DECOMPOSITION OF * - CONTINUITY IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and investigate the notions of Ioω-open and Ioω -irresolute maps in ideal topological spaces. Also we introduce the notions of slc*-I-sets, ωs*-sets, ωs - I -closed sets, slc* - I -continuous maps and ωs - I -continuous maps. Finally, we obtain the decompositions of *-continuity.


Keywords: Ioω -open set, slc* - I - set, ωs*-set, ωs - I -closed set, slc*- I - continuous and ωs - I -continuous.

1 INTRODUCTION AND PRELIMINARIES

The concept of ideals in topological spaces is treated in the classic text by Kuratowski [10] and Vaidyanathaswamy [16]. The notion of I-open sets in topological spaces was introduced by Jankovic and Hamlett [8]. Dontchev et al. [4] introduced and studied the notion of Ig -closed sets. Recently, Navaneethakrishnan and Paulraj Joseph [13] have studied further the properties of Ig -closed sets and Ig -open sets. An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties: (1) A ∈ I and B ⊆ A imply B ∈ I (heredity), (2) A ∈ I and B ∈ I imply A ∪ B ∈ I (finite additivity). A topological space (X, τ) with an ideal I on X is called an ideal topological space and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I) = {x ∈ X : U ∩ A ∈ I for every U ∈ τ(x)} is called the local function [10] of A with respect to I and τ. We simply write A* in case there is no chance for confusion. A Kuratowski closure operator cl*(.) for a topology τ*(I) called the *-topology finer than τ is defined by cl*(A) = A ∪ A* [16]. Let (X, τ) denote a topological space on which no separation axioms are assumed unless explicitly stated. In a topological space (X, τ), the closure and the interior of any subset A of X will be denoted by cl(A) and int(A), respectively. A subset A of a space is said to be semi-open [12] if A ⊆ cl(int(A)). A subset A of a topological space (X, τ) is said to be g-closed [11] (resp. ω-closed [15]) if cl(A) ⊆ U whenever A ⊆ U and U is open (resp. semi-open) in X. A subset A of a topological space (X, τ) is locally closed [5] (briefly LC) if A = ∪ V where U is open and V is closed.

A subset A of a topological space (X, τ) is semi-locally closed [6] (briefly scl) if A = U ∩ V where U is semi-open and V is semi-closed. A subset A of a topological space (X, τ) is called ssc*- set if A = U ∩ V where U is semi-open and V is closed. A function f : (X, τ) → (Y, σ) is g-continuous [11] (resp. ω-continuous [15]) if f -1(V) is g-closed (resp. ω-closed) in (X,τ) for every closed set V of (Y, σ).


Lemma 1.1: [8] Let (X, τ, I) be a space with an ideal I on X, and A is a subset of X. Then,
1. Every * -closed set is Ioω -closed,
2. Every Ioω -closed set is Igω -closed.

We denote the family of all Ioω -closed (resp. Ioω -open) subsets of an ideal space (X, τ, I) by Ioω(X) (resp. IoOω(X)).

Definition 1.2: A subset A of an ideal space (X, τ, I) is called

1. strongly- I -locally closed [7] (briefly, strongly- I -LC) if A = U ∩ V where U is regular open and V is *-closed.

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2. weakly-I-locally closed \[9\] (briefly, weakly-I-LC) if \(A = U \cap V\), where \(U\) is open and \(V\) is \(*\)-closed.
3. mildly-I-locally closed \[2\] (briefly, mildly-I-LC) if \(A = U \cap V\), where \(U\) is \(\pi\)-open and \(V\) is \(*\)-closed.

**Definition 1.3:** A function \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is said to be
1. \(*\)-continuous \[7\] if \(f^{-1}(A)\) is \(*\)-closed in \(X\) for every closed set \(A\) in \(Y\).
2. Ig-continuous \[7\] if \(f^{-1}(A)\) is Ig-closed in \(X\) for every closed set \(A\) in \(Y\).
3. strongly-I-LC-continuous \[7\] if \(f^{-1}(A)\) is a strongly-I-LC-set in \(X\) for every closed set \(A\) in \(Y\).
4. weakly-I-LC-continuous \[9\] if \(f^{-1}(A)\) is a weakly-I-LC-set in \(X\) for every closed set \(A\) in \(Y\).
5. mildly-I-LC-continuous \[2\] if \(f^{-1}(A)\) is a mildly-I-LC-set in \(X\) for every closed set \(A\) in \(Y\).

**Lemma 1.4:** \[7\] Let \((X, \tau, I)\) be an ideal space and \(A\) be a subset of \(X\). If \(A\) is \(*\)-closed then \(A\) is strongly-I-LC-set but not conversely.

**2. \(I_0\)-continuity and \(I_0\)-irresoluteness**

**Definition 2.1:** A function \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is said to be \(I_0\)-continuous if \(f^{-1}(A)\) is \(I_0\)-closed in \(X\) for every closed set \(A\) in \(Y\).

**Remark 2.2:** If \(I = \{\phi\}\) in the above definition, then the notion of \(I_0\)-continuity coincides with the notion of \(\omega\)-continuity.

**Definition 2.3:** A function \(f: (X, \tau, I) \rightarrow (Y, \sigma, J)\) is said to be \(I_0\)-irresolute if \(f^{-1}(A)\) is \(I_0\)-closed in \((X, \tau, I)\) for every \(I_0\)-closed set \(A\) in \((Y, \sigma, J)\).

**Theorem 2.4:** For a function \(f: (X, \tau, I) \rightarrow (Y, \sigma)\), the following hold:
1. Every continuous function is \(I_0\)-continuous,
2. Every \(*\)-continuous function is \(I_0\)-continuous,
3. Every \(\omega\)-continuous function is \(I_0\)-continuous,
4. Every \(I_0\)-continuous function is Ig-continuous.

**Proof:**

1. Let \(f\) be a continuous function and \(V\) be a closed set in \((Y, \sigma)\). Then \(f^{-1}(V)\) is closed in \((X, \tau, I)\). Since every closed set is \(*\)-closed and hence \(I_0\)-closed, \(f^{-1}(V)\) is \(I_0\)-closed in \((X, \tau, I)\). Therefore, \(f\) is \(I_0\)-continuous.

2. Let \(V\) be a closed set in \((Y, \sigma)\). Then \(f^{-1}(V)\) is \(*\)-closed in \((X, \tau, I)\) because \(f\) is \(*\)-continuous in \(X\). Since every \(*\)-closed set is \(I_0\)-closed, \(f^{-1}(V)\) is \(I_0\)-closed in \((X, \tau, I)\). Therefore, \(f\) is \(I_0\)-continuous.

3. Let \(f\) be an \(\omega\)-continuous function. Then \(f^{-1}(V)\) is \(\omega\)-closed in \((X, \tau, I)\) for every closed set \(V\) in \((Y, \sigma)\). Since every \(\omega\)-closed set is \(I_0\)-closed \[14\], \(f^{-1}(V)\) is \(I_0\)-closed in \((X, \tau, I)\). Therefore, \(f\) is \(I_0\)-continuous.

4. Let \(V\) be a closed set in \((Y, \sigma)\) and \(f\) be an \(I_0\)-continuous function. Then \(f^{-1}(V)\) is \(I_0\)-closed in \((X, \tau, I)\). Since every \(I_0\)-closed set is Ig-continuous, \(f^{-1}(V)\) is Ig-continuous in \((X, \tau, I)\). Therefore, \(f\) is Ig-continuous.

**Remark 2.5:** The relationships defined above, are shown in the following diagram:

\[
\begin{align*}
\text{Continuity} & \quad \rightarrow \quad \omega\text{-continuity} \quad \rightarrow \quad \text{g-continuity} \\
\downarrow & \quad \downarrow & \quad \downarrow \\
\ast\text{-continuity} & \quad \rightarrow \quad I_0\text{-continuity} \quad \rightarrow \quad \text{Ig-continuity}
\end{align*}
\]

None of these implications is reversible as shown by the following examples.

**Example 2.6:** Let \(X = \{a, b, c\}\), \(\tau = \{\phi, \{a\}, \{b, c\}, X\}\), \(I = \{\phi, \{c\}\}\) and \(\sigma = \{\phi, \{a\}, X\}\). Define \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) as \(f(a) = b\), \(f(b) = a\), \(f(c) = c\). Then \(f\) is \(I_0\)-continuous but not continuous.

**Example 2.7:** Let \(X = \{a, b, c\}\), \(\tau = \{\phi, \{a\}, \{b, c\}, X\}\), \(I = \{\phi, \{c\}\}\) and \(\sigma = \{\phi, \{a, c\}, X\}\). The identity map \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is \(I_0\)-continuous but not \(*\)-continuous.

**Example 2.8:** Let \(X = \{a, b, c, d\}\), \(\tau = \{\phi, \{b\}, \{b, c, d\}, X\}\), \(I = \{\phi, \{a\}\}\) and \(\sigma = \{\phi, \{c, d\}, X\}\). Then the identity map \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is Ig-continuous but not \(I_0\)-continuous.
Theorem 2.9: A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is $I$-continuous if and only if $f^{-1}(V)$ is $I$-open in $(X, \tau, I)$ for every open set $V$ in $(Y, \sigma)$.

Proof: Let $V$ be an open set in $(Y, \sigma)$ and $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be $I$-continuous. Then $V^c$ is closed in $(Y, \sigma)$ and $f^{-1}(V^c)$ is $I$-closed in $(X, \tau, I)$. But $f^{-1}(V^c) = (f^{-1}(V))^c$ and so $f^{-1}(V)$ is $I$-open in $(X, \tau, I)$.

Conversely, Suppose that $f^{-1}(V)$ is $I$-open in $(X, \tau, I)$ for each open set $V$ in $(Y, \sigma)$. Let $F$ be a closed set in $(Y, \sigma)$. Then $F^c$ is open in $(Y, \sigma)$ and by hypothesis $f^{-1}(F^c)$ is $I$-open in $(X, \tau, I)$. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is $I$-closed in $(X, \tau, I)$ and so $f$ is $I$-continuous.

Definition 2.10: For a function $f : (X, \tau, I) \rightarrow (Y, \sigma)$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of $f$ and is denoted by $\text{G}(f)$.

Lemma 2.11: The following properties are equivalent for a graph $G(f)$ of a function $f$:
1. $G(f)$ is $I$-continuous.
2. For each $(x, f(x)) \in X \times Y \setminus G(f)$, there exists an $I$-open set $U_0$ of $X$ containing $x$ and an open set $V$ containing $f(x)$ such that $g(U_0) \cap V \neq \emptyset$.

Theorem 2.12: Let $f : (X, \tau, I) \rightarrow (Y, \sigma)$ be a function and $g : X \rightarrow X \times Y$ be the graph function of $f$. If $g$ is $I$-continuous, then $f$ is $I$-continuous.

Proof: Suppose that $g$ is $I$-continuous. Let $x \in X$ and $V$ be any open set of $Y$ containing $f(x)$. Then $X \times V$ is open in $X \times Y$ and by $I$-continuity of $g$, there exists $U \in \text{IoO}(X, \tau)$ containing $x$ such that $g(U) \subset X \times V$. Therefore we obtain $f(U) \subset V$. This shows that $f$ is $I$-continuous.

Theorem 2.13: Let $(X, \tau, I)$ be an ideal topological space. If $A \in \text{Io}(X)$ and $A \subset X_0 \subset \text{Io}(X)$ then $A \in \text{Io}(X_0)$.

Proof: Let $A \in \text{Io}(X)$ and $A \subset X_0 \subset \text{Io}(X)$ then $\text{cl}^*(A) \cap X_0 \subset U$ whenever $A \subset U$ and $U$ is semi-open. Also $\text{cl}^*X_0 (A) = \text{cl}^*(A) \cap X_0$ (Lemma 3.2 [11]). Hence $A \in \text{Io}(X_0)$.

Theorem 2.14: If $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is an $I$-continuous function and $X_0 \in \text{IoO}(X)$. Then the restriction $f / X_0 : (X_0, \tau / X_0, I)$ is $I$-continuous.

Proof: Let $V$ be any open set of $(Y, \sigma)$. Since $f$ is $I$-continuous, $f^I(V)$ is $I$-open in $(X, \tau, I)$ and $f^{-1}(V) \cap X_0 = (f / X_0)^{-1}(V) \in \text{IoO}(X)$. Moreover by Theorem 2.13 we have $(f / X_0)^{-1}(V) \in \text{IoO}(X_0)$. This shows that $f / X_0$ is $I$-continuous.

Remark 2.15: The composition of two $I$-continuous maps need not be $I$-continuous as seen from the following example:

Example 2.16: Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{c\}, Y\}$, $\eta = \{\phi, \{b, d\}, Z\}$, $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, $J = \{\phi, \{c\}\}$. Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g : (Y, \sigma, J) \rightarrow (Z, \eta)$ be identity maps. Then the maps $f$ and $g$ are $I$-continuous but $g \circ f$ is not $I$-continuous.

Theorem 2.17: Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be $I$-continuous and $g : (Y, \sigma, J) \rightarrow (Z, \eta)$ be continuous. Then $g \circ f : (X, \tau, I) \rightarrow (Z, \eta)$ is $I$-continuous.

Theorem 2.18: A function $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is $I$-irresolute if and only if the inverse image of every $I$-open set in $(Y, \sigma, J)$ is $I$-open in $(X, \tau, I)$.

Theorem 2.19: Let $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ and $g : (Y, \sigma, J) \rightarrow (Z, \eta, K)$ be $I$-irresolute. Then $(g \circ f) : (X, \tau, I) \rightarrow (Z, \eta, K)$ is $I$-irresolute.

Proof: Let $g$ be an $I$-irresolute function and $V$ be any $I$-open set in $(Z, \eta, K)$. Then $g^{-1}(V)$ is $I$-open in $(Y, \sigma, J)$. Since $f$ is $I$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $I$-open in $(X, \tau, I)$. Hence $g \circ f$ is $I$-irresolute.

Theorem 2.20: If $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is $I$-irresolute and $g : (Y, \sigma, J) \rightarrow (Z, \eta)$ is $*$-continuous then $g \circ f : (X, \tau, I) \rightarrow (Z, \eta)$ is $I$-continuous.

Proof: Let $V$ be any closed set of $(Z, \eta)$. Then $g^{-1}(V)$ is $*$-closed in $(Y, \sigma, J)$. Therefore, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $I$-closed in $(X, \tau, I)$, since every $*$-closed set is $I$-closed. Hence $g \circ f$ is $I$-continuous.
Theorem 2.21: Let \((X, \tau, I)\) be an ideal topological space, \((Y, \sigma, J)\) be a TI-\(\omega\) space and \((Z, \eta)\) be a topological space. If \(f: (X, \tau, I) \to (Y, \sigma, J)\) is \(I\omega\)-irresolute and \(g: (Y, \sigma, J) \to (Z, \eta)\) is \(I\omega\)-continuous then \(g \circ f\) is \(I\omega\)-continuous.

Theorem 2.22: Let \((X, \tau, I)\) be an ideal topological space, \((Y, \sigma, J)\) be a TI-\(\omega\) space and \((Z, \eta)\) be a topological space. If \(f: (X, \tau, I) \to (Y, \sigma, J)\) is \(I\omega\)-irresolute and \(g: (Y, \sigma, J) \to (Z, \eta)\) is \(Ig\)-continuous then \(g \circ f\) is \(I\omega\)-continuous.

3 slc*- I –sets

In this section, we introduce the notions of slc* - I -sets in ideal topological spaces and study some of its properties.

Definition 3.1: A subset \(A\) of an ideal topological space \((X, \tau, I)\) is called slc* - I -set if \(A = U \cap F\) where \(U\) is semi-open and \(F\) is \(\ast\)-closed.

Proposition 3.2: Let \((X, \tau, I)\) be an ideal space and \(A\) be a subset of \(X\). Then the following holds.
1. If \(A\) is semi-open then \(A\) is slc* - I -set.
2. If \(A\) is \(\ast\)-closed then \(A\) is slc* - I -set.
3. If \(A\) is weakly- \(I\) -LC-set then \(A\) is slc* - I -set.

Example 3.3: Let \(X = \{a, b, c, d\}\), \(\tau = \{\phi, \{a\}, \{a, b, c\}, X\}\) and \(I = \{\phi, \{a\}, \{b\}, \{a, b\}\}\). Then the set \(A = \{c\}\) is an slc* - I -set but not a \(\ast\)-closed set.

Example 3.4: Let \(X = \{a, b, c, d\}\), \(\tau = \{\phi, \{a, b\}, X\}\) and \(I = \{\phi, \{a\}\}\). Then the set \(A = \{a, c\}\) is an slc* - I -set but not a weakly- \(I\) -LC-set.

Remark 3.5: The notions of \(I\omega\)-closed sets and slc* - I -sets are independent.

Example 3.6: Let \(X = \{a, b, c\}\), \(\tau = \{\phi, \{a\}, \{b, c\}, X\}\) and \(I = \{\phi, \{c\}\}\). Then the set \(A = \{b\}\) is \(I\omega\)-closed but not an slc* - I -set.

Example 3.7: Let \(X = \{a, b, c, d\}\), \(\tau = \{\phi, \{a, b, c\}, X\}\) and \(I = \{\phi, \{a\}, \{b\}, \{a, b\}\}\). Then the set \(A = \{c\}\) is slc* - I -set but not an \(I\omega\)-closed set.

Theorem 3.8: A subset of an ideal topological space \((X, \tau, I)\) is \(\ast\)-closed if and only if it is both \(I\omega\)-closed and an slc* - I -set.

Proof: Necessity is trivial. To prove the sufficiency, assume that \(A\) is both \(I\omega\)-closed and an slc* - I -set. Then \(A = U \cap F\) where \(U\) is semi-open and \(F\) is \(\ast\)-closed. Therefore \(A \subseteq U\) and \(A \subseteq F\) and so by hypothesis, \(A^* \subseteq U\) and \(A^* \subseteq F\). Thus \(A^* \subseteq U \cap F = A\). Hence \(A\) is \(\ast\)-closed.

Remark 3.9: From [2] and Proposition 3.2, We have the following implications:

- strongly- \(I\) -LC-set \(\implies\) mildly- \(I\) -LC-set \(\implies\) weakly- \(I\) -LC-set \(\implies\) slc* - I -set.

Theorem 3.10: For a subset \(A\) of an ideal topological space \((X, \tau, I)\), the following are equivalent.

1. \(A\) is a \(\ast\)-closed set.
2. \(A\) is a strongly- \(I\) -LC-set and an \(I\omega\) -closed set.
3. \(A\) is a mildly- \(I\) -LC-set and an \(I\omega\) -closed set.
4. \(A\) is a weakly- \(I\) -LC-set and an \(I\omega\) -closed set.
5. \(A\) is an slc* - I -set and an \(I\omega\) -closed set.

Corollary 3.11: For a subset \(A\) of an ideal topological space \((X, \tau, I)\), the following are equivalent.

1. \(A\) is a \(\ast\)-closed set.
2. \(A\) is a weakly- \(I\) -LC-set and an \(I\omega\) -closed set.
3. \(A\) is a weakly- \(I\) -LC-set and an \(Ig\) -closed set. [7]

4 A new subset of an ideal topological space

Definition 4.1: [3] Let \(A\) be a subset of a topological space \((X, \tau)\). Then the \(s\) - kernel of the set \(A\), denoted by \(s\) - \(\ker(A)\) is the intersection of all semi-open supersets of \(A\).
Definition 4.2: [3] A subset A of a topological space \((X, \tau)\) is called \(\wedge s\)-set if \(A = s - \ker(A)\).

Definition 4.3: A subset A of an ideal space \((X, \tau, I)\) is called \(\lambda s\)-I-closed if \(A = U \cap V\) where \(U\) is a \(\wedge s\)-set and \(V\) is \(*\)-closed.

Proposition 4.4: In an ideal space \((X, \tau, I)\), every \(*\)-closed set is \(\lambda s\)-I-closed but not conversely.

Example 4.5: Let \(X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}\) and \(I = \{\emptyset, \{a\}\}\). Then the set \(A = \{b\}\) is \(\lambda s\)-I-closed but not \(*\)-closed.

Lemma 4.6: For a subset A of an ideal space \((X, \tau, I)\), the following are equivalent.
1. A is \(\lambda s\)-I-closed.
2. A = \(U \cap \cl^*(A)\) where \(U\) is a \(\wedge s\)-set.
3. A = \(s - \ker(A) \cap \cl^*(A)\).

Lemma 4.7: A subset A of an ideal space \((X, \tau, I)\) is I\(\omega\)-closed if and only if \(\cl^*(A) \subset s - \ker(A)\).

Remark 4.8: The notions of I\(\omega\)-closed sets and \(\lambda s\)-I-closed sets are independent.

Example 4.9: Let \(X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}\) and \(I = \{\emptyset, \{a\}\}\). Then the set \(A = \{c\}\) is \(\lambda s\)-I-closed but not I\(\omega\)-closed.

Example 4.10: Let \(X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b, c\}, X\}\) and \(I = \{\emptyset, \{c\}\}\). Then the set \(A = \{b\}\) is I\(\omega\)-closed but not \(\lambda s\)-I-closed.

Theorem 4.11: A subset of an ideal topological space \((X, \tau, I)\) is \(*\)-closed if and only if it is both I\(\omega\)-closed and \(\lambda s\)-I-closed.

5. Decompositions of \(*\)-continuity

In this section, we obtain decompositions of \(*\)-continuity in ideal topological spaces. In order to obtain the decompositions of \(*\)-continuity we introduce the notion of slc\(^*-\)I-continuity and \(\lambda s\)-I-continuity in ideal topological spaces.

Definition 5.1: A function \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is said to be slc\(^*-\)I-continuous (resp. \(\lambda s\)-I-continuous) if \(f^{-1}(V)\) is an slc\(^*-\)I-set (resp. \(\lambda s\)-I-closed set) in \((X, \tau, I)\) for every closed set \(V\) in \((Y, \sigma)\).

Example 5.2: Let \(X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{a, b, c\}, X\}\) and \(I = \{\emptyset, \{a\}\}\). Then the identity map \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is an slc\(^*-\)I-continuous map.

Remark 5.3: Every \(*\)-continuous map is slc\(^*-\)I-continuous, but the converse is not true.

Example 5.4: Let \(X = \{a, b, c, d\}, \tau = \{\emptyset, \{a\}, \{b, c, d\}, X\}\) and \(I = \{\emptyset, \{a\}\}\). Let \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) be the identity map. Then \(f\) is slc\(^*-\)I-continuous but not \(*\)-continuous.

Remark 5.5: The concepts of I\(\omega\)-continuity and slc\(^*-\)I-continuity are independent as seen from the following examples.

Example 5.6: Let \(X, \tau, \sigma\) and \(I\) be defined as in Example 5.4. Then the map \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) is slc\(^*-\)I-continuous but not I\(\omega\)-continuous.

Example 5.7: Let \(X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{b, c\}, X\}\) and \(I = \{\emptyset, \{c\}\}\). Let \(f: (X, \tau, I) \rightarrow (Y, \sigma)\) be the identity map. Then \(f\) is I\(\omega\)-continuous but not slc\(^*-\)I-continuous.

Theorem 5.8: For a function \(f: (X, \tau, I) \rightarrow (Y, \sigma)\), the following are equivalent.
1. \(f\) is \(*\)-continuous.
2. \(f\) is strongly- I-LC-continuous and I\(\omega\)-continuous.
3. \(f\) is mildly- I-LC-continuous and I\(\omega\)-continuous.
4. \(f\) is weakly- I-LC-continuous and I\(\omega\)-continuous.
5. \(f\) is slc\(^*-\)I-continuous and I\(\omega\)-continuous.
Corollary 5.9: For a function $f: (X, \tau, I) \rightarrow (Y, \sigma)$, the following are equivalent.

1. $f$ is $\ast$-continuous.
2. $f$ is weakly-$I$-LC-continuous and $I\omega$-continuous.
3. $f$ is weakly-$I$-LC-continuous and $Ig$-continuous. [7]

Theorem 5.10: A function $f: (X, \tau, I) \rightarrow (Y, \sigma)$ is $\ast$-continuous if and only if it is both $I\omega$-continuous and $\lambda s$-I-continuous.

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