

**SIMILARITY SOLUTION OF THE INCOMPRESSIBLE BOUNDARY LAYER SLIPS  
CONDITIONS FOR VERTICAL SURFACE WITH INTERNAL HEAT GENERATION  
AND VARIABLE SUCTION**

**M. Ferdows<sup>1\*</sup>, Hamad M. A. A<sup>2</sup>. and S. M. Chapal<sup>3</sup>**

<sup>1</sup>*Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh*

<sup>2</sup>*Mathematics Departments, Faculty of Science, Assiut University, Assiut 71516, Egypt*

<sup>3</sup>*Department of Mathematics, Jagannath University, Dhaka, Bangladesh*

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**ABSTRACT**

*The problem of viscous flow and heat transfer over an 1-D unsteady free convection boundary layer from vertical surface has been studied by taking slip conditions & variable suction into consideration. The exponential decay with heat generation term was incorporated in the energy equation. The governing equations of the problem the unsteady nonlinear momentum & energy equations are reduced to be similar by introducing a time dependent length scale & by the usual method of similarity transformation. Maple solutions are obtained for a range value of suction parameter ( $V_0$ ), Prandtl number ( $Pr$ ), velocity slip parameter ( $a$ ) & temperature slip parameter ( $b$ ) with & without internal heat generation. Flow & heat transfer characteristics are discussed & are given in figures & tables. Numerical result show that the internal heat generation effects have significant impacts on velocity profiles, temperature profiles & heat transfer rates from vertical surface.*

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**INTRODUCTION**

The heat, mass and momentum transfer over an unsteady free convection boundary layer from vertical surface are important from theoretical as well as practical points of view because of their wide applications to Aerodynamics, Astrophysics, Geophysics and Engineering. Several authors have analyzed physical problems in this field. Nandha and Sharma [1] analyzed the unsteady free convection flow with suction along an infinite permeable plate. Messiha [2] investigated on unsteady flow past an infinite porous plate with variable suction. Pop [3] studied unsteady flow past an infinite porous plate with variable suction for hydromagnetic case. Soundalgekar [4] analyzed the effects of variable suction and the horizontal magnetic field on the free convection flow past an infinite vertical porous plate and made a comparative discussion of different parameters and the free convection flow of mercury and ionized air. A study on steady laminar free convection flow in an electrically conducting fluid along a porous vertical plate in the presence of heat source was carried out Sharma and Pankaj Mathur [5]. The effect of thermal diffusion on steady laminar free convective flow along a moving porous hot vertical plate in the presence of heat source with mass transfer was studied by Varshney and Shilendra kumar [6]. Heat transfer in MHD free convection flow over an infinite vertical plate with time-dependent suction was investigated in detail by Basant Kumar Mishra [7]. Sharma and Singh [8] examined unsteady MHD free convection flow and heat transfer along a vertical porous plate with variable suction and internal heat generation.

Rarefaction effects must be considered in gases in which the molecular mean free path is comparable to the plate's characteristics domain. The continuum assumption is no longer valid and the gas exhibits non-continuum effects such as velocity slip and temperature jump. Traditional examples of non-continuum as flows such as high altitude aircraft are vacuum technology. Sharma and Chaudhary [9] analyzed the effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. A study of vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime was made by Mittal and Mukesh Bijalwan [10]. Rajesh Johari et al. [11] analyzed unsteady MHD flow through porous medium and heat transfer past a porous vertical moving plate with heat source. Free convection flow of magnetopolar fluid through porous medium in slip flow regime with mass transfer was studied by Rajput et al. [12] Unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical porous plate with variable viscosity and thermal conductivity was investigated by Reddy et al. [13] Recently, Dulal Pal Babulal Talukdar [14] reported perturbation

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**\*Corresponding author: M. Ferdows<sup>1\*</sup>,**

<sup>1</sup>*Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh.*

analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past vertical permeable plate with thermal radiation and chemical reaction neglecting the Soret effect and slip due to jump in temperature. In all the above mentioned investigations it is noted however that they have not considered slip flow due to jump in temperature which is practically important.

## ANALYSIS

The unsteady free convection flow of a viscous, incompressible, electrically conducting fluid with heat and mass transfer past a vertical plate with time dependent suction in a slip flow is considered in the presence of internal heat generation and variable suction. The  $x$  axis is taken along the plate in the upward direction and  $y$  axis is normal to it. The governing equations for the problem are based on the balances of mass, linear momentum and energy species. These equations are as given below:

**Mass equation:**

$$\frac{\partial v}{\partial y} = 0$$

**Momentum equation:**

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

**Energy equation:**

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + q'''$$

where  $u$  and  $v$  are the velocity components in the  $x$  and  $y$  directions respectively,  $g$  is the acceleration due to gravity,  $\rho$  is the density,  $\varepsilon$  is the porosity,  $\mu$  is the absolute viscosity,  $\beta$  is the coefficient of volume expansion,  $T$ ,  $T_w$  and  $T_\infty$  are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream respectively,  $\alpha$  is the thermal diffusivity.

Slip boundary conditions are as follows:

$$u = U_w + N_1 v \frac{\partial u}{\partial y}, \quad v = 0 \quad \text{at } y = 0$$

$$T = T_w + D_1 \frac{\partial T}{\partial y},$$

$$u = 0, T = T_\infty, \quad \text{as } y \rightarrow \infty$$

Let us introduce the following transformations

$$\eta = \frac{y}{\sigma}, \quad u = U_w f(\eta), \quad v = v(t) = -v_0 \frac{\nu}{\sigma}, \quad \sigma = \sigma(t), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Finally, we get the following transformed equations

$$f'' + 2 \left( \eta + \frac{V_0}{2} \right) f' = 0$$

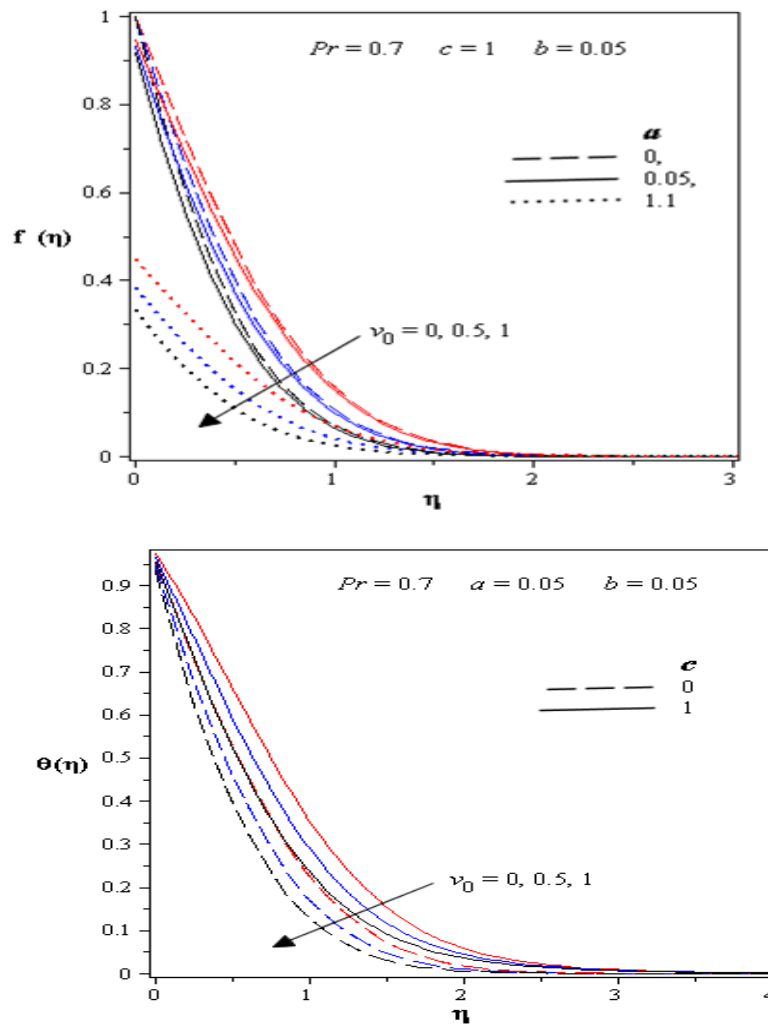
$$\theta'' + 2 \left( \eta + \frac{V_0}{2} \right) \text{Pr} \theta' + c \text{Pr} e^{-\eta} = 0$$

where  $\text{Pr} = \frac{\nu}{\alpha}$  is the Prandtl number.

The transformed boundary conditions are given by

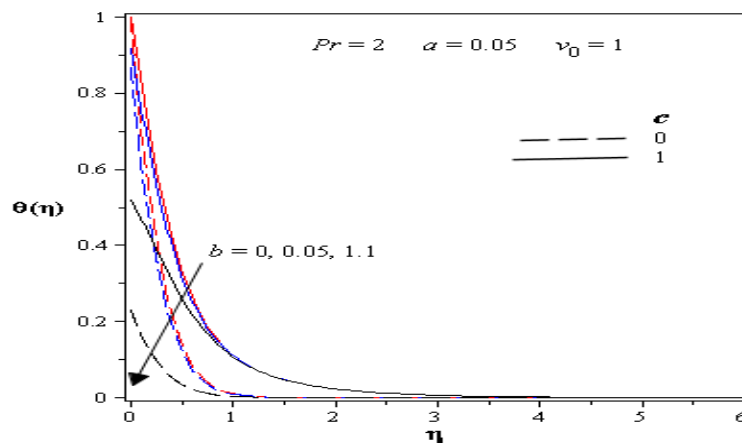
$$f = 1 + af', \theta = 1 + b\theta', f' = 1 + af'' \text{ at } \eta = 0$$

$$f' = 0, \theta = 0 \text{ as } \eta \rightarrow \infty$$



**Fig1:** Distance functions & temperature profiles for various values of  $V_0$ .

Figure 1 illustrates the influence of Suction parameter ( $V_0$ ) on the distance functions & temperature profiles with & without internal heat generation. It can be seen that with increasing  $V_0$ , the distance & temperature inside the boundary layer become smaller. Also internal heat generation induced more flow than that of without internal heat generation.



**Fig2:** Temperature profiles for various values of  $b$ .

Figure2 illustrates the influence of temperature slip parameter ( $b$ ) on the temperature profiles with & without internal heat generation. It is seen that mere & far from the wall temperature profiles decreases with increasing  $b$ . Also more flow induced in the temperature boundary in the presence of internal heat generation.

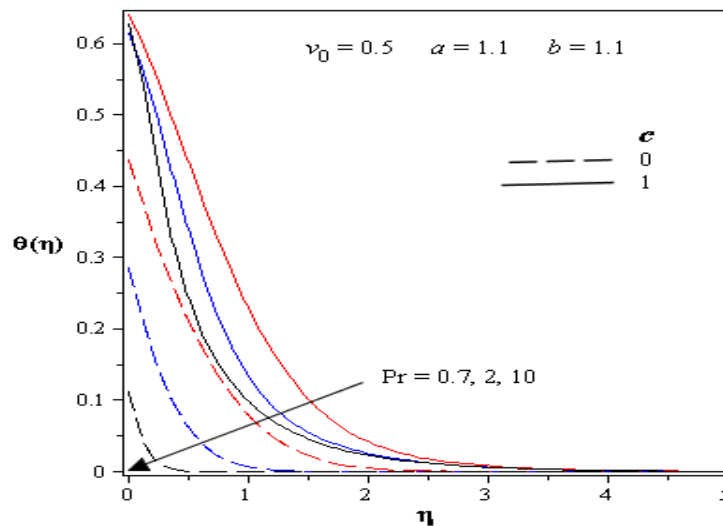


Fig3: Temperature profiles for various values of  $Pr$ .

Figure 3 illustrates the influence of prandtl number ( $Pr$ ) on the temperature profiles with & without internal heat generation. It can be seen that with increasing  $Pr$ , the temperature inside the boundary layer become smaller. Also internal heat generation induced more flow than that of without internal heat generation, i.e., the mechanical strength in the fluid motion is increased. Note that temperature has a significant impact due to prandtl number.

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