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ON STEADY PLANE MHD VISCOUS FLUID FLOW THROUGH POROUS MEDIA

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ABSTRACT

Plane steady MHD flow of a viscous incompressible fluid with infinite electrical conductivity through porous media has been considered. A new system of flow equations has been obtained in the case of constantly inclined magnetic field. A system of orthogonal curvilinear coordinates in the plane of flow has been taken. Lastly, the solutions have been obtained when the streamlines and their orthogonal trajectories generate an isometric net.

Key words: *porous medium, isometric net, streamlines, magnetic field.*

1. INTRODUCTION

The geometric theory in the studies of flow machinery furnishes the possible information of flow field and structures in general, simplifying the mathematical complexities to a greater extent. The mathematical complexity of the equations governing the flow of an electrically conducting fluid has prohibited a thorough analysis since the early theoretical work of Alfven[1]. All methods of analysis to date have required imposing some restriction on the angle between the velocity vector field and the magnetic vector field. Hence, in the literature investigations deal with aligned, orthogonal and constantly inclined flows. Kingston and Talbot[2] have undertaken the analysis of the flow equations for incompressible plane MHD flow of an inviscid fluid for the orthogonal case. Chandna and Toews[3] studied plane constantly inclined MHD flow and obtained the solutions when the streamlines and their orthogonal trajectories generate an isometric net. Barron and Chandna[4] developed a technique based on the hodograph method, for the study of steady plane, viscous, incompressible, constantly inclined MHD flows. Also, Thakur and Mishra[5] studied the problem of constantly inclined two-phase MHD flows applying hodograph transformation. Further, Singh and Singh[6] and Thakur and Singh[7] have discussed the problems of steady plane MHD flows through porous media. Makinde and Mhone[8] have investigated the combined effects of a transverse magnetic field and radiative heat transfer on unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous media.

This paper deals with plane steady MHD flow of a viscous, incompressible fluid with infinite electrical conductivity through porous media. It is considered that the magnetic field lines meeting the streamlines at a constant angle and a new system of flow equations governing these flows are obtained. This system is used to obtain the solutions when the streamlines and their orthogonal trajectories generate an isometric net.

2. BASIC EQUATIONS

The steady plane MHD flow of a homogeneous, incompressible, viscous fluid with infinite electrical conductivity through porous media is governed by system of equations (Ram and Mishra[9]):

$$\mathrm{div}\,\mathbf{v} = 0 \tag{2.1}$$

$$\rho\left[\left(\mathbf{v}.\mathrm{grad}\right)\mathbf{v}\right] = -\mathrm{grad}\,p + \eta\nabla^2\mathbf{v} + \mu j \times H - \frac{\eta}{k}v \tag{2.2}$$

$$\mathrm{curl}\left(\mathbf{v}\times\mathbf{H}\right) = \mathbf{0} \tag{2.3}$$

and

$$\mathrm{div}\,\mathbf{H} = 0\,, \tag{2.4}$$

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where, $\mathbf{v}$= velocity vector, $\mathbf{H}$ = magnetic field vector, $\mathbf{j} = \mathrm{curl}\mathbf{H}$ = current density vector, p = fluid pressure, ρ = fluid density, η = coefficient of viscosity, μ = magnetic permeability, k = the permeability of the medium.

In this paper, we study non-aligned plane flows for which the magnetic lines lie in the flow plane and are constantly inclined to the streamlines everywhere in the flow region. Let $\theta \neq 0$ denote the constant angle between $\mathbf{v} = (v_1,v_2)$ and $\mathbf{H} = (H_1,H_2)$ in the (x, y) plane and employing (2.3), we find

$$v_1H_2 - v_2H_1 = VH\sin\theta = A, \tag{2.5}$$

where V and H are the magnitude of velocity and magnetic intensity vector respectively and A is an arbitrary constant which is non-zero due to the exclusion of aligned flows. Since θ is constant, the scalar product of $\mathbf{v}$ and $\mathbf{H}$,

$$v_1H_1 + v_2H_2 = VH\cos\theta = A\cot\theta = B, \tag{2.6}$$

whence

$$V^2H^2 = A^2 + B^2 = A^2\mathrm{cosec}^2\theta. \tag{2.7}$$

The constant B is zero if and only if the flow is a crossed flow.

Solving (2.5) and (2.6) for $\mathbf{v}$ and $\mathbf{H}$, we have

$$\mathbf{v} = \frac{A}{H^2}\mathbf{H}\times\hat{n} + \frac{B}{H^2}\mathbf{H} \tag{2.8}$$

$$\mathbf{H} = \frac{A}{V^2}\hat{n}\times\mathbf{v} + \frac{B}{V^2}\mathbf{v}, \tag{2.9}$$

where $\hat{n}$ is the unit vector normal to the plane of flow.

Employing (2.8) in (2.1) and using (2.4) we get

$$\hat{n}.\mathrm{curl}\mathbf{H} = 2\hat{n}.(\mathrm{grad}\ln H\times\mathbf{H}) + \frac{2B}{A}\mathrm{grad}\ln H.\mathbf{H}. \tag{2.10}$$

Using (2.7) and (2.8) in (2.10), we find

$$\mathrm{curl}\mathbf{H} = \left[\frac{B^2 + A^2}{A}\mathrm{grad}\left(\frac{1}{V^2}\right).\mathbf{v}\right]\hat{n}. \tag{2.11}$$

Equation (2.2) together with equation (2.9) and (2.11), yields

$$\rho(\mathbf{v}.\mathrm{grad})\mathbf{v} + \mathrm{grad}\,p = \eta\nabla^2\mathbf{v} + \mu\left[\frac{B^2 + A^2}{2A}\mathrm{grad}\left(\frac{1}{V^4}\right).\mathbf{v}\right]\left[B\hat{n}\times\mathbf{v} - A\mathbf{v} - \frac{\eta}{k}\mathbf{v}\right] \tag{2.12}$$

Employing (2.9) in (2.4) and using (2.1), we find that $\mathbf{v}$ satisfies

$$\left[\frac{2}{A}\mathrm{grad}\ln V.(A\hat{n}\times\mathbf{v} + B\mathbf{v})\right]\hat{n}\times\mathrm{curl}\,\mathbf{v} = \mathbf{0}. \tag{2.13}$$

Equations (2.1) and (2.13) are two equations in $\mathbf{v}$ and can be employed to solve for the velocity field. However, the solutions thus obtained must satisfy the integrability condition for the pressure function which is derived by taking curl of (2.12). Having obtained $\mathbf{v}$, we solve for the pressure function and the magnetic field by employing (2.12) and (2.9) respectively.

3. SOLUTIONS OF FLOWS HAVING AN ISOMETRIC STREAMLINE PATTERN

$$\text{Let } x = x(\alpha,\beta),\ \ y = y(\alpha,\beta) \tag{3.1}$$

define a system of orthogonal curvilinear coordinates in the plane of flow such that the curves $\beta(x, y)$ = constant represent the stream lines and $\alpha(x, y)$ = constant represent their orthogonal trajectories. Letting $\hat{e}_1$ be the unit tangent vector to β = constant in the direction of increasing α, $\hat{e}_2$ the unit tangent vector to α = constant, $h_1(\alpha,\beta)d\alpha$ and $h_2(\alpha,\beta)d\beta$ the components of a vector elements of arc length, we have

$$\mathbf{v} = V(\alpha,\beta)\hat{e}_1$$
$$ds^2 = h_1^2(\alpha,\beta)d\alpha^2 + h_2^2(\alpha,\beta)d\beta^2. \tag{3.2}$$

In the present section we enquire what possible solutions for constantly inclined non aligned flows are possible when flow streamlines and their orthogonal trajectories form an isometric net. Therefore, we search for solutions when metric coefficients of the natural, i.e. streamline, coordinates satisfy the condition

$$h_1^2(\alpha,\beta) = h_2^2(\alpha,\beta) = h(\alpha,\beta)\ (\text{say}), \tag{3.3}$$

where

$$h(\alpha,\beta) = \left(\frac{\partial x}{\partial \alpha}\right)^2 + \left(\frac{\partial y}{\partial \alpha}\right)^2 = \left(\frac{\partial x}{\partial \beta}\right)^2 + \left(\frac{\partial y}{\partial \beta}\right)^2 \tag{3.4}$$

and $h(\alpha,\beta)$ satisfies the Gauss's equation[10]

$$\left(\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2}\right)\ln\sqrt{h} = 0 \tag{3.5}$$

For our analysis, we emply some properties of isometric orthogonal net and these are stated in the following lemma[3],
Lemma: If $f(z) = \alpha(x,y) + i\beta(x,y)$ is a complex function so that the curves α = constant and β = constant generate an orthogonal isometric net, then $f(z) = \alpha(x,y) + i\beta(x,y)$, $z(\xi) = x(\alpha,\beta) + iy(\alpha,\beta)$ are analytic functions of $z = x + iy$, $\xi = \alpha + i\beta$ respectively and furthermore,

(i) $$\frac{f''(z)}{[f'(z)]} = -\frac{1}{2h}\frac{\partial h}{\partial \alpha} + i\frac{1}{2h}\frac{\partial h}{\partial \beta} \tag{3.6}$$

(ii) $$\frac{\partial \omega_1}{\partial \alpha} = \frac{\partial \omega_2}{\partial \beta}, \frac{\partial \omega_2}{\partial \alpha} = -\frac{\partial \omega_1}{\partial \beta}, \tag{3.7}$$

where

$$\omega_1 = -\frac{1}{2h}\frac{\partial h}{\partial \alpha}, \omega_2 = \frac{1}{2h}\frac{\partial h}{\partial \beta} \tag{3.8}$$

and $h(\alpha, \beta)$ is defined by (3.4).

Now, returning to our constantly inclined non-aligned flows governed by the system of equations (2.1), (2.12), (2.13), and (2.9), we find that, relative to the chosen orthogonal natural isometric net, the flow is governed by

$$\frac{\partial}{\partial \alpha}\left(\sqrt{h}V\right) = 0 \tag{3.9}$$

$$\rho V\frac{\partial V}{\partial \alpha} + \frac{\partial p}{\partial \alpha} = \eta\frac{\partial}{\partial \beta}\left\{\frac{1}{h}\frac{\partial}{\partial \beta}\left(\sqrt{h}V\right)\right\} + \frac{2\mu\left(A^2 + B^2\right)}{v^3}\frac{\partial V}{\partial \alpha} - \frac{\eta}{k}\sqrt{h}V \tag{3.10}$$

$$\rho\frac{V}{\sqrt{h}}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right)-\rho V\frac{\partial V}{\partial\beta}-\frac{\partial p}{\partial\beta}=\eta\frac{\partial}{\partial\alpha}\left\{\frac{1}{h}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right)\right\}+\frac{2\mu\beta\left(A^2+B^2\right)}{AV^3}\frac{\partial V}{\partial\alpha} \tag{3.11}$$

$$B\frac{\partial V}{\partial\alpha}+A\frac{\partial V}{\partial\beta}=\frac{1}{2\sqrt{h}}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right) \tag{3.12}$$

$$\mathbf{H}=\frac{B}{V}\hat{e}_1+\frac{A}{V}\hat{e}_2. \tag{3.13}$$

From equation (3.9), it follows that

$$V(\alpha,\beta)=\frac{\psi(\beta)}{\sqrt{h}}, \tag{3.14}$$

where $\psi(\beta)$ is an arbitrary differentiable function of β.

Employing (3.14) in (3.12), we obtain

$$A\omega_2-B\omega_1=\frac{A\psi'(\beta)}{2\psi(\beta)}. \tag{3.15}$$

where $\omega_1(\alpha,\beta)$ and $\omega_2(\alpha,\beta)$ are defined in (3.8).

Differentiating (3.15) with respect to α and using the cauchy-Rieman conditions satisfied by $\omega_1(\alpha,\beta)$ and $\omega_2(\alpha,\beta)$ as given in (3.8), we obtain the following first order uncoupled partial differential equations:

$$A\frac{\partial\omega_1}{\partial\beta}+B\frac{\partial\omega_1}{\partial\alpha}=0 \tag{3.16}$$

$$B\frac{\partial\omega_2}{\partial\beta}-A\frac{\partial\omega_2}{\partial\alpha}=0. \tag{3.17}$$

Solving (3.16) and (3.17), we obtain the general solutions for ω_1 and ω_2

$$\omega_1(\alpha,\beta)=f(A\alpha-B\beta)$$

$$\omega_2(\alpha,\beta)=g(A\beta+B\alpha), \tag{3.18}$$

where f and g are arbitrary functions of their arguments. If we define $\xi=A\alpha-B\beta$ and $\lambda=A\beta+B\alpha$, it follows that the transformation jacobian from the (α,β)-plane to the (ξ,λ)-plane is

$\left|\frac{\partial(\xi,\lambda)}{\partial(\alpha,\beta)}\right|=A^2+B^2\neq 0.$ Therefore we can regard ξ and λ as two independent variables.

Substituting (3.18) in (3.8) and taking ξ, λ as two independent variables, we find that $\omega_1'(\xi)=\omega_2'(\lambda)$. Therefore we have,

$$\omega_1=C(A\alpha-B\beta)+C_1,\quad \omega_2=C(A\beta+B\alpha)+C_2, \tag{3.19}$$

where C, C_1 and C_2 are arbitrary constants.

Employing (3.19) in (3.8) and (3.15), we obtain

$$\frac{\partial}{\partial\alpha}(\ln h) = 2C(B\beta - A\alpha) - 2C_1$$
$$\frac{\partial}{\partial\beta}(\ln h) = 2C(A\beta + B\alpha) + 2C_2$$

and

$$\frac{\partial}{\partial\beta}(\ln\psi) = \frac{2C}{A}\left(A^2 + B^2\right)\beta - \frac{2BC_1}{A} + 2C_2. \tag{3.20}$$

Integration of (3.20) yields

$$h(\alpha,\beta) = \exp[AC\left(\beta^2 - \alpha^2\right) + 2BC\alpha B + 2C_2\beta - 2C_1\alpha + C_3] \tag{3.21}$$

$$\psi(\beta) = \exp\left[\frac{C}{A}\left(A^2 + B^2\right)\beta^2 + \left(2C_2 - \frac{2BC_1}{A}\right)\beta + C_4\right], \tag{3.22}$$

where C_3 and C_4 are arbitrary constants. Therefore, for an isometric streamline pattern, solutions for $V(\alpha,\beta)$ and $h(\alpha,\beta)$ satisfying (3.9) and (3.12) are given by (3.14) and (3.21) with $\psi(\beta)$ given by (3.22). However, these solutions must satisfy the integrability condition for $p(\alpha,\beta)$ obtained from (3.10), (3.11) and given by

$$\rho\frac{\partial}{\partial\alpha}\left\{\frac{V}{\sqrt{h}}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right)\right\} - \eta\left(\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\beta^2}\right)\left\{\frac{1}{h}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right)\right\} - 2\mu\left(A^2 + B^2\right)$$
$$\left\{\frac{\partial}{\partial\beta}\left(\frac{1}{V^3}\frac{\partial V}{\partial\alpha}\right) + \frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{V^3}\frac{\partial V}{\partial\alpha}\right)\right\} + \frac{\eta}{k}\frac{\partial}{\partial\beta}\left(\sqrt{h}V\right) = 0 \tag{3.23}$$

Eliminating $V(\alpha,\beta)$ between (3.14) and (3.23), we find that $\psi(\beta)$ and $h(\alpha,\beta)$ must satisfy

$$\rho\psi\psi'\left[\frac{1}{h}\frac{\partial h}{\partial\alpha}\right] + \eta\left[\psi''' - 2\psi''\left\{\frac{1}{h}\frac{\partial h}{\partial\beta}\right\} + \psi'\left\{2\left(\frac{1}{h}\frac{\partial h}{\partial\alpha}\right)^2 + 2\left(\frac{1}{h}\frac{\partial h}{\partial\beta}\right)^2 - \frac{1}{h}\frac{\partial^2 h}{\partial\alpha^2} - \frac{1}{h}\frac{\partial^2 h}{\partial\beta^2} - \frac{h}{k}\right\}\right]$$
$$-\frac{\mu\left(A^2 + B^2\right)}{2A}\frac{h^2}{\psi^2}\left(\frac{2A}{h}\frac{\partial^2 h}{\partial\alpha\partial\beta} + \frac{2B}{h}\frac{\partial^2 h}{\partial\alpha^2} - \frac{4A}{h}\frac{\partial h}{\partial\alpha}\frac{\psi'}{\psi}\right) = 0 \tag{3.24}$$

Using (3.21) in (3.24), we have

$$\rho\psi\psi'(2CB\beta - 2AC\alpha - 2C_1) + \eta\psi''' - 2\eta\psi''(2AC\beta + 2BC\alpha + 2C_2)$$
$$+\eta\psi'\left\{(2CB\beta - 2AC\alpha + 2C_1)^2 + (2AC\beta + 2BC\alpha + 2C_2)^2\right\} - \frac{\eta\psi' h}{k}$$
$$-\frac{4\mu\left(A^2 + B^2\right)}{A\psi^2}\left\{h^2(BC\beta - AC\alpha - C_1)\left(A^2C\beta + B^2C\beta + AC_2 - BC_1\right) - \frac{A\psi'}{\psi}h^2(BC\beta - AC\alpha - C_1)\right\} = 0 \tag{3.25}$$

Differentiating (3.25) thrice with respect to α, we obtain

$$-\frac{\eta}{k}\psi'\frac{\partial^3 h}{\partial\alpha^3} + \frac{4\mu\left(A^2 + B^2\right)}{A\psi^2}\left\{\frac{\partial^3}{\partial\alpha^3}\left[h^2(BC\beta - AC\alpha - C_1)\left(A^2C\beta + B^2C\beta + AC_2 - BC_1\right)\right]\right.$$
$$\left.\frac{-A\psi'}{\psi}\frac{\partial^3}{\partial\alpha^3}\left[h^2(BC\beta - AC\alpha - C_1)\right]\right\} = 0 \tag{3.26}$$

Equation (3.26) will satisfy if and only if

$$\frac{\partial^3 h}{\partial \alpha^3} = 0 \tag{3.27}$$

and

$$\frac{\partial^3}{\partial \alpha^3}\left[h^2\left(BC\beta - AC\alpha - C_1\right)\left(A^2C\beta + B^2C\beta + AC_2 - BC_1\right)\right] = 0$$
$$-\frac{A\psi'}{\psi}\frac{\partial^3}{\partial \alpha^3}\left[h^2\left(BC\beta - AC\alpha - C_1\right)\right] = 0\,. \tag{3.28}$$

Using (3.21), (3.22) in (3.27) and (3.28) and simplifying, we obtain

$$\left(AC\alpha - BC\beta + C_1\right)\left[\left(AC\alpha - BC\beta + C_1\right)^2 - \frac{3}{2}AC\right] = 0 \tag{3.29}$$

and

$$\left\{C\left(A^2 + B^2\right)\beta + AC_2 - BC_1\right\}\left\{16\left(BC\beta - AC\alpha - C_1\right)^4 - 24AC\left(BC\beta - AC\alpha - C_1\right)^2 + 3A^2C^2\right\} = 0. \tag{3.30}$$

These are two system of equations in α and β and are satisfied throughout the flow region.

4. CLASSIFICATION AND GEOMETRY OF ISOMETRIC FLOWS

For flows with isometric streamline pattern, the system of equations (3.29) and (3.30) are satisfied for one of the following two possibilities:

(i) $C = 0$, $C_1 = 0$ and $C_2 \neq 0$
(ii) $C = C_1 = C_2 = 0$

Employing (3.20) in (3.6) and taking $C = 0$ and $C_1 = 0$ which is true for both possibilities, we have

$$\frac{f''(z)}{\{f'(z)\}^2} = iC_2 \tag{4.1}$$

For possibility (i) $C_2 \neq 0$ and integration of (4.1) gives

$$f(z) = -\frac{1}{iC_2}\left[\ln(z + D) + E\right], \tag{4.2}$$

where $D = D_1 + iD_2$ and $E = E_1 + iE_2$ are two arbitrary complex constants.

Let $Z+D = r \exp(i\theta)$, where (r, θ) are polar coordinates and recalling that $f(z) = \alpha + i\beta$, separation of real and imaginary parts of (4.2) yields

$$\alpha(r,\theta) = -\frac{1}{C_2}\left(\theta + E_2\right)$$
$$\beta(r,\theta) = \frac{1}{C_2}\left(\ln r + E_1\right). \tag{4.3}$$

For possibility (ii) when $C_2 = 0$, integration of (4.1) and separation into real and imaginary parts yields

$$\alpha(x,y) = L_1x - L_2y + M_1$$
$$\beta(x,y) = L_2x + L_1y + M_2, \tag{4.4}$$

where L_1, L_2, M_1 and M_2 are real arbitrary constants.

Type (1): $C = 0$, $C_1 = 0$ $C_2 \neq 0$

From equation (4.3), we find that the streamlines are given by ln r = constant. Therefore, the streamlines in this case are a family of concentric circles. Equations (3.21) and (3.22) give

$$h(\alpha,\beta) = \exp(2C_2\beta + C_3),$$

and

$$\psi(\beta) = \exp(2C_2\beta + C_4). \tag{4.5}$$

From equations (3.14) and (3.13), and using equation (4.5), we have

$$V(\alpha,\beta) = \exp(C_2\beta - C_3/2 + C_4) \tag{4.6}$$

and

$$H(\alpha,\beta) = [\exp(C_2\beta - C_3/2 + C_4)][B\hat{e}_1 + A\hat{e}_2].$$

The pressure function is given by

$$P = \frac{\rho}{2}\exp(2C_2\beta + 2C_4 - C_3) - \frac{\eta}{k}\int \exp(2C_2\beta + C_4)d\alpha + C_5, \tag{4.7}$$

where C_5 is an arbitrary constant.

TYPE (II): $C = C_1 = C_2 = 0$.

For this type of flows, from equation (4.4) the streamlines are given by $L_2x + L_1y$ = constant. These are a family of parallel straight lines. The solutions of these flow are given by (4.5), (4.6) and (4.7) with $C_2 = 0$.

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