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EFFECT OF VARIABLE VISCOSITY ON THE PERISTALTIC FLOW OF A JEFFREY FLUID IN A TUBE UNDER THE EFFECT OF A MAGNETIC FIELD: APPLICATION TO ADOMIAN DECOMPOSITION METHOD

P. Gangavathi[a], Dr. A. Ramakrishna Prasad[b] and Dr. M. V. Subba Reddy*[c]

[a]Department of Mathematics, Shiridi Sai Engineering College, Anekal, Bangalore-562106, India
[b]Professor & Head, Department of Mathematics, JNTUH-500085, A.P., India
[c]Professor, Department of Computer Science and Engineering, Sri Venkatesa Perumal College of Engineering &Technology, Puttur-517583, A.P, India

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ABSTRACT

In this paper, the effect of magnetic peristaltic flow of Jeffrey fluid with variable viscosity under the assumption of long wavelength and low Reynolds number using Adomian decomposition method is investigated. The closed form solution is obtained for velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient and pumping characteristics are discussed through graphs.

Keywords: *Adomian decomposition method; Hartmann number; Jeffrey fluid; variable viscosity.*

1. INTRODUCTION

Peristalsis is a the wormlike movement by which the alimentary canal or other tubular organs with both longitudinal and circular muscle fibers propel their contents, consisting of a wave of contraction passing along the tube. Peristalsis is also seen in the tubular organs that connect the kidneys to the bladder, bile through the bile duct and urine through the ureters. A complete survey of peristaltic transport in the domain of biomechanics has been presented by Jaffrin and Shapiro [11] and subsequently by Rath [13].

In addition many of the biological fluids are known to be non-Newtonian. Peristaltic transport of blood in small vessels was investigated using the power-law, visco-elastic, Casson fluid, micropolar, models by (Radhakrishnamacharya [14]; Bohme and Friedrich [7)]; Srivastava and Srivastava [17]; Srinivasacharya et al. [16]) respectively. Peristaltic transport of a power-law fluid with variable consistency has been studied by Shukla and Gupta [15]. Srivastava et al. [17] studied the peristaltic transport of a fluid with variable viscosity through a non-uniform tube. Abd El Hakeem et al. [2] have investigated the effect of endoscope and fluid with variable viscosity on peristaltic motion. Recently, Sudhakar Reddy et al. [18] have studied the effect of variable viscosity on the peristaltic flow of a Jeffrey fluid in a tube. Moreover, magnetohydrodynamics (MHD) is the science which deals with the motion of conducting fluids in the presence of a magnetic field. Mekheimer [12] have investigated the MHD peristaltic flow in a non-uniform channel under the assumption of small wave number. Therefore, at least in an initial study, this motivates an analytic study of MHD peristaltic non-Newtonian tube flow that holds for all non-Newtonian parameters. By choosing the Jeffrey fluid model it become possible to treat both the MHD Newtonian and non-Newtonian problems analytically under long wavelength and low Reynolds number considerations by considering the blood as a MHD fluid, it may be possible to control blood pressure and its flow behavior by using an appropriate magnetic field. The influence of magnetic field may also be utilized as a blood pump for cardiac operations for blood flow in arterial stenosis or arteriosclerosis. Hayat and Ali [10] studied peristaltic flow of Jeffrey fluid under the effect of a magnetic field in tube. Abd El Hakeem et al. [1] have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic field. Ali et al. [6] have investigated peristaltic flow of MHD fluid in a channel with variable viscosity under the effect of slip condition.

The Adomian decomposition method in applied mathematics is an effective procedure to obtain analytic and approximate solutions for different types of operator equations (Adomian, 3; Adomian [4]; Adomian [5]; Eldabe et al. [8]. It is based on the search for a solution in the form of a series. The hydromagnetic peristaltic flow of a bio-fluid with variable viscosity in a circular cylindrical tube using Adomian decomposition method was studied by Ebaid [9].

In view of these, we studied the effect of magnetic peristaltic flow of Jeffrey fluid with variable viscosity under the assumption of long wavelength and low Reynolds number using Adomian decomposition method. The closed form solution is obtained for velocity and pressure gradient. The effects of various emerging parameters on the pressure gradient and pumping characteristics are discussed through graphs.

2. MATHEMATICAL FORMULATION

We consider the axisymmetric flow of a Jeffery fluid in a uniform circular tube with a sinusoidal peristaltic wave of small amplitude traveling down its wall. We further assume that wall is extensible and fluid is electrically conducting. A uniform magnetic field B_o is applied in the transverse direction to the flow. The magnetic Reynolds member is taken small so that the induced magnetic field is neglected. The geometry of wall surface is therefore described as

$$R = H(Z,t) \;=\; a + b\ \sin\left(\frac{2\pi}{\lambda}(Z - ct)\right) \tag{2.1}$$

in which a is the average radius of the undisturbed tube, b is the amplitude of the peristaltic wave, λ is the wavelength, c is the wave propagation speed, and t is the time, R and Z are the cylindrical coordinate with Z measures along the axis of the tube and R is in the radial direction. Let (U, W) be the velocity components in fixed frame of reference (R, Z). Fig. 1 shows the physical model of the tube.

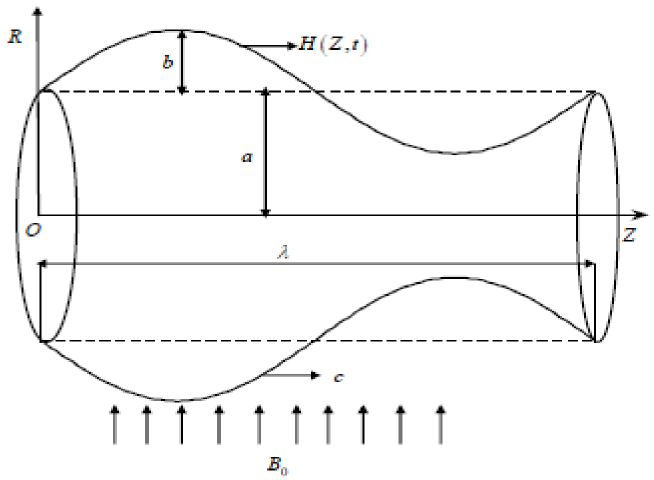

Fig. 1. Physical model.

The constitutive equations for an incompressible Jeffery fluid are

$$T = -pI + S \tag{2.2}$$

$$S = \frac{\mu(R)}{1+\lambda_1}(\dot{\gamma} + \lambda_2\ddot{\gamma}) \tag{2.3}$$

where T and S are Cauchy stress tensor and extra stress tensor respectively, p is the pressure I is the identity tensor, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, μ is the dynamic viscosity, $\dot{\gamma}$ is the shear rate and dots over the quantity indicate differentiation with respect to time.

In the fixed frame of reference (R, Z) the flow is unsteady. However, in a coordinate frame moving with the wave speed c (wave frame) (r, z) the boundary shape is stationary.

The transformation from fixed frame to wave frame is given below as

$$z = Z - ct,\ r \;=\; R\,, w(r,\ z) = W - c,\ u(r,\ z) = U\,. \tag{2.4}$$

where u and w being the velocity components in the wave frame.

The governing hydrodynamic equations are the equations of conservation of mass and momentum. The momentum equation here is modified to account for the interaction between magnetic field and fluid flow through the ponder motive force.

The governing equations in the wave frame are given as follows

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.5}$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}(rS_{rr})+\frac{\partial}{\partial z}(S_{rz})-\frac{S_{\theta\theta}}{r} \tag{2.6}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rS_{rz})+\frac{\partial}{\partial z}(S_{zz})-\sigma B_0^2(w+c) \tag{2.7}$$

where ρ is the density, σ is the electrical conductivity of the fluid and

$$S=\frac{\mu(r)}{1+\lambda_1}\left(1+\lambda_2\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\dot{\gamma}\,. \tag{2.8}$$

Introducing the following non-dimensional variables

$$\bar{r}=\frac{r}{a},\bar{z}=\frac{z}{\lambda},\bar{u}=\frac{u}{c\delta},\bar{p}=\frac{pa^2}{\mu_0 c\lambda}\;\bar{\mu}(r)=\frac{\mu(r)}{\mu_0},\bar{w}=\frac{w}{c},\delta=\frac{a}{\lambda},S=\frac{aS}{\mu_0 c},$$

$$\phi=\frac{b}{a},h=1+\phi\sin 2\pi z\,,$$

where ϕ is the amplitude ratio, μ_0 is the viscosity and δ is the wave number, in the equations (2.5) – (2.8), we get

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{2.9}$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\delta}{r}\frac{\partial}{\partial r}(rS_{rr})+\delta^2\frac{\partial}{\partial z}(S_{rz})-\frac{\partial}{r}(S_{\theta\theta}) \tag{2.10}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rS_{rz})+\delta\frac{\partial}{\partial z}(S_{zz})-m^2(w+1) \tag{2.11}$$

where

$$S_{rr}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{\partial u}{\partial r},$$

$$S_{rz}=\frac{1}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\left(\frac{\partial w}{\partial r}+\delta^2\frac{\partial u}{\partial z}\right),$$

$$S_{\theta\theta}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{u}{r},$$

$$S_{zz}=\frac{2\delta}{1+\lambda_1}\mu(r)\left(1+\frac{\lambda_2 c\delta}{a}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\right)\frac{\partial w}{\partial z},$$

$m=aB_0\sqrt{\dfrac{\sigma}{\mu_0}}$ is the Hartmann number and $\mathrm{Re}=\dfrac{\rho ac}{\mu_0}$ is the Reynolds member.

The corresponding non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r}=0,u=0\quad\text{at}\quad r=0\,, \tag{2.12}$$

$$w=-1\qquad\text{at}\qquad r=h=1+\phi\sin 2\pi z\,. \tag{2.13}$$

Using the long wavelength approximation ($\delta<<1$) and low Reynolds number ($\mathrm{Re}\to 0$) assumptions, the equations (2.10) and (2.11) becomes

***Corresponding author:** *Dr. M. V. Subba Reddy*[c], [c]Professor, Department of Computer Science and Engineering, Sri Venkatesa Perumal College of Engineering &Technology, Puttur-517583, A.P, India*

$$\frac{\partial p}{\partial r}=0 \tag{2.14}$$

$$\frac{\partial p}{\partial z}=\frac{1}{(1+\lambda_1)r}\frac{\partial}{\partial r}\left(r\mu(r)\frac{\partial w}{\partial r}\right)-m^2(w+1)\,. \tag{2.15}$$

From the equations (2.14) and (2.15), we have

$$\mu(r)\frac{\partial^2 w}{\partial r^2}+\left(\frac{\mu(r)}{r}+\frac{d\mu(r)}{dr}\right)\frac{\partial w}{\partial r}=M^2 w+(1+\lambda_1)\frac{dp}{dz}+M^2 \tag{2.16}$$

here $M^2=m^2(1+\lambda_1)$.

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(r)$. For the present investigation, we assume that viscosity variation in the dimensionless form

$$\mu(r)=e^{-\alpha r} \quad \text{or} \quad \mu(r)=1-\alpha r \quad \text{for} \quad \alpha<<1. \tag{2.17}$$

The dimensionless volume flow rate in the wave frame is given by

$$q=2\int_0^h wrdr\,. \tag{2.18}$$

The dimensionless instantaneous volume flow rate in the fixed frame of reference is given by

$$Q(x,t)=2\int_0^h wrdr=2\int_0^h (w+1)rdr=q+h^2\,. \tag{2.19}$$

The dimensionless time mean flow over a period $T(=\lambda/c)$ of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q(x,t)dt=q+1+\frac{\phi^2}{2}. \tag{2.20}$$

From Eq. (2.20), we have $q=\overline{Q}-1-\frac{\phi^2}{2}$.

The non-dimensional expressions for the pressure rise Δp per one wave length and friction force F (on the wall) are respectively given as

$$\Delta p=\int_0^1\frac{dp}{dz}dz \tag{2.21}$$

3. SOLUTION

For the solution of Eq. (2.16), we use Adomian decomposition method, we write Eq. (2.16) in operator form

$$L_r w=(1+\lambda_1)\frac{dp}{dz}+M^2(w+1) \tag{3.1}$$

where the differential operator L_r employs the first two derivatives in the form

$$L_r=\frac{1}{r}\frac{\partial}{\partial r}\left[r\mu(r)\frac{\partial}{\partial r}\right]=\mu(r)\frac{\partial^2}{\partial r^2}+\left[\frac{\mu(r)}{r}+\frac{d\mu(r)}{dr}\right]\frac{\partial}{\partial r} \tag{3.2}$$

and the inverse operator L_r^{-1} is defined by

$$L_r^{-1}[.]=\int_0^r\left[\frac{1}{r\mu(r)}\int_0^r r[.]dr\right]dr=\int_0^r\left[\frac{1}{r(1-\alpha r)}\int_0^r r[.]dr\right]dr \tag{3.3}$$

Applying L_r^{-1} to the LHS of Eq. (2.16), we obtain

$$L_r^{-1}\left[\mu(r)\frac{\partial^2 w}{\partial r^2}+\left(\frac{\mu(r)}{r}+\frac{d\mu(r)}{dr}\right)\frac{\partial w}{\partial r}\right]=\int_0^r\left[\frac{1}{r\mu(r)}\int_0^r\left[r\mu(r)\frac{\partial^2 w}{\partial r^2}+\left(\frac{\mu(r)}{r}+r\frac{d\mu(r)}{dr}\right)\frac{\partial w}{\partial r}\right]dr\right]dr$$
$$=\int_0^r\frac{1}{r\mu(r)}\left[r\mu(r)\frac{\partial w}{\partial r}-\int_0^r\left(r\frac{d\mu(r)}{dr}+\mu(r)\right)\frac{\partial w}{\partial r}dr+\int_0^r\left(r\frac{d\mu(r)}{dr}+\mu(r)\right)\frac{\partial w}{\partial r}dr\right]dr$$
$$=w(r,z)-w(0,z)$$
$$=w(r,z)-\gamma(z)\ ,\quad w(0,z)=\gamma(z) \tag{3.4}$$

Note that only $\gamma(z)=w(0,z)$ is sufficient to carry out the solution and it is to be evaluated from the boundary condition $w(h,z)=-1$ and the other condition ($\partial w/\partial r=0$ at $r=0$) can be used to show that the obtained solution satisfies this given condition.

Operating L_r^{-1} with on (3.1) it then follows

$$w(r,z)=\gamma(z)-\frac{(1+\lambda_1)}{2\alpha^2}\frac{dp}{dz}\left[\alpha r+\ln(1-\alpha r)\right]+N^2\int_0^r\left[\frac{1}{r(1-\alpha r)}\int_0^r rw\,dr\right]dr \tag{3.5}$$

Now we decompose $w(r,z)$ as $w=\sum_{n=0}^{\infty}w_n$ and according to the modified decomposition method, the solution $w(r,z)$ can be computed by using the recurrence relation

$$w_0=\gamma(z)$$
$$w_1=-\frac{(1+\lambda_1)}{2\alpha^2}\frac{dp}{dz}\left[\alpha r+\ln(1-\alpha r)\right]+M^2\int_0^r\left[\frac{1}{r(1-\alpha r)}\int_0^r rw_0\,dr\right]dr$$
$$w_{n+2}=M^2\int_0^r\left[\frac{1}{r(1-\alpha r)}\int_0^r rw_{n+1}dr\right]dr,\quad n\ge 0 \tag{3.6}$$

which gives

$$w_0=\gamma(z)$$
$$w_1=-\left((1+\lambda_1)\frac{dp}{dz}+M^2\gamma(z)\right)\chi_1(r),$$
$$w_2=-\left(M^2(1+\lambda_1)\frac{dp}{dz}+M^4\gamma(z)\right)\chi_2(r),$$
$$w_n=-\left(M^{2n-2}(1+\lambda_1)\frac{dp}{dz}+M^{2n}\gamma(z)\right)\chi_n(r),\quad n\ge 1 \tag{3.7}$$
.

where $\chi_1(r)$ and $\chi_2(r)$ are given by

$$\chi_1(r)=\frac{1}{2\alpha^2}\left[\alpha r+\ln(1-\alpha r)\right],$$
$$\chi_2(r)=\frac{1}{4!\alpha^4}\left[5\alpha r-2\alpha^2r^2+11\ln(1-\alpha r)+6\left(\sum_{i=1}^{\infty}\frac{(\alpha r)^i}{i^2}-\alpha r\ln(1-\alpha r)\right)\right].$$

Using the first three components w_0, w_1 and w_2, then the approximate solution is given by

$$w(r,z) = w_0 + w_1 + w_2$$

$$= -\frac{dp}{dz}\chi_1(r) - M^2\frac{dp}{dz}\chi_2(r) + \gamma(z)\left[1 - M^2\chi_1(r) - M^4\chi_2(r)\right]$$

$$= -\frac{(1+\lambda_1)}{M^2}\frac{dp}{dz} + \left(\gamma(z) + \frac{(1+\lambda_1)}{M^2}\frac{dp}{dz}\right)\left[1 - M^2\chi_1(r) - M^4\chi_2(r)\right] \tag{3.8}$$

The volume flow rate q in a wave frame is given by

$$q = -\frac{h^2}{2} + \left(1 - \frac{(1+\lambda_1)}{M^2}\frac{dp}{dz}\right)\left[\frac{h^2}{2} - \frac{\int_0^h r\chi(r)}{\chi(h)}\right] = \frac{\int_0^h r\chi(r)}{\chi(h)} - \frac{(1+\lambda_1)}{M^2}\frac{dp}{dz}\left[\frac{h^2}{2} - \frac{\int_0^h r\chi(r)}{\chi(h)}\right] \tag{3.9}$$

Solving this equation for $\frac{dp}{dz}$, we obtain

$$\frac{dp}{dz} = \frac{M^2}{(1+\lambda_1)}\left[\frac{1}{2}\left(1 + \frac{\phi^2}{2}\right) - \frac{hI_1(Mh)}{MI_0(Mh)} - \overline{Q}\right] \Bigg/ \left[\frac{h^2}{2} - \frac{\int_0^h r\chi(r)}{\chi(h)}\right] \tag{3.10}$$

4. DISCUSSION OF THE RESULTS

Fig. 2 illustrates the variation of axial pressure gradient $\frac{dp}{dx}$ with material parameter λ_1 for $\alpha = 0.1,\ m = 1$ and $\phi = 0.5$. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing λ_1.

The variation of axial pressure gradient $\frac{dp}{dx}$ with viscosity parameter α for $\lambda_1 = 0.3,\ m = 1$ and $\phi = 0.5$ is shown in Fig. 3. It is found that, the axial pressure gradient $\frac{dp}{dx}$ decreases on increasing α.

Fig. 4 shows the variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number m for $\alpha = 0.1, \lambda_1 = 0.3$ and $\phi = 0.5$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing m.

The variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio ϕ for $\alpha = 0.1, m = 1$ and $\lambda_1 = 0.3$ is depicted in Fig. 5. It is found that, the axial pressure gradient $\frac{dp}{dx}$ initially decreases and then increases with increasing ϕ.

Fig. 6 depicts the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of material parameter λ_1 with $\alpha = 0.1, m = 1$ and $\phi = 0.5$. It is observed that, the time averaged flux $\overline{Q}$ decreases in both the pumping region $(\Delta p > 0)$ and free-pumping region $(\Delta p = 0)$ with increasing λ_1, while it increases in the co-pumping region $(\Delta p < 0)$ with increasing λ_1.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of viscosity parameter α with $\lambda_1 = 0.3, m = 1$ and $\phi = 0.5$ is presented in Fig. 7. It is found that, the time averaged flux $\overline{Q}$ decreases in the pumping region with an increase in α, while it increases in both the free-pumping and co-pumping regions with increasing α.

Fig. 8 shows the variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of Hartmann number m with $\alpha = 0.1, \lambda_1 = 0.3$ and $\phi = 0.5$. It is noted that, the time averaged flux $\overline{Q}$ increases in the pumping region on increasing m, while it decreases in both the free-pumping and co-pumping regions with increasing m.

The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of ϕ with $\alpha = 0.1, m = 1$ and $\lambda_1 = 0.3$ is shown in Fig. 9. It is observed that, the time averaged flux $\overline{Q}$ increases in both the pumping region and free-pumping region with increasing ϕ, while it decreases the co-pumping region with increasing ϕ.

5. CONCLUSIONS

In this paper, We studied the effect of magnetic peristaltic flow of Jeffrey fluid with variable viscosity under the assumption of long wavelength and low Reynolds number using Adomian decomposition method. The closed form solution is obtained for velocity and pressure gradient. It is found that, the axial pressure gradient increases with increasing m and ϕ, while it decreases with increasing α and λ_1. Also the time averaged flux in pumping region increases with increasing m and ϕ, while it decreases with increasing α and λ_1.

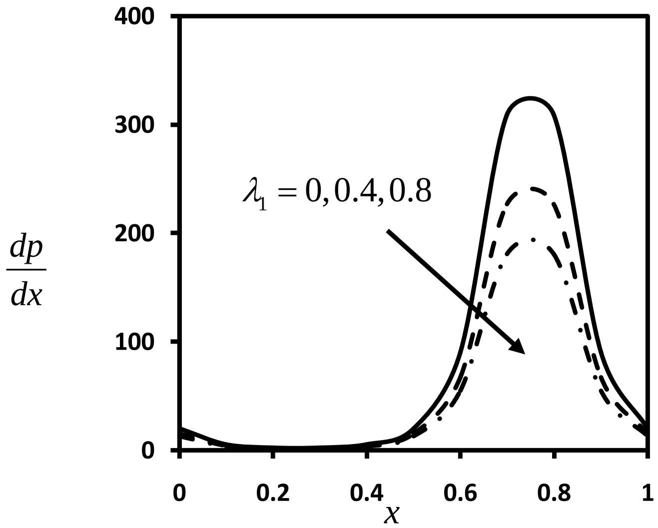

Fig. 2 The variation of axial pressure gradient $\frac{dp}{dx}$ with λ_1 for $\alpha = 0.1, M = 1$ and $\phi = 0.5$.

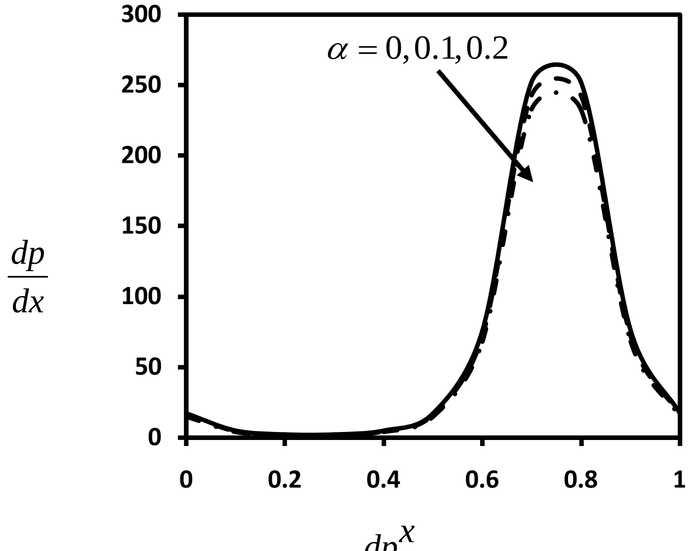

Fig. 3 The variation of axial pressure gradient $\frac{dp}{dx}$ with α for $\lambda_1 = 0.3, M = 1$ and $\phi = 0.5$.

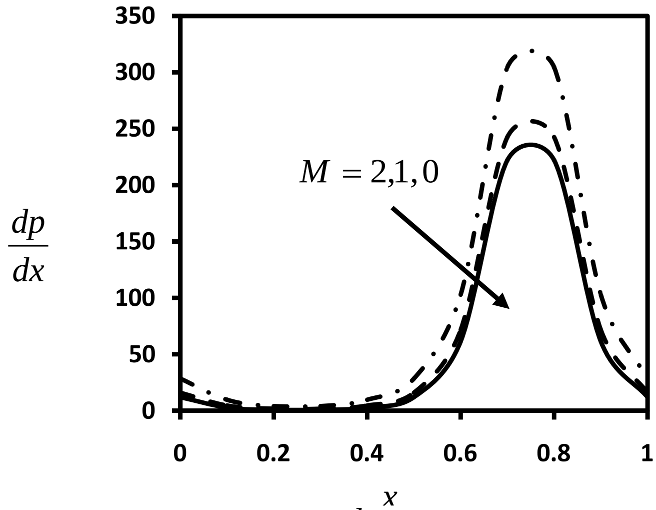

Fig. 4 The variation of axial pressure gradient $\frac{dp}{dx}$ with M for $\alpha = 0.1, \lambda_1 = 0.3$ and $\phi = 0.5$.

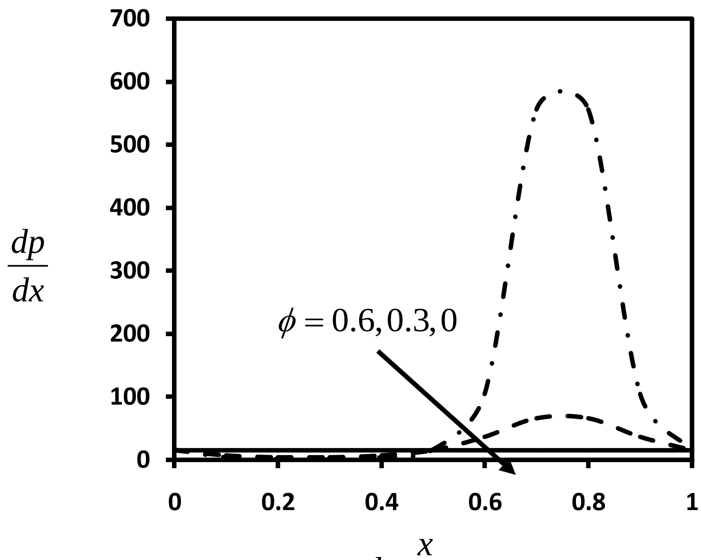

Fig. 5 The variation of axial pressure gradient $\frac{dp}{dx}$ with ϕ for $\alpha = 0.1, M = 1$ and $\lambda_1 = 0.3$.

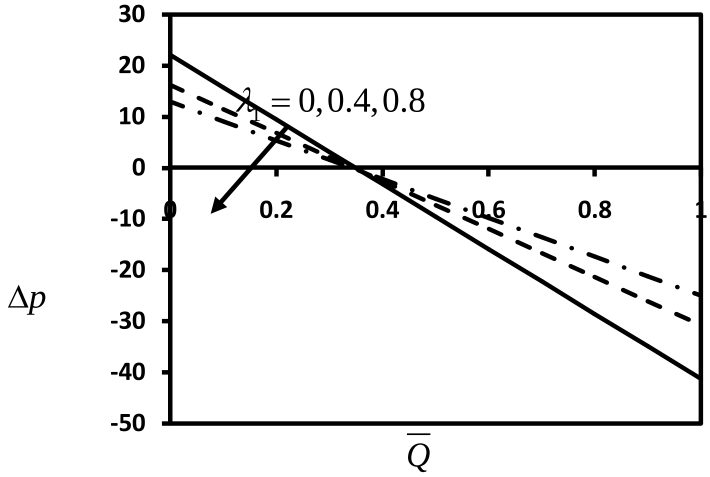

Fig. 6 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of λ_1 with $\alpha = 0.1, M = 1$ and $\phi = 0.5$.

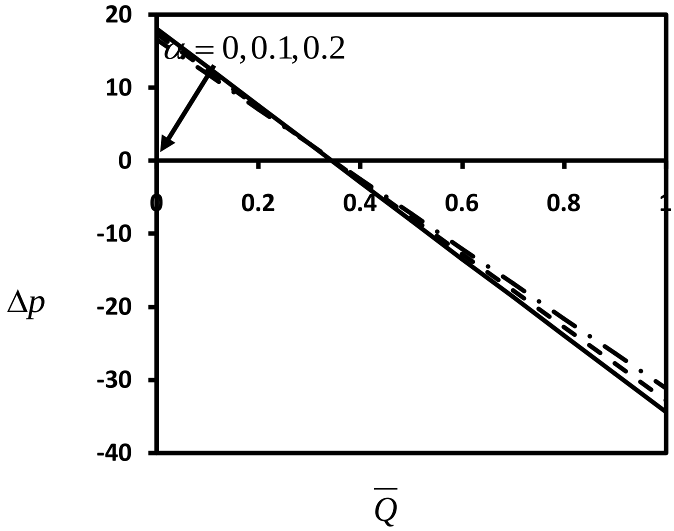

Fig. 7 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of α with $\lambda_1 = 0.3, M = 1$ and $\phi = 0.5$.

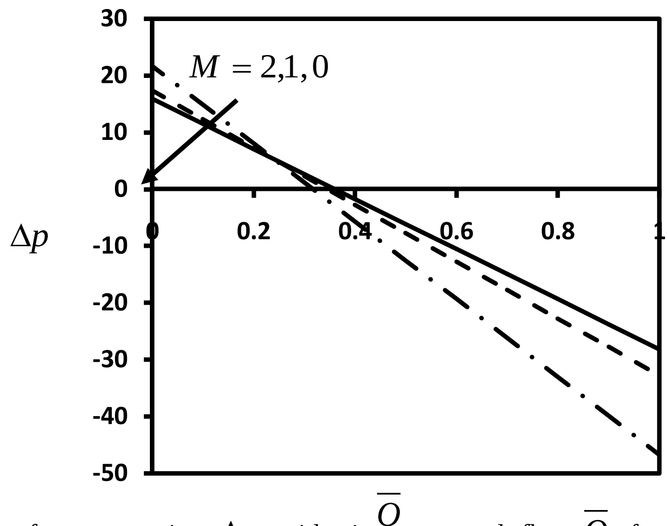

Fig. 8 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of M with $\alpha = 0.1, \lambda_1 = 0.3$ and $\phi = 0.5$.

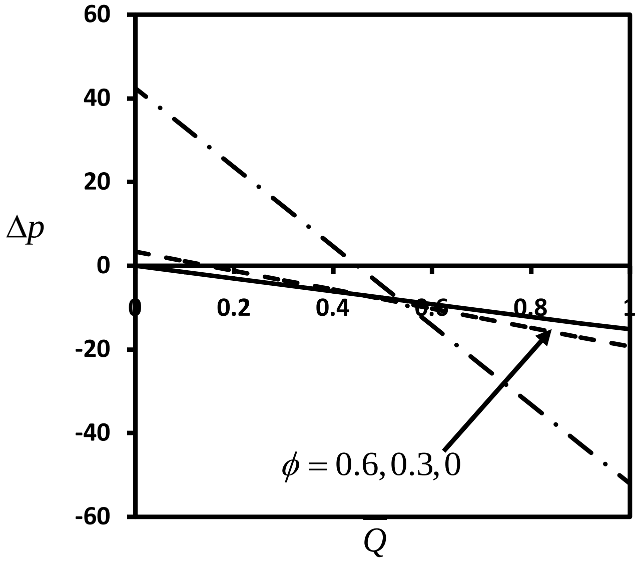

Fig. 9 The variation of pressure rise Δp with time averaged flux $\overline{Q}$ for different values of ϕ with $\alpha = 0.1, M = 1$ and $\lambda_1 = 0.3$.

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