open functions

S. Balasubramanian*[1], C. Sandhya[2] and P. Aruna Swathi Vyjayanthi[3]

[1]Department of Mathematics, Govt. Arts College (A), Karur - 639 005, (TN), India
[2]Department of Mathematics, C.S.R. Sarma College, Ongole–523 001, (AP), India
[3]Research Scholar, Dravidian University, Kuppam - 517 425, (AP), India

(Received on: 27-03-12; Revised & Accepted on: 30-05-12)

ABSTRACT

In this paper we discuss new type of continuous functions called slightly sg-continuous; somewhat sg-continuous and somewhat sg-open functions; its properties and interrelation with other such functions are studied.

Keywords: *slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β-continuous functions; slightly γ-continuous functions and slightly ν-continuous functions; somewhat continuous functions; somewhat semi-continuous functions; somewhat pre-continuous; somewhat β-continuous functions; somewhat γ-continuous functions and somewhat ν-continuous functions; somewhat open functions; somewhat semi-open functions; somewhat pre-open; somewhat β-open functions; somewhat γ-open functions and somewhat ν–open functions*

AMS-Classification Numbers: *54C10; 54C08; 54C05.*

1. Introduction

In 1995 T.M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G.I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly β−continuous functionsin 2001. C.W. Baker introduced slightly precontinuous functions in 2002. Erdal Ekici and M. Caldas studied slightly γ-continuous functions in 2004. Arse Nagli Uresin and others studied slightly δ−continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly ν-continuous functions in 2011.

b-open sets are introduced by Andrijevic in 1996. K.R. Gentry introduced somewhat continuous functions in the year 1971. V.K. Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T. Noiri and N. Rajesh introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in this paper slightly *sg*−continuous, somewhat *sg*-continuous functions and somewhat *sg*-open functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X,τ) and (Y,σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. Preliminaries

Definition 2.1: $A\subset X$ is called
(i) closed if its complement is open.

(ii) rα-open [*v*-open] if $\exists\ U\in \alpha O(X)[RO(X)]$ such that $U\subset A\subset \alpha cl(U)[\ U\subset A\subset cl(U)]$.

(iii) semi-θ-open if it is the union of semi-regular sets and its complement is semi-θ-closed.

(iv) Regular closed[α-closed; pre-closed; β−closed] if $A = cl\{A^o\}$[resp:$(cl(A^o))^o\subseteq A$; $cl(A^o)\subseteq A$; $cl((cl(A))^o)\subseteq A$].

(v) Semi closed [*v*-closed] if its complement if semi open [*v*-open].

Corresponding author: *S. Balasubramanian**
[1]Department of Mathematics, Govt. Arts College (A), Karur – 639 005, (T.N.), India

(vi) g-closed [rg-closed] if cl A$\subseteq$ U whenever A$\subseteq$ U and U is open in X.

(vii) sg-closed [gs-closed] if s(cl A) $\subseteq$ U whenever A$\subseteq$ U and U is semi-open{open} in X.

(viii) *v*g-closed if *v*cl(A) $\subseteq$ U whenever A$\subseteq$ U and U is *v*-open in X.

(ix) b-open if A$\subset$(cl{A})$^o\cap$ cl{A^o}.

Definition 2.2: A function *f***:** X$\to$ Y is said to be
(i) continuous [resp**:** nearly-continuous; rα-continuous; *v*-continuous; α-continuous; semi-continuous; β-continuous; pre-continuous] if inverse image of each open set is open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β -open; preopen].

(ii) nearly-irresolute [resp**:** rα-irresolute; *v*-irresolute; α-irresolute; irresolute; β-irresolute; pre -irresolute] if inverse image of each regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen] set is regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen].

(iii) almost continuous[resp**:** almost nearly-continuous; almost rα-continuous; almost *v*-continuous; almost α-continuous; almost semi-continuous; almost β-continuous; almost pre-continuous] if for each x in X and each open set (V, *f*(x)), $\exists$ an open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that *f*(U) $\subset$ (cl(V))^o.

(iv) weakly continuous[resp**:** weakly nearly-continuous; weakly rα-continuous; weakly *v*-continuous; weakly α-continuous; weakly semi-continuous; weakly β-continuous; weakly pre-continuous] if for each x in X and each open set (V, *f*(x)), $\exists$ an open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that *f*(U) $\subset$ cl(V).

(v) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β-continuous; slightly γ-continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at x in X if for each clopen subset V in Y containing *f*(x), $\exists$ U$\in$ τ (X)[$\exists$ U$\in$ SO(X); $\exists$ U$\in$ PO(X); $\exists$ U$\in$ βO(X); $\exists$ U$\in$ γ O(X); $\exists$ U$\in$ α O(X); $\exists$ U$\in$ RO(X); $\exists$ U$\in$ *v* O(X)] containing x such that *f*(U) $\subseteq$ V.

(vi) slightly continuous[resp**:** slightly semi-continuous; slightly pre-continuous; slightly β-continuous; slightly γ-continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] if it is slightly-continuous [resp**:** slightly semi-continuous; slightly pre-continuous; slightly β-continuous; slightly γ-continuous; slightly α-continuous; slightly r-continuous; slightly *v*-continuous] at each x in X.

(vii) almost strongly θ-semi-continuous[resp**:** strongly θ-semi-continuous] if for each x in X and for each V$\in$ σ(Y, *f*(x)), $\exists$ U$\in$ SO(X, x) such that *f*(scl(U)) $\subset$ scl(V)[resp**:** *f*(scl(U)) $\subset$ V].

(viii) somewhat continuous[resp: somewhat b-continuous; somewhat *v*-continuous] if for U$\in\sigma$ and f^{-1}(U) $\neq$ φ, there exists a non empty open[resp: non empty b-open; non empty *v*-open] set V in X such that V$\neq$ φ and V$\subset$ f^{-1}(U).

(ix) somewhat-open[resp: somewhat b--open; somewhat *v*-open] provided that if U$\in\tau$ and U$\neq$ φ, then there exists a non empty b-open set[resp: non empty b-open; non empty *v*-open] V in Y such that V $\neq$ φ and V$\subset$ *f*(U).

(x) somewhat *v*-irresolute if for U$\in$ *v*O(σ) and f^{-1}(U) $\neq$ φ , there exists a non-empty *v*-open set V in X such that V$\subset$ f^{-1}(U).

Definition 2.3: X is said to be a
(i) compact [resp**:** nearly-compact; rα-compact; *v*-compact; α-compact; semi-compact; β-compact; pre -compact; mildly-compact] space if every open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen; clopen] cover has a finite subcover.

(ii) countably-compact[resp**:** countably-nearly-compact; countably-rα-compact; countably-*v*-compact; countably-α-compact; countably-semi-compact; countably-β-compact; countably-pre-compact; mildly-countably compact] space if every countable open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen; clopen] cover has a finite subcover.

(iii) closed-compact [resp**:** closed-nearly-compact; closed-rα-compact; closed-ν-compact; closed-α-compact; closed-semi-compact; closed-β–compact; closed-pre-compact] space if every closed [resp**:** regular-closed; rα-closed; ν-closed; α-closed; semi-closed; β-closed; preclosed] cover has a finite subcover.

(iv) Lindeloff[resp**:** nearly-Lindeloff; rα-Lindeloff; ν-Lindeloff; α-Lindeloff; semi-Lindeloff; β-Lindeloff; pre - Lindeloff; mildly-Lindeloff] space if every open[resp**:** regular-open; rα-open; ν-open; α-open; semi-open; β-open; preopen; clopen] cover has a countable subcover.

(v) Extremally disconnected [briefly e.d] if the closure of each open set is open.

Lemma 2.1:
(i) Let A and B be subsets of a space X, if $A\in SGO(X)$ and $B\in RO(X)$, then $A\cap B\in SGO(B)$.
(ii) Let $A\subset B\subset X$, if $A\in SGO(B)$ and $B\in RO(X)$, then $A\in SGO(X)$.

3. Slightly *sg*-continuous functions

Definition 3.1: A function f**:** $X\to Y$ is said to be
(i) slightly *sg*-continuous function at x in X if for each clopen subset V in Y containing $f(x)$, $\exists$ $U\in SGO(X)$ containing x such that $f(U)\subseteq V$.
(ii) slightly *sg*-continuous function if it is slightly *sg*-continuous at each x in X.

Note 1: Here after we call slightly *sg*-continuous function as sl.*sg*.c function shortly.

Example 3.1: $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f**:** $X\to Y$ defined as $f(a) = b$; $f(b) = c$ and $f(c) = c$, then f is sl.sg.c.

Example 3.2: $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let f**:** $X\to Y$ defined as follows:

(i) $f(a) = b$; $f(b) = c$ and $f(c) = a$, then f is not sl.sg.c.
(ii) $f(a) = b$; $f(b) = a$ and $f(c) = c$, then f is not sl.sg.c.

Theorem 3.1: The following are equivalent:
(i) f**:** $X\to Y$ is sl.sg.c.
(ii) $f^{-1}(V)$ is *sg*-open for every clopen set V in Y.
(iii) $f^{-1}(V)$ is *sg*-closed for every clopen set V in Y.
(iv) $f(sgcl(A))\subseteq sgcl(f(A))$.

Corollary 3.1: The following are equivalent.
(i) f**:** $X\to Y$ is sl.sg.c.
(ii) For each x in X and each clopen subset $V\in (Y, f(x))$ $\exists$ $U\in SGO(X, x)$ such that $f(U)\subseteq V$.

Theorem 3.2: Let $\sum = \{U_i{:}i\in I\}$ be any cover of X by regular open sets in X. A function f is sl.sg.c. iff $f_{/U_i}$**:** is sl.sg.c., for each $i\in I$.

Proof: Let $i\in I$ be an arbitrarily fixed index and $U_i\in RO(X)$. Let $x\in U_i$ and $V\in CO(Y, f_{U_i}(x))$ Since f is sl.sg.c, $\exists$ $U\in SGO(X, x)$ such that $f(U)\subset V$. Since $U_i\in RO(X)$, by Lemma 2.1 $x\in U\cap U_i\in SGO(U_i)$ and $(f_{/U_i})U\cap U_i = f(U\cap U_i)\subset f(U)\subset V$. Hence $f_{/U_i}$ is sl.sg.c.

Conversely Let x in X and $V\in CO(Y, f(x))$, $\exists$ $i\in I$ such that $x\in U_i$. Since $f_{/U_i}$ is sl.*sg*.c, $\exists$ $U\in SGO(U_i, x)$ such that $f_{/U_i}(U)\subset V$. By Lemma 2.1, $U\in SGO(X)$ and $f(U)\subset V$. Hence f is sl.sg.c.

Theorem 3.3:
(i) If f**:** $X\to Y$ is *sg*-irresolute and g**:** $Y\to Z$ is sl.sg.c.[slightly-continuous], then $g\bullet f$ is sl.sg.c.
(ii) If f**:** $X\to Y$ is *sg*-irresolute and g**:** $Y\to Z$ is *g*.-continuous, then $g\bullet f$ is sl.sg.c.
(iii) If f**:** $X\to Y$ is *sg*-continuous and g**:** $Y\to Z$ is slightly-continuous, then $g\bullet f$ is sl.sg.c.
(iv) If f**:** $X\to Y$ is rg-continuous and g**:** $Y\to Z$ is sl.sg.c. [slightly-continuous], then $g\bullet f$ is sl.sg.c.

Theorem 3.4: If f**:** $X\to Y$ is *sg*-irresolute, *sg*-open and $SGO(X) = \tau$ and g**:** $Y\to Z$ be any function, then $g\bullet f$**:** $X\to Z$ is sl.sg.c iff g**:** $Y\to Z$ is sl.sg.c.

Proof: If part**:** Theorem 3.3(i)

Only if part**:** Let A be clopen subset of Z. Then $(g\bullet f)^{-1}(A)$ is a *sg*-open subset of X and hence open in X[by assumption]. Since *f* is *sg*-open $f(g\bullet f)^{-1}(A)$ is *sg*-open Y $\Rightarrow g^{-1}(A)$ is *sg*-open in Y. Thus g**:** Y→ Z is sl.sg.c.

Corollary 3.2: If *f***:** X→ Y is *sg*-irresolute, *sg*-open and bijective, *g***:** Y→ Z is a function. Then *g***:** Y→ Z is sl.sg.c. iff $g\bullet f$ is sl.sg.c.

Theorem 3.5: If *g***:** X → X× Y, defined by *g*(x) = (x, *f*(x)) for all x in X be the graph function of *f***:** X → Y. Then *g***:** X→ X×Y is sl.*sg*.c iff *f* is sl.*sg*.c.

Proof: Let V∈ CO(Y), then X× V is clopen in X× Y. Since *g***:** X→ Y is sl.sg.c., $f^{-1}(V) = f^{-1}(X\times V) \in SGO(X)$. Thus *f* is sl.sg.c.

Conversely, let x in X and F be a clopen subset of X× Y containing *g*(x). Then $F\cap(\{x\}\times Y)$ is clopen in $\{x\}\times Y$ containing *g*(x). Also $\{x\}\times Y$ is homeomorphic to Y. Hence $\{y\in Y:(x, y) \in F\}$ is clopen subset of Y. Since *f* is sl.sg.c. $\cup\{f^{-1}(y) :(x, y) \in F\}$ is *sg*-open in X. Further $x\in \cup\{f^{-1}(y) :(x, y) \in F\}\subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is *sg*-open. Thus *g***:** X→ Y is sl.sg.c.

Theorem 3.6:
(i) If *f***:** $X\to \Pi Y_\lambda$ is sl.*sg*.c, then $P_\lambda\bullet f$: $X\to Y_\lambda$ is sl.*sg*.c for each $\lambda\in\Gamma$, where P_λ is the projection of ΠY_λ onto Y_λ.
(ii) *f***:** $\Pi X_\lambda\to \Pi Y_\lambda$ is sl.*sg*.c, iff f_λ**:** $X_\lambda\to Y_\lambda$ is sl.*sg*.c for each $\lambda\in\Gamma$.

Remark 1:
(i) Composition of two sl.*sg*.c functions is not in general sl.*sg*.c.
(ii) Algebraic sum and product of sl.*sg*.c functions is not in general sl.*sg*.c.
(iii) The pointwise limit of a sequence of sl.*sg*.c functions is not in general sl.*sg*.c.

Example 3.3: Let X = Y = [0, 1]. Let f_n**:** X→ Y is defined as follows $f_n(x) = x_n$ for n = 1, 2, 3, . ., then *f***:**X→ Y defined by $f(x) = 0$ if $0 \le x < 1$ and $f(x) = 1$ if x = 1. Therefore each f_n is sl.*sg*.c but *f* is not sl.*sg*.c. For (1/2, 1] is clopen in Y, but $f^{-1}((1/2, 1]) = \{1\}$ is not *sg*-open in X.

However we can prove the following**:**

Theorem 3.7: The uniform limit of a sequence of sl.*sg*.c functions is sl.*sg*.c.

Note 2: Pasting Lemma is not true for sl.*sg*.c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that X = A∪ B and let $f_{/A}$**:** A→ Y and $g_{/B}$**:** B → Y are sl.r.c maps such that *f*(x) = *g*(x) for all x∈ A∩B. Suppose A and B are r-open sets in X and RO(X) is closed under finite unions, then the combination α**:** X→ Y is sl.*sg*.c continuous.

Theorem 3.9: Pasting Lemma Let X and Y be spaces such that X = A∪ B and let $f_{/A}$**:** A→ Y and $g_{/B}$**:** B → Y are sl.*sg*.c maps such that *f*(x) = *g*(x) for all x∈ A∩ B. Suppose A, B are r-open sets in X and *SGO*(X) is closed under finite unions, then the combination α**:** X→ Y is sl.*sg*.c.

Proof: Let F∈CO(Y), then $\alpha^{-1}(F) = f^{-1}(F)\cup g^{-1}(F)$, where $f^{-1}(F)\in SGO(A)$ and $g^{-1}(F)\in SGO(B) \Rightarrow f^{-1}(F)$; $g^{-1}(F)\in SGO(X) \Rightarrow f^{-1}(F)\cup g^{-1}(F)\in SGO(X)$[by assumption]. Therefore $\alpha^{-1}(F)\in SGO(X)$. Hence α**:** X→Y is sl.*sg*.c.

4. Somewhat sg-continuous function:

Definition 4.1: A function *f* is said to be somewhat sg-continuous if for U∈σ and $f^{-1}(U) \ne \varphi$, there exists a non-empty sg-open set V in X such that $V\subset f^{-1}(U)$.

It is clear that every continuous function is somewhat continuous and every somewhat continuous is somewhat sg-continuous. But the converses are not true by Example 1 of [17] and the following example.

Example 4.1: Let X = {a, b, c}, τ = {φ, {b},{a, b},{b,c},X} and σ = {φ, {a}, {b, c}, X}. The function *f* :(X,τ)→ (X,σ) defined by *f*(a) = b, *f*(b) = c and *f*(c) = a is somewhat sg-continuous.

Note 3: Every somewhat *sg* continuous function is slightly *sg* continuous.

Example 4.2: Let X = {a, b, c}, τ = {φ, {b, c}, X}, σ = {φ, {b}, {a, c}, X} and η = {φ, {a}, X}. Then the identity functions f :(X, τ)$\to$ (X, σ) and g:(X, σ)$\to$ (X; η) and g•f are somewhat sg-continuous.

However, we have the following

Theorem 4.1: If f is somewhat sg-continuous and g is continuous, then g•f is somewhat sg-continuous.

Corollary 4.1:
(i) If f is somewhat sg-continuous and g is r-continuous, then g•f is somewhat sg-continuous.
(ii) If f is somewhat sg-continuous and g is r-irresolute, then g•f is somewhat sg-continuous.
(iii)If f is somewhat sg-continuous and g is r-continuous, then g•f is somewhat sg-continuous.

Theorem 4.2: For a surjective function f, the following statements are equivalent:
(i) f is somewhat sg-continuous.

(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper sg-closed subset D of X such that $f^{-1}(C) \subset D$.

(iii) If M is a sg-dense subset of X, then f(M) is a dense subset of Y.

Proof: (i) ⇒(ii): Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then Y-C is an open set in Y such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists a sg- open set V$\in$ SGO(X) such that V $\neq \varphi$ and V$\subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. This means that X-V$\supset f^{-1}(C)$ and X - V = D is a proper sg-closed set in X.

(ii) ⇒(i): Let U$\in\sigma$ and $f^{-1}(U) \neq \varphi$ Then Y-U is closed and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper sg-closed set D such that D$\supset f^{-1}(Y\text{-}U)$. This implies that X-D $\subset f^{-1}(U)$ and X-D is sg-open and X-D $\neq \varphi$.

(ii) ⇒(iii): Let M be a sg-dense set in X. Suppose that f(M) is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper sg-closed set D such that M$\subset f^{-1}(C) \subset D \subset$ X. This is a contradiction to the fact that M is sg-dense in X.

(iii) ⇒(ii): Suppose (ii) is not true. there exists a closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper sg-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is sg-dense in X. But by (iii), $f(f^{-1}(C))$ = C must be dense in Y, which is a contradiction to the choice of C.

Theorem 4.3: Let f be a function and X = A$\cup$ B, where A, B$\in\tau$(X). If the restriction functions $f_{/A}$: (A; $\tau_{/A}$) $\to$ (Y,σ) and $f_{/B}$:(B; $\tau_{/B}$) $\to$ (Y, σ) are somewhat sg-continuous, then f is somewhat sg-continuous.

Proof: Let U$\in\sigma$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat sg-continuous, there exists a sg-open set V in A such that V$\neq \varphi$ and V$\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V is sg-open in A and A is r-open in X, V is sg-open in X.

Thus f is somewhat sg-continuous.

The proof of other cases are similar.

Definition 4.2: If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: sg- equivalent] to σ provided if U$\in\tau$ and U $\neq \varphi$, then there is an open[resp:sg-open] set V in X such that V $\neq \varphi$ and V $\subset$ U and if U$\in\sigma$ and U $\neq \varphi$, then there is an open[resp:sg-open] set V in (X, τ) such that V$\neq \varphi$ and U $\supset$ V.

Definition 4.3: A$\subset$ X is said to be sg-dense in X if there is no proper sg-closed set C in X such that M$\subset$ C$\subset$ X.

Now, consider the identity function f and assume that τ and σ are sg-equivalent. Then f and f^{-1} are somewhat sg-continuous. Conversely, if the identity functions f is somewhat sg-continuous in both directions, then τ and σ are sg-equivalent.

Theorem 4.4: Let f :(X, τ)$\to$ (Y, σ) be a somewhat sg-continuous surjection and τ* be a topology for X, which is sg-equivalent to τ. Then f : (X, τ*)$\to$ (Y, σ) is somewhat sg-continuous.

Proof: Let V$\in\sigma$ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat sg-continuous, there exists a nonempty sg-open set U in (X,τ) such that U$\subset f^{-1}(V)$. But by hypothesis τ* is sg-equivalent to τ.

Therefore, there exists a sg-open set $U^* \in (X; \tau^*)$ such that $U^* \subset U$. But $U \subset f^{-1}(V)$. Then $U^* \subset f^{-1}(V)$; hence $f:(X, \tau^*) \rightarrow (Y, \sigma)$ is somewhat sg-continuous.

Theorem 4.5: Let $f:(X,\tau) \rightarrow (Y, \sigma)$ be a somewhat sg-continuous surjection and σ^* be a topology for Y, which is equivalent to σ. Then $f:(X, \tau) \rightarrow (Y, \sigma^*)$ is somewhat sg-continuous.

Proof: Let $V^* \in \sigma^*$ such that $f^{-1}(V^*) \neq \varphi$. Since σ^* is equivalent to σ, there exists a nonempty open set V in (Y, σ) such that $V \subset V^*$. Now $\varphi = f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat sg-continuous, there exists a nonempty sg-open set U in (X,τ) such that $U \subset f^{-1}(V)$. Then $U \subset f^{-1}(V^*)$; hence $f:(X, \tau) \rightarrow (Y, \sigma^*)$ is somewhat sg-continuous.

5. Somewhat sg-open function

Definition 5.1: A function f is said to be somewhat sg-open provided that if $U \in \tau$ and $U \neq \varphi$, then there exists a non-empty sg-open set V in Y such that $V \subset f(U)$.

Example 5.1: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function $f: (X,\tau) \rightarrow (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat sg-open, somewhat sg-open and somewhat open.

Example 5.2: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. The function f: $(X,\tau) \rightarrow (X,\sigma)$ defined by $f(a) = c$, $f(b) = a$ and $f(c) = b$ is not somewhat sg-open.

Theorem 5.1: Let f be an r-open function and g somewhat sg-open. Then $g \bullet f$ is somewhat sg-open.

Theorem 5.2: For a bijective function f, the following are equivalent:
(i) f is somewhat sg-open.
(ii) If C is a closed subset of X, such that $f(C) \neq Y$, then there is a sg-closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.

Proof: (i) ⇒(ii): Let C be any closed subset of X such that $f(C) \neq Y$. Then X-C is open in X and $X\text{-}C \neq \varphi$.

Since f is somewhat sg-open, there exists a sg-open set $V \neq \varphi$ in Y such that $V \subset f(X\text{-}C)$. Put $D = Y\text{-}V$.

Clearly D is sg-closed in Y and we claim $D \neq Y$. If $D = Y$, then $V = \varphi$, which is a contradiction.

Since $V \subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.

(ii) ⇒(i): Let U be any nonempty open subset of X. Then $C = X\text{-}U$ is a closed set in X and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a sg-closed set D of Y such that $D \neq Y$ and $f(C) \subset D$.

Clearly $V = Y\text{-}D$ is a sg-open set and $V \neq \varphi$. Also, $V = Y\text{-}D \subset Y\text{-} f(C) = Y\text{-} f(X\text{-}U) = f(U)$.

Theorem 5.3: The following statements are equivalent:
(i) f is somewhat sg-open.
(ii) If A is a sg-dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.

Proof: (i) ⇒(ii): Suppose A is a sg-dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists a closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since f is somewhat sg-open and X-B is open, there exists a nonempty sg-open set C in Y such that $C \subset f(X\text{-}B)$. Therefore, $C \subset f(X\text{-}B) \subset f(f^{-1}(Y\text{-}A)) \subset Y\text{-}A$. That is, $A \subset Y\text{-}C \subset Y$.

Now, Y-C is a sg-closed set and $A \subset Y\text{-}C \subset Y$. This implies that A is not a sg-dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.

(ii) ⇒(i): Suppose A is a nonempty open subset of X. We want to show that $sg(f(A))^o \neq \varphi$. Suppose $sg(f(A))^o = \varphi$. Then, $sgcl(f(A)) = Y$. Therefore, by (ii), $f^{-1}(Y - f(A))$ is dense in X. But $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$. Now, X-A is closed. Therefore, $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$ gives $X = cl\{(f^{-1}(Y\text{-} f(A)))\} \subset X\text{-}A$. This implies that $A = \varphi$, which is contrary to $A \neq \varphi$. Therefore, $sg(f(A))^o \neq \varphi$. Hence f is somewhat sg-open.

Theorem 5.4: Let f be somewhat sg-open and A be any r-open subset of X. Then $f_{/A}:(A; \tau_{/A}) \rightarrow (Y, \sigma)$ is somewhat sg-open.

Proof: Let $U \in \tau_{/A}$ such that $U \neq \varphi$. Since U is r-open in A and A is r-open in X, U is r-open in X and since by hypothesis f is somewhat sg-open function, there exists a sg-open set V in Y, such that $V \subset f(U)$. Thus, for any open set of A with U≠ φ, there exists a sg-open set V in Y such that V⊂ f(U) which implies $f_{/A}$ is a somewhat sg-open function.

Theorem 5.5: Let f be a function and X = A∪ B, where A, B∈τ(X). If the restriction functions $f_{/A}$ and $f_{/B}$ are somewhat sg-open, then f is somewhat sg-open.

Proof: Let U be any open subset of X such that U≠ φ. Since X = A∪ B, either A∩ U ≠ φ or B∩U ≠ φ or both A∩U ≠ φ and B∩U ≠ φ. Since U is open in X, U is open in both A and B.

Case (i): Suppose that A∩U ≠ φ, where U∩A is open in A. Since $f_{/A}$ is somewhat sg-open function, there exists a sg-open set V of Y such that V⊂f(U∩A) ⊂ f(U), which implies that f is a somewhat sg-open function.

Case (ii): Suppose that B∩U ≠ φ, where U∩B is r-open in B. Since $f_{/B}$ is somewhat sg-open function, there exists a sg-open set V in Y such that V⊂ f(U∩B) ⊂ f(U), which implies that f is also a somewhat sg-open function.

Case (iii): Suppose that both A∩U ≠ φ and B∩U ≠ φ. Then by case (i) and (ii) f is a somewhat sg-open function.

Remark 3: Two topologies τ and σ for X are said to be sg-equivalent if and only if the identity function f: (X,τ)→ (Y,σ) is somewhat sg-open in both directions.

Theorem 5.6: Let f: (X,τ)→ (Y,σ) be a somewhat almost open function. Let τ* and σ* be topologies for X and Y, respectively such that τ* is equivalent to τ and σ* is sg-equivalent to σ. Then f: (X; τ*)→ (Y; σ*) is somewhat sg-open.

6. Covering and Separation properties of sl.*sg*.c. and swt.*sg*.c. functions

Theorem 6.1: If f**:** X→ Y is sl.sg.c.[resp**:** sl.r.c] surjection and X is *sg*-compact, then Y is compact.

Proof: Let {G_i**:**i∈ I} be any open cover for Y. Then each G_i is open in Y and hence each G_i is clopen in Y. Since f**:** X→ Y is sl.sg.c., $f^{-1}(G_i)$ is *sg*-open in X. Thus {$f^{-1}(G_i)$} forms a *sg*-open cover for X and hence have a finite subcover, since X is *sg*-compact. Since f is surjection, Y = f(X) = $\cup^n_{i=1}G_i$. Therefore Y is compact.

Corollary 6.1: If f**:**X→ Y is sl.g.c.[resp**:** sl.r.c] surjection and X is *sg*-compact, then Y is compact.

Theorem 6.2: If f**:** X→ Y is sl.sg.c., surjection and X is *sg*-compact[*sg*-lindeloff] then Y is mildly compact[mildly lindeloff].

Proof: Let {U_i**:**i∈ I} be clopen cover for Y. For each x in X, ∃ α_x∈ I such that f(x) ∈ $U_{\alpha x}$ and ∃ V_x∈ *SGO*(X, x) such that $f(V_x)$⊂ $U_{\alpha x}$. Since the family {V_i**:**i∈ I} is a cover of X by *sg*-open sets of X, ∃ a finite subset I_0 of I such that X⊂ {V_x**:**x∈ I_0}. Therefore Y⊂ ∪ {$f(V_x)$**:**x∈ I_0}⊂ ∪ {$U_{\alpha x}$**:**x∈ I_0}. Hence Y is mildly compact.

Corollary 6.2:

(i) If f**:** X→ Y is sl.rg.c[resp**:** sl.*sp*.c.; sl.r.c] surjection and X is *sg*-compact[*sg*-lindeloff] then Y is mildly compact[mildly lindeloff].

(ii) If f**:** X→ Y is sl.sg.c.[resp**:** sl.rg.c; sl.*sp*.c.; sl.r.c] surjection and X is locally *sg*-compact{resp**:***sg*-Lindeloff; locally *sg*-lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff}.

(iii) If f**:** X→ Y is sl.sg.c., surjection and X is semi-compact[semi-lindeloff] then Y is mildly compact[mildly lindeloff].

(iv) If f**:** X→ Y is sl.sg.c., surjection and X is β−compact[β−lindeloff] then Y is mildly compact[mildly lindeloff].

(v) If f**:** X→ Y is sl.sg.c.[sl.r.c.], surjection and X is locally *sg*-compact{resp**:** *sg*-lindeloff; locally *sg*-lindeloff} then Y is locally mildly compact{resp**:** locally mildly lindeloff}.

Theorem 6.3: If f**:** X→ Y is sl.sg.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Proof: Let {V_i **:** V_i∈ CO(Y); i∈ I} be a cover of Y, then {$f^{-1}(V_i)$ **:** i∈ I} is *sg*-open cover of X[by Thm 3.1] and so there is finite subset I_0 of I, such that {$f^{-1}(V_i)$**:**i∈ I_0} covers X. Therefore {V_i **:** i∈ I_0} covers Y since f is surjection. Hence Y is mildly compact.

Corollary 6.3: If f**:** X→ Y is sl.c[resp**:** sl.*s*.c.; sl.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.4: If f**:** X→ Y is sl.sg.c.,[resp**:** sl.c.; sl.*s*.c.; sl.r.c.] surjection and X is *sg*-connected, then Y is connected.

Proof: If Y is disconnected, then Y = A∪ B where A and B are disjoint clopen sets in Y. Since f is sl.sg.c. surjection, X = f^{-1}(Y) = f^{-1}(A) ∪ f^{-1}(B) where f^{-1}(A) f^{-1}(B) are disjoint *sg*-open sets in X, which is a contradiction for X is *sg*-connected. Hence Y is connected.

Corollary 6.4: The inverse image of a disconnected space under a sl.sg.c.,[resp**:** sl.rg.c.; sl.*sp*.c.; sl.r.c.] surjection is *sg*-disconnected.

Theorem 6.5: If f**:** X→ Y is sl.sg.c..[resp**:** sl.c.; sl.*s*.c.], injection and Y is UT_i, then X is sg_i i = 0, 1, 2.

Proof: Let $x_1 \neq x_2 \in$ X. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is UT_2 $\exists$ $V_j \in CO(Y)$ such that $f(x_j) \in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in SGO(X)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is sg_2.

Theorem 6.6: If f**:** X→Y is sl.sg.c.[resp**:** sl.c.; sl.r.c.] injection; closed and Y is UT_i, then X is sgg_i i = 3, 4.

Proof:
(i) Let x in X and F be disjoint closed subset of X not containing x, then f(x) and f(F) be disjoint closed subset of Y not containing f(x), since f is closed and injection. Since Y is ultraregular, f(x) and f(F) are separated by disjoint clopen sets U and V respectively. Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in SGO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is sgg_3.

(ii) Let F_j and $f(F_j)$ are disjoint closed subsets of X and Y respectively for j = 1,2, since f is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in SGO(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is sgg_4.

Theorem 6.7: If f**:** X→ Y is sl.sg.c.[resp**:** sl.r.c.; sl.c.], injection and
(i) Y is UC_i[resp**:** UD_i] then X is sgC_i[resp**:** sgD_i] i = 0, 1, 2.
(ii)Y is UR_i, then X is sg-R_i i = 0, 1.

Theorem 6.8: If f**:** X→ Y is sl.sg.c.[resp**:** sl.c; sl.r.c] and Y is UT_2, then the graph G(f) of f is *sg*-closed in the product space X× Y.

Proof: Let $(x_1, x_2) \notin G(f)$ implies $y \neq f(x)$ implies $\exists$ disjoint V; W∈CO(Y) such that $f(x) \in V$ and $y \in W$. Since f is sl.sg.c., $\exists$ $U \in SGO(X)$ such that $x \in U$ and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is *sg*-closed in X×Y.

Theorem 6.9: If f**:** X→ Y is sl.sg.c.[resp**:** sl.c; sl.r.c] and Y is UT_2, then A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is *sg*-closed in the product space X× X.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in CO(Y)$ such that $f(x_j) \in V_j$, and since f is sl.sg.c., $f^{-1}(V_j) \in SGO(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in SGO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is *sg*-closed.

Theorem 6.10: If f**:** X→ Y is sl.r.c.[resp**:** sl.c.]; g**:** X→ Y is sl.sg.c[resp**:** sl.r.c; sl.c]; and Y is UT_2, then E = {x in X**:** $f(x) = g(x)$} is *sg*-closed in X.

Following definitions 3.1; 4.1 and Note 3, we have the following consequences of theorems 6.1 to 6.10:

Theorem 6.11: If f**:** X→ Y is swt.sg.c.[resp**:** swt.r.c] surjection and X is *sg*-compact, then Y is compact.

Corollary 6.5: If f**:** X→ Y is swt.g.c.[resp**:** swt.r.c] surjection and X is *sg*-compact, then Y is compact.

Theorem 6.12: If f**:** X→ Y is swt.sg.c., surjection and X is *sg*-compact[*sg*-lindeloff] then Y is mildly compact[mildly lindeloff].

Corollary 6.6:
(i) If f**:** X→ Y is swt.rg.c[resp**:** swt.*sp*.c.; swt.r.c] surjection and X is *sg*-compact[*sg*-lindeloff] then Y is mildly compact[mildly lindeloff].

(ii) If *f*: X→ Y is swt.sg.c.[resp: swt.rg.c; swt.*sp*.c.; swt.r.c] surjection and X is locally *sg*-compact{resp:*sg*-Lindeloff; locally *sg*-lindeloff}, then Y is locally compact{resp: Lindeloff; locally lindeloff}.

(iii)If *f*: X→ Y is swt.sg.c., surjection and X is semi-compact[semi-lindeloff] then Y is mildly compact[mildly lindeloff].

(iv) If *f*: X→ Y is swt.sg.c., surjection and X is β−compact[β−lindeloff] then Y is mildly compact[mildly lindeloff].

(v) If *f*: X→ Y is swt.sg.c.[swt.r.c.], surjection and X is locally *sg*-compact{resp: *sg*-lindeloff; locally *sg*-lindeloff} then Y is locally mildly compact{resp: locally mildly lindeloff}.

Theorem 6.13: If *f*: X→ Y is swt.sg.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Corollary 6.7: If *f*: X→ Y is swt.c[resp: swt.*s*.c.; swt.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.14: If *f*: X→ Y is swt.sg.c.,[resp: swt.c.; swt.*s*.c.; swt.r.c.] surjection and X is *sg*-connected, then Y is connected.

Corollary 6.8: The inverse image of a disconnected space under a swt.sg.c.,[resp: swt.rg.c.; swt.*sp*.c.; swt.r.c.] surjection is *sg*-disconnected.

Theorem 6.15: If *f*: X→ Y is swt.sg.c..[resp: swt.c.; swt.*s*.c.], injection and Y is UT_i, then X is sg_i i = 0, 1, 2.

Theorem 6.16: If *f*:X→Y is swt.sg.c.[resp: swt.c.; swt.r.c.] injection; closed and Y is UT_i, then X is sgg_i i = 3, 4.

Theorem 6.17: If *f*: X→ Y is swt.sg.c.[resp: swt.r.c.; swt.c.], injection and
(i) Y is UC_i[resp: UD_i] then X is sgC_i[resp: sgD_i] i = 0, 1, 2.
(ii)Y is UR_i, then X is sg-R_i i = 0, 1.

Theorem 6.18: If *f*: X→ Y is swt.sg.c.[resp: swt.c; swt.r.c] and Y is UT_2, then
(i) the graph G(*f*) of *f* is *sg*-closed in the product space X× Y.
(ii) A = $\{(x_1, x_2)| f(x_1) = f(x_2)\}$ is *sg*-closed in the product space X× X.

Theorem 6.19: If *f*: X→ Y is swt.r.c.[resp: swt.c.]; *g*: X→ Y is swt.sg.c[resp: swt.r.c; swt.c]; and Y is UT_2, then E = {x in X : *f*(x) = *g*(x)} is *sg*-closed in X.

CONCLUSION

In this paper we defined slightly-*sg*-continuous functions, studied its properties and their interrelations with other types of slightly-continuous functions.

REFERENCES

[1] Abd El-Monsef. M.E., S.N. Eldeeb and R.A. Mahmoud, β−open sets and β−continuous mappings, Bull. Fac. Sci. Assiut. Chiv. A.12,no.1(1983) 77-90.

[2] Abd El-Monsef. M.E., R.A. Mahmoud and E.R. Lashin, β−closure and β−interior, J. Fac. Educ. Soc A, Ain Shams Univ.10(1986)235-245.

[3] Andreivic. D., β−open sets, Math. Vestnick. 38(1986)24-32.

[4] D. Andrijevic. On b-open sets. Math. Vesnik, 1996, 48: 59 - 64.

[5] Arse Nagli Uresin, Aynur kerkin, T. Noiri, slightly δ-precontinuous funtions, Commen, Fac. Sci. Univ. Ark. Series 41.56(2) (2007)1-9.

[6] Arya. S. P., and M.P. Bhamini, Some weaker forms of semi-continuous functions, Ganita 33(1-2) (1982) 124-134.

[7] A.A. El-Atik. A study of some types of mappings on topological spaces. M. Sc. Thesis, Tanta University, Egypt, 1997.

[8] Baker. C.W., Slightly precontinuous funtions, Acta Math Hung, 94(1-6) (2002) 45-52.

[9] Balasubramanian. S., Slightly *vg*-continuous functions, Inter. J. Math. Archive, Vol. 2(8) (2011)1455 – 1463.

[10] Balasubramanian. S., and P.A.S. Vyjayanthi, Slightly *v*-continuous functions, J. Adv. Res. Pure Math., Vol.4, No.1(2012)100–112.

[11] Balasubramanian. S., and P.A.S. Vyjayanthi, Slightly *gpr*-continuous functions – Scientia Magna, Vol.7, No.3 (2011) 46 – 52.

[12] Beceron. Y., S. Yukseh and E. Hatir, on almost strongly θ-semi continuous functions, Bull. Cal. Math. Soc., 87., 329-

[13] Davis. A., Indexed system of neighbourhoods for general topological spaces, Amer. Math. Monthly 68(1961)886-893.

[14] Di. Maio. G., A separation axiom weaker than R_0, Indian J. Pure and Appl. Math.16 (1983)373-375.

[15] Di.Maio. G., and T. Noiri, on s-closed spaces, Indian J. Pure and Appl. Math (11)226.

[16] Dunham. W., $T_{1/2}$ Spaces, Kyungpook Math. J.17 (1977) 161-169.

[17] E. Ekici and M. Caldas, Slightly γ−continuous functions, Bol. Sac. Paran. Mat (38) V.22.2, (2004)63-74.

[18] M. Ganster. Preopen sets and resolvable spaces. Kyungpook Math. J., 1987, 27(2):135-143.

[19] K.R. Gentry, H.B. Hoyle. Somewhat continuous functions. Czech. Math. J., 1971, 21(96):5-12.

[20] Maheswari. S.N., and R. Prasad, on R_0 spaces, Portugal Math., 34 (1975) 213-217.

[21] Maheswari. S.N., and R. Prasad, some new separation axioms, Ann. Soc. Sci, Bruxelle, 89(1975)395-

[22] Maheswari. S.N., and R. Prasad, on s-normal spaces, Bull. Math. Soc. Sci. R. S. Roumania, 22(70) (1978)27-

[23] Maheswari. S.N., and S.S. Thakur, on α-iresolute mappings, Tamkang J. Math.11, (1980) 201-214.

[24] Mahmoud. R.A., and M.E. Abd El-Monsef, β−irresolute and β−topological invariant, Proc. Pak. Acad. Sci, 27(3) (1990)285-296.

[25] Mashhour. A.S., M.E. Abd El-Monsef and S.N. El-Deep, on precontinuous and weak precontinuous functions, Proc. Math. Phy. Soc. Egypt, 3, (1982) 47-53.

[26] Mashhour. A.S., M.E. Abd El-Monsef and S.N. El-Deep, α-continuous and α-open mappings, Acta Math Hung. 41(3-4) (1983) 231-218.

[27] Njastad. O., On some class of nearly open sets, Pacific J. Math 15(1965) 961-970.

[28] Noiri. T., & G.I. Chae, A Note on slightly semi continuous functions Bull. Cal. Math. Soc 92(2) (2000) 87-92.

[29] Noiri. T., Slightly β−continuous functions, Internat. J. Math. & Math. Sci. 28(8) (2001)469-478.

[30] T. Noiri, N. Rajesh. Somewhat b-continuous functions. J. Adv. Res. in Pure Math., 2011, 3(3):1-7.doi: 10.5373/jarpm.515.072810.

[31] Nour. T.M., Slightly semi continuous functions Bull. Cal. Math. Soc 87, (1995) 187-190.

[32] Singhal & Singhal, Almost continuous mappings, Yokohama J.Math.16, (1968) 63-73.

Source of support: Nil, Conflict of interest: None Declared