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The Near ring $(G,+,*)$ on a Finite Cyclic Group $(G,+)$ with a function on G

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ABSTRACT

Let $(G,+,*)$ be a left near ring on a finite cyclic group $(G,+)$. Let Π be a function defined on G : $p * 1 = p$ for every $p \in G$ .This paper describes some theorems: (i) If the left distribution of $*$ over $+$ is also a right distributive over $+$ in $(G,+,*)$, the function Π on $(G,+)$ is a homomorphism and vice versa. (ii) If the operation $*$ is commutative on $(G,+,*)$, $\Pi(p).q = p.\Pi(q)$ for all $p,q \in G$ and vice versa.(iii) If the near ring $(G,+,*)$ is a non-identity integral domain, the function Π is a homomorphism on $(G,+,*)$ such that $\Pi(p.\Pi(q)) = \Pi(p).\Pi(q) \neq 0$ whenever $p.q \neq 0$ and vice versa.

Key words: *Left near ring, Right near ring.*

INTRODUCTION

The triple $(G,+,*)$ is said to be a left(right) near ring if $(G,+)$ is a group (not necessarily abelian), $(G,*)$ is a semigroup , and $*$ is left(right) distributive over $+$ .Throughout this paper , the term near ring will mean left near ring, the set $G = Z_n = \{0,1,\dots,n-1\}$. Also,+ and . will denote addition and multiplication modulo n , respectively, on our set G, and Π will denote a function on Z_n such that $p * q = \Pi(p).q$. All of these assumptions are taken like Clay [1].

OUR CONTRIBUTION

Theorem 1: The left distribution of $*$ over $+$ is also a right distributive in $(Z_n,+,*)$ iff the function Π on $(Z_n,+)$ is a homomorphism.

Proof: Assume that the left distribution of $*$ over $+$ is also a right distributive in $(Z_n,+,*)$. Thus, for all $p,q,r \in Zn$: $(p+q)*r = p*r + q*r \Rightarrow \Pi(p+q).r = \Pi(p).r + \Pi(q).r$ Hence, when $r = 1$.

we have $\Pi(p+q) = \Pi(p) + \Pi(q)$.

Conversely, Assume that the function Π is a homomorphism on $(Z_n,+)$. For all, $q \in Z_n$,

$\Pi(p+q) = \Pi(p) + \Pi(q)$. Then

$\Pi(p+q) = \Pi(p) + \Pi(q) \Rightarrow \Pi(p+q).r = (\Pi(p)+\Pi(q)).r \Rightarrow \Pi(p+q).r = \Pi(p).r + \Pi(q).r \Rightarrow$ $(p+q)*r = p*r + q*r$

Corollary 1: The near ring $(Z_n,+,*)$ is a ring iff the function Π on$(Z_n,+)$ is homomorphism such that $\Pi(p.\Pi(q)) = \Pi(p).\Pi(q)$.

Proof: Clay [1] proved that a necessary and sufficient condition for a function Π to define an associative left distributive operation is that $\Pi(p.\Pi(q)) = \Pi(p).\Pi(q)$.

Theorem 2: The binary operation $*$ on $(Z_n,+,*)$ is commutative iff $\Pi(p).q = p.\Pi(q)$ for all $p,q \in Z_n$.

Proof: Assume that the operation $*$ is commutative. But we have $\Pi(p).q = p*q$ and $p.\Pi(q) = \Pi(q).p = q*p$.

Since the operation $*$ is commutative, $\Pi(p).q = p.\Pi(q)$ for all $p,q \in Z_n$.

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Conversely, assume that $\Pi(p).q = p.\Pi(q)$ for all $p,q \in Z_n$. From above two expressions; we have $p*q = q*p$ for all $p;\ q \in Z_n$.

Corollary 2: The operation $*$ is commutative and associative operation in Z_n iff the function Π such that $\Pi\big(p.\Pi(q)\big) = \Pi(p).\Pi(q)$ and $p.\Pi(q) = \Pi(p).q$ for all, $q \in Z_n$.

Proof: Clay [1] proved that $*$ on Z_n is an associative left distributive operation iff $\Pi\big(p.\Pi(q)\big) = \Pi(p).\Pi(q)$.

Thus, Theorem 2 shows that the operation $*$ is commutative and associative operation in Z_n iff the function Π such that $\Pi\big(p.\Pi(q)\big) = \Pi(p).\Pi(q)$ and $p.\Pi(q) = \Pi(p).q$ for all $p;\ q \in Z_n$.

Thus, a necessary and sufficient conditions that the operation $*$ is commutative and associative operation in Z_n iff the function Π such that $\Pi(p.\Pi(q)) = \Pi(p).\Pi(q)$ and $p.\Pi(q) = \Pi(p).q$ for all , $q \in Z_n$.

Theorem 3: If the function Π is a one-one such that $\Pi\big(p.\Pi(q)\big) = \Pi(p).\Pi(q)$ for all p, $q \in Z_n$, then the operation $*$ satisfies the associative and commutative laws in Z_n.

Proof: Associative property was proved by clay [1] We know that $\Pi(p.\Pi(q) = \Pi(p).\Pi(q)$ and $\big(q.\Pi(p)\big) = \Pi(q).\Pi(p)$. Thus $\Pi(p.\Pi(q)) = \Pi(q.\Pi(p))$. Since Π is one-one function, so $p.\Pi(q) = q.\Pi(p)$ for all $p,q \in Z_n$. Thus $p * q = q * p$ for all , $q \in Z_n$.

Corollary 3: If the function Π is a one-one homomorphism on$(Z_n,+)$ such that$\Pi(p.\Pi(q) = \Pi(p).\Pi(q)$ for all $p,q \in Z_n$, then the near ring $(Z_n,+,*)$ becomes a commutative ring.

Proof: Theorem 1 shows that a necessary and sufficient condition that the left distribution of $*$ over $+$ is also a right distributive over + in $(Z_n,+,*)$ iff the function Π on $(Z_n,+)$ is a homomorphism.

Theorem 3 shows that ,If the function Π is a one-one such that $\Pi\big(p.\Pi(q)\big) = \Pi(p).\Pi(q)$ for all $p,q \in Z_n$, then the operation $*$ satisfies the associative and commutative laws in Z_n.

Thus, If the function Π is a one-one homomorphism on $(Z_n,+)$ such that $\Pi(p.\Pi(q) = \Pi(p).\Pi(q)$ for all $p,q \in Z_n$, then the near ring $(Z_n,+,*)$ becomes a commutative ring.

Theorem 4: The near ring $(Z_n,+,*)$ is a non-identity integral domain iff the function Π is a homomorphism on $(Z_n,+)$ such that $\Pi(p.\Pi(q) = \Pi(p).\Pi(q) \neq 0$ whenever $p,q \neq 0$.

Proof: The proof of the theorem follows from the above theorem 1, the theorem II in [1] proved by clay except the near ring $(Z_n,+,*)$ is without zero divisors iff $\Pi(p.\Pi(q) = \Pi(p).\Pi(q) \neq 0$ whenever $p,q \neq 0$. Since $(Z_n,+,*)$ is a non-identity integral domain, $p * q \neq 0$ whenever $p,q \neq 0$. Then $\Pi(p).q \neq 0 \Rightarrow \Pi(p) \neq 0$ Similarly, if we interchange $p,q \neq 0$; we get $\Pi(q) \neq 0$. Conversely, suppose that $\Pi(p.\Pi(q) = \Pi(p).\Pi(q) \neq 0$ whenever $p,q \neq 0$. Then it follows that $\Pi(p) \neq 0$ whenever $p \neq 0$

Therefore,$\Pi(p.\Pi(q)) \neq 0$ whenever, $q \neq 0$. It implies that $p.\Pi(q) \neq 0$ whenever $p,q \neq 0$. Then $q * p \neq 0$ whenever $p,q \neq 0$.

Theorem 5: The set Z_n has an identity element with respect to binary operation $*$ iff there exist an element n such that $\Pi(p) = np$ for all $p \in Z_n$.

Proof: Assume that the identity element of Z_n with respect to binary operation $*$ is e(say). Therefore ,$p * e = e * p = p$ for all $p \in Z_n$. It implies that $\Pi(p).e = \Pi(e).p = p \Rightarrow \Pi(p) = p$ for all $p \in Z_n$.

Conversely, assume that $p \in Zn \Rightarrow 1.\Pi(p) = p, \Pi(1).p = p$

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