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NEW TOOLS FOR ROUGH APPROXIMATIONS IN BITOPOLOGICAL SPACES

Hamdi M. Genedi

Mathematics and Statistics Department, Faculty of Science, Taif University, Kingdom of Saudi Arabia
E-mail: hmgenedy1954@hotmail.com

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ABSTRACT

The purpose of this paper is to introduce a class of open sets in bitopological spaces. This clan is based on closure and interior operators with respect to the two topologies. Properties and characterization of the new clan are obtained and some examples and counter examples are given. The suggested clan can be used in the context of generalized rough set approximation which is widely applied in the fields of artificial intelligence and knowledge discovery.

Keywords: *Bitopological spaces, Rough sets, Information systems.*

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1. INTRODUCTION:

Recent view of topological structure [6, 8] on a set is to consider topology on a set as a knowledge base to extract rules and decisions in information systems. In the case of using two information systems to extract knowledge, the suitable model is bitopological space [1]. The aim of this mark is to initiate a class of subsets which help in obtaining new approximations in information systems. For example, the following are examples of two information systems in the sense of Pawlak [7],

	H	MP	C	F
p	Y	N	Y	Y
q	N	Y	N	N
r	N	N	Y	N
s	Y	Y	N	Y
t	Y	Y	Y	Y

	H	MP	C	F
p	N	N	Y	N
q	N	N	Y	N
r	Y	N	Y	Y
s	Y	N	N	N
t	Y	N	Y	Y

where $X = \{p, q, r, s, t\}$ is a set of a patient, $\{H, MP, C\}$ is the set of conditional attributes, $\{F\}$ is the decision attributes, $Y = yes$, $N = no$, $H = headech$, $MP = muscels\ pain$, $C = caugh$ and $F = flu$. By $A(p) = (Y, N, Y)$ we mean that the values of p corresponding to H, MP, C respectively.
Using the tables and the relation R on X ; $R = \{(x, y) : A(x) = A(y)\}$, then we get two topologies on X and we can use the bitopological concepts to get knowledge about patients and diseases.

Definition: 1.1 A subset A in a bitopological space (X, τ_1, τ_2) will be termed by I-open if there exists an τ_1-open U such that $U \subset A \subset cl_2 U$, the family of all I-open set in bitopological space was not necessary topology on X and it is denoted by $I.O.(X)$. The complement of I-open sets will be called I-closed set. It is clear that every τ_1- open is I-open but the converse is not true as shown by the following example.

Example: 1.2 Let $X = \{\alpha, \beta, \gamma\}$, $\tau_1 = \{X, \emptyset, \{\alpha\}, \{\beta, \gamma\}\}$ and $\tau_2 = \{X, \emptyset, \{\beta\}, \{\alpha, \gamma\}\}$. Then (X, τ_1, τ_2) is bitopological space and $I.O.(X) = \{X, \emptyset, \{\alpha\}, \{\beta, \gamma\}, \{\alpha, \gamma\}\}$. Take $A = \{\alpha, \gamma\}$, then A is I-open set but it is not τ_1-open set.

The following theorems give some properties of I-open sets.

Proposition: 1.3 Let A be a subset in a bitopological space (X, τ_1, τ_2), then A is I-open if and only if $A \subset cl_2(int_1(A)$.
Proof: Let $A \subset cl_2(int_1(A))$ and $V = int_1(A)$, we have $V \subset A \subset cl_2(V)$. Conversely, let A be I-open set. Then $V \subset A \subset cl_2(V)$ for some τ_1-open set V. But $V \subset int_1(A)$ and thus $cl_2(V) \subset cl_2(int_1(A))$. Hence $A \subset cl_2(V) \subset cl_2(int_1(A))$.

Theorem: 1.4 Let (X, τ_1, τ_2) be a bitopological space and let A, B be two subsets of X. Then:
(1) Let $\{A_\alpha\}_{\alpha \in J}$ be a collection of I-open sets in bitopological space X. Then $\bigcup_{\alpha \in J} A_\alpha$ is I-open.
(2) Let A be a I-open set in the bitopological space X and $A \subset B \subset cl_2(A)$, then B is I-open set.

Proof: **(1)** For each $\alpha \in J$, we have an τ_1-open set V_α such that $V_\alpha \subset A_\alpha \subset cl_2(V_\alpha)$. Then $\bigcup_{\alpha \in J} V_\alpha \subset \bigcup_{\alpha \in J} A_\alpha \subset \bigcup_{\alpha \in J} cl_2(V_\alpha) \subset cl_2(\bigcup_{\alpha \in J} V_\alpha)$. Hence, if $V = \bigcup_{\alpha \in J} V_\alpha$, then $V \subset \bigcup_{\alpha \in J} A_\alpha \subset cl_\alpha(V)$.

**Corresponding author: Hamdi M. Genedi, E-mail: hmgenedy1954@hotmail.com*

(2) Since A is I-open set, then there exists an τ_1-open set V such that $V \subset A \subset cl_\alpha(V)$. Then $V \subset B$. But $cl_2(A) \subset cl_2(V)$ and thus $B \subset cl_2(V)$. Hence $V \subset B \subset cl_2(V)$ and B is I-open set.

Theorem: 1.5 Let $\mu = \{B_\alpha\}, \alpha \epsilon J$, be a collection of sets in a bitopological space (X, τ_1, τ_2) such that

(1) $\tau_1 \subset \tau_2$.

(2) If $B \in \mu$ and $B \subset V \subset cl_2(B)$, then $V \subset \mu$.

Then $I.O.(X) \subset \mu$ and $I.O.(X)$ is the smallest class of sets in X satisfying (1) and (2).

Proof: Let $A \in I.O.(X)$, then $U \subset A \subset cl_2(U)$ for some τ_1- open U. Then $U \in \mu$ by (1) and thus $A \in \mu$ by (2).

Theorem: 1.6 Let $A \subset Y \subset X$ where Y is a subspace of a bitopological space (X, τ_1, τ_2) and $A \in I.O.(X)$, then $A \in I.O.(Y)$.

Proof: Let V be a τ_1-open in X, then $V \subset A \subset cl_2(V)$. Now $V \subset Y$ and thus $V = V \cap Y \subset A \cap Y \subset Y \cap cl_2(V)$ *or* $V \subset A \subset cl_{2Y}(V)$, since $V = V \cap Y$, V is τ_1/Y - open in Y and the theorem is proved.

We introduce the following definitions of the I-neighbourhood, I- derived, I- closure, I-interior of a set which is similar to that of standard notions of neighbourhood, derived, closure and interior [2, 3, 4].

Definition: 1.7 A set $N_x \subset X$ is said to be I-neighbourhood of a point $x \in X$ if there exists a set $A \in I.O.(X)$ such that $x \in A \subset N_x$ .The next theorem is obvious:

Proposition: 1.8 $A \in I.O.(X)$ if and only if A is I-neighbourhood of each $x \in A$.

Definition: 1.9 A point $x \in X$ is said to be I-limit point of A if and only if for each $U \in I.O.(X)$, $x \in U$ and $(U - \{x\}) \cap A \neq \emptyset$. The set of all I-limit points of A is said to be I-derived set of A and is denoted by $I\text{-}der\ (A)$. We can prove the following theorem directly from the definition.

Theorem: 1.11 A is I- closed if and only if it contains the set of all its I- limit points.

Theorem: 1.12 Let A and B be two subsets of a bitopological space (X, τ_1, τ_2), then:

(1) If $A \subset B$, then $I\text{-}der\ (A) \subset I\text{-}der\ (B)$,

(2) $I\text{-}der\ (A) \cup I\text{-}der\ (B) \subset I\text{-}der\ (A \cup B)$,

(3) $I\text{-}der\ (A \cap B) \subset I\text{-}der\ (A) \cap I\text{-}der\ (B)$,

(4) $I\text{-}der\ (I\text{-}der\ (A)) \setminus A \subset I\text{-}der\ (A)$,

(5) $I\text{-}der(A \cup I\text{-}der\ (A)) \subset A \cup I\text{-}der\ (A)$.

Proof: We prove parts (4), (5) and the others follow directly from the definitions.

(4) Let $x \in I\text{-}der(I\text{-}der(A))\setminus A$ and $U \in IO(X)$. Then $U \cap I\text{-}der(A)\setminus\{x\} \neq \emptyset$. Let $y \in U \cap I\text{-}der(A)\setminus\{x\}$, then $U \cap A\setminus \{y\} \neq \emptyset$. Let $z \in U \cap A\setminus\{y\}$, then $z \neq x$ for $z \in A$ and $x \notin A$. Therefore $U \cap A\setminus\{x\} \neq \emptyset$, implies that $x \in I\text{-}der(A)$.

(5) Let $x \in I\text{-}der(A \cup I\text{-}der(A))$. If $x \in A$, then the result is obvious. So, let $x \in I\text{-}der(A \cup I\text{-}der(A))\setminus A$ and if $x \in U \in I.O.(X)$, $U \cap I\text{-}der(A \cup I\text{-}der(A))\setminus\{x\} \neq \emptyset$, then $U \cap I\text{-}der(A)\setminus\{x\} \neq \emptyset$. Now it follows similarly from (4), that $U \cap A\setminus\{x\} \neq \emptyset$. Therefore $x \in I\text{-}der(A)$. Thus in any case $I\text{-}der(A \cup I\text{-}der(A)) \subset A \cup I\text{-}der(A)$.

The reverse inclusion in theorem 1.12, parts (2), (3) are not true as shown by the following example:

Example: 1.13 Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{b, c\}\}$. Then (X, τ_1, τ_2) is bitopological space and $I.O.(x). = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}$. Take $A = \{a\}$ and $B = \{c\}$, then $I\text{-}der(A) = \emptyset$, $I\text{-}der(B) = \emptyset$. Also $I\text{-}der(A \cup B) = I\text{-}der\{a, b\} = \{c\} \not\subset I\text{-}der(A) \cup I\text{-}der(B) = \emptyset$. Now take $A = \{a, c\}$ and $B = \{b, c\}$, then $I\text{-}der(A) = \{c\}$ and $I\text{-}der(B) = \{c\}$. Again $I\text{-}der(A \cap B) = I\text{-}der\{c\} = \emptyset$. Therefore $I\text{-}der(A) \cap I\text{-}der(B) = \{c\} \not\subset I\text{-}der(A \cap B) = \emptyset$. Take $A = \{b, c\} \neq B = \{b\}$, then $I\text{-}cl(A) = \{b, c\} = I\text{-}cl(B) = \{b, c\}$. Now take $A = \{b\}$, then $I\text{-}cl(A) = \{b, c\}$ and $I\text{-}der(I\text{-}\mathrm{cl}(A)) = I\text{-}\mathrm{der}\{b, c\} = \{c\}$. Hence $I\text{-}cl(A) = \{b, c\} \not\subset I\text{-}der(I\text{-}cl(A)) = \{c\}$.

Definition: 1.14 Let A be a subset of a bitopological space (X, τ_1, τ_2), $A \cup I\text{-}der\ (A)$ is defined to be the I-closure of A and is denoted by $I\text{-}cl\ (A)$.

The following theorem gives some properties of I-closure sets

Theorem: 1.15 Let A and B be two subsets of a bitopological space (X, τ_1, τ_2), , then:

(1) $A \subset I\text{-}cl(A) = A \cup I\text{-}cl(A)$,

(2) If $A \subset B$, then $I\text{-}cl\ (A) \subset I\text{-}cl\ (B)$,

(3) $I\text{-}cl\ (A) \cup I\text{-}cl\ (B) \subset I\text{-}cl\ (A \cup B)$,

(4) $I\text{-}cl\ (A \cap B) \subset I\text{-}cl\ (A) \cap I\text{-}cl\ (B)$,

(5) $I\text{-}cl(\emptyset) = \emptyset$, $I\text{-}cl(X) = X$,

(6) $I\text{-}cl\ (I\text{-}cl\ (A)) = I\text{-}cl\ (A)$,

(7) A is a I- closed if and only if $I\text{-}cl\ (A) = A$, $I\text{-}cl\ (A)$ is I-closed,
(8) $I\text{-}cl\ (A) = \cap\ \{F; F$ is I-closed and $A \subset F\}$, $I\text{-}cl\ (A)$ is the smallest I-closed set containing A.

Proof: We prove parts (2), (6), (7) and the others follow directly from the definitions.
(2) Since $A \subset B$ and by theorem 1.10, part (1) $I\text{-}der(A) = I\text{-}der(B)$. Then $A \cup I\text{-}der(A) \subset B \cup I\text{-}der(B)$, Therefore $I\text{-}cl(A) \subset I\text{-}cl(B)$.

(6) $I\text{-}cl(I\text{-}cl(A)) = I\text{-}cl(A \cup I\text{-}der(A)) = (A \cup I\text{-}der(A)) \cup I\text{-}der(A \cup I\text{-}der(A)) = A \cup I\text{-}der(A) = I\text{-}cl(A)$.

(7) By theorem 1.11, A is I-closed if and only if $I\text{-}der(A) \subset A$, i. e. if and only if $I\text{-}cl(A) = A$.

The reverse inclusion in theorem 1.15, parts (3) and (4) are not true as shown by the following example.

Example: 1.16 Consider the bitopological space (X, τ_1, τ_2) in example 1.2. Take $A = \{\alpha\}$ and $B = \{\beta\}$, then $I\text{-}cl(A) = \{\alpha\}$, $I\text{-}cl(B) = \{\beta\}$ and $I\text{-}cl(A \cup B) = X$. Therefore $I\text{-}cl(A \cup B) \not\subset I\text{-}cl(A) \cup I\text{-}cl(B)$. Now take $A = \{\beta, \gamma\}$ and $B = \{\alpha, \beta\}$, then $I\text{-}cl(A) = \{\beta, \gamma\}$, $I\text{-}cl(B) = X$ and $I\text{-}cl(A \cap B) = \{\beta\}$. Therefore $I\text{-}cl(A) \cap I\text{-}cl(B) \not\subset I\text{-}cl(A \cap B)$.

We introduce the following definition of the I-interior of a set which is similar to that of standard interior.

Definition: 1.17 A point $x \in X$ is said to be a I- interior point of $A \subset X$ if and only if there exist $U \in I.O.(X)$ containing x, such that $U \subset A$. The set of all I- interior of A is said to be the I-interior of A and is denoted by $I\text{-}int(A)$.

The following theorem gives some properties of I- interior sets.

Theorem: 1.18 Let A and B be two subsets of a bitopological space (X, τ_1, τ_2) , then:
(1) $I\text{-}int(A)$ is I-open
(2) $I\text{-}int\ (A)$ is the largest I- open set contained in A,
(3) A is I- open if and only if $A = I\text{-}int\ (A)$,
(4) $I\text{-}int(I\text{-}int\ (A)) = I\text{-}int(A)$,
(5) $I\text{-}int(A) = A \backslash I\text{-}der(X \backslash A)$,
(6) (a) $X \backslash I\text{-}int(A) = I\text{-}cl(X \backslash A)$, (b) $I\text{-}int(X \backslash A) \subset X \backslash I\text{-}int(A)$,
(7) (a) $X \backslash I\text{-}cl(A) = I\text{-}int(X \backslash A)$ (b) If $A \subset B$, then $I - int\ (A) \subset I - int\ (B)$,
(8) (a) $I\text{-}int(A) \cup I\text{-}int(B) \subset I\text{-}int(A \cup B)$, (b) $I\text{-}int(A \cap B) \subset I\text{-}int(A) \cap I\text{-}int(B)$.
(9) (a) $\bigcup_{\alpha \in J} I\text{-}int(A_\alpha) \subset I\text{-}int\left(\bigcup_{\alpha \in J} A_\alpha\right)$, (b) $I\text{-}int(A \backslash B) \subset I\text{-}int(A) \backslash B \subset I\text{-}int(A) \backslash I\text{-}int(B)$.

Proof: We prove parts (1), (2), (5), (6) and the others follow directly from the definitions.
(1) Let $x \in I\text{-}int(A)$, then $U \subset A$ for some $U \in I.O.(X)$ containing x. Also $y \in U$, then $y \in I\text{-}int(A)$, therefore $U \subset I\text{-}int(A)$. Hence $I\text{-}int(A)$ is $I\text{-}neighbourhood$ of x. Therefore by theorem 1.7, $I\text{-}int(A)$ is I-open.

(2) Let $V \in I.O.(X)$, $V \subset A$. Then $y \in V$, implies that $y \in A$, so that $y \in I\text{-}int(A)$. Therefore $V \subset I\text{-}int(A)$. Now the result follow from part (1).

(5) Let $x \in A$, implies that $x \notin I\text{-}der(X \backslash A)$. Then there exist I-open set U containing x such that $U \cap (X \backslash A) = \emptyset$, implies that $x \in U \subset A$, then $x \in I\text{-}int(A)$. Conversely, let $x \in I\text{-}int(A)$, then $x \in I\text{-}der(X \backslash A)$ for $I\text{-}int(A)$ is I-open and $I\text{-}int(A) \cap (X \backslash A) = \emptyset$. Therefore $I\text{-}int(A) = A \backslash I\text{-}der(X \backslash A)$.

(6) **(a)** $X \backslash I\text{-}int(A) = X \backslash (A \backslash I\text{-}der(X \backslash A) = (X \backslash A) \cup I\text{-}der(X \backslash A) = I\text{-}cl(X \backslash A)$.
(b) $I\text{-}int(X \backslash A) \subset X \backslash A \subset I\text{-}cl(X \backslash A) = X \backslash I\text{-}int(A)$.

The reverse inclusion in theorem 1.18, parts (8a) and (8b) are not true as shown by the following example.
Example: 1.19. Consider the bitopological space (X, τ_1, τ_2) defined in example 1.13, take $A = \{c\}$ and $B = \{b\}$. Then $I\text{-}int(A) = \emptyset$ and $I\text{-}int(B) = \{b\}$, but $I\text{-}int(A \cup B) = \{b, c\}$. Therefore $I\text{-}int(A \cup B) \not\subset I\text{-}int(A) \cup I\text{-}int(B)$. Also if $A = \{a\}$ and $B = \{a, c\}$, then $I\text{-}int(A) = \{a\}$ and $I\text{-}int(B) = \{a\}$. Therefore $I\text{-}int(A) = I\text{-}int(B)$ does not imply $A = B$. Now let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\tau_2 = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, c\}, \{a, c, d\}\}$. So (X, τ_1, τ_2) is a bitopological space and $I.O.(X) = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b\}, \{b, c, d\}$. Take $A = \{b, c\}$, $B = \{a, b, d\}$, then $I\text{-}int(A) = \{b, c\}$, $I\text{-}int(B) = \{a, b\}$ and $I\text{-}int(A \cap B) = \emptyset$. Therefore $I\text{-}int(A) \cap I\text{-}int(B) \not\subset I\text{-}int(A \cap B)$.

2 *I*-boundary and *I*-exterior operators:
In this section we define I-boundary (I-co-dense) and I-exterior of a set. We will study these three operators and prove some standard results.
Definition: 2.1 Let A be a subset of a bitopological space (X, τ_1, τ_2).
(i) $A \backslash I\text{-}int(A)$ is said to be the I-boundary of A and is denoted by $I\text{-}fr(A)$.
(ii) $I\text{-}int(X \backslash A)$ is said to be the I-exterior of A and is denoted by I-ext(A).

The most important properties of the boundary and exterior operators are listed in the following theorems.

Theorem: 2.2. For any subsets A and B of bitopological space (X, τ_1, τ_2). Then:
(1) (i) $A = I\text{-}int(A) \cup I\text{-}fr(A)$, (ii) $I\text{-}int(A) \cap I\text{-}fr(A) = \emptyset$,
(2) A is I-open if and only if $I\text{-}fr(A) = \emptyset$,
(3) (i) $I\text{-}fr(I\text{-}int(A)) = \emptyset$, (ii) $I\text{-}int(I\text{-}fr(A)) = \emptyset$,
(4) $I\text{-}fr(I\text{-}fr(A)) = I\text{-}fr(A)$,
(5) $I\text{-}fr(A) = A \cap I\text{-}fr(A)$,
(6) (i) $I\text{-}fr(A) = I\text{-}der(X\backslash A)$, (ii) $I\text{-}der(A) = I\text{-}fr(X\backslash A)$,
(7) If $A \subset B$, then $I\text{-}fr(A) \subset I\text{-}fr(B)$,
(8) (i) $I\text{-}fr(A \cup B) \subset I\text{-}fr(A) \cup I\text{-}fr(B)$ (ii) $I\text{-}fr(A) \cap I\text{-}fr(B) \subset I\text{-}fr(A \cap B)$.

Proof: We prove parts (3ii), (5), (6i), (7) and the others follow directly from the definitions and above theorems.
(3ii) Let $x \in I\text{-}int(I\text{-}fr(A)$, then $x \in I\text{-}fr(A) \subset A$. Therefore $x \in I\text{-}int(I\text{-}fr(A)) \subset I\text{-}int(A)$. Hence $x \in I\text{-}int(A) \cap I\text{-}fr(A)$, which contradicts part (1ii). Consequently $I\text{-}int(I\text{-}fr(A)) = \emptyset$,
(5) $I\text{-}fr(A) = A\backslash I\text{-}int(A) = A\backslash(X\backslash I\text{-}cl(X\backslash A)) = A \cap I\text{-}cl(X\backslash A)$,
(6i) $I\text{-}fr(A) = A\backslash I\text{-}int(A) = A\backslash(A\backslash I\text{-}der(X\backslash A)) = I\text{-}der(X\backslash A)$,
(7) Let $x \in A$, *and* $x \in I\text{-}fr(B)$, then $x \notin I\text{-}int(B)$, implies that $x \notin I\text{-}int(A)$. Then $x \in I\text{-}fr(A)$, therefore $I\text{-}fr(B) \subset I\text{-}fr(A)$.

The reverse inclusion in theorem 2.2, parts (8 i, ii) are not true as shown by the following example.

Example: 2.3 Consider the bitopological space (X, τ_1, τ_2) defined in example 1.19. Take $A = \{a, c\}$, $B = \{b, d\}$, then $I\text{-}fr(A) = \{c\}$ and $I\text{-}fr(B)$. Therefore $I\text{-}fr(A) \cup I\text{-}fr(B) = \{c, b, d\} \not\subset I\text{-}fr(A \cup B) = \emptyset$. Now take $A = \{b\}$ and $B = \{a, b, c\}$, then $I\text{-}fr(A) = \{b\}$, $I\text{-}fr(B) = \emptyset$ and $I\text{-}fr(A \cap B) = \{b\}$. Then $I\text{-}fr(A \cap B) \not\subset I\text{-}fr(A) \cap I\text{-}fr(B)$.

Theorem: 2.4 For any two subsets A and B of bitopological space (X, τ_1, τ_2). Then
(1) $I\text{-}ext(A)$ is I-open set,
(2) $I\text{-}ext(A) = X\backslash I\text{-}cl(A)$,
(3) $I\text{-}ext(L\text{-}ext(A)) = I\text{-}int(I\text{-}cl(A))$,
(4) (i) $I\text{-}ext(A \cup B) \subset I\text{-}int(A) \cap I\text{-}ext(B)$, (ii) $I\text{-}ext(A) \cup I\text{-}ext(B) \subset I\text{-}ext(A \cup B)$.

Proof: We prove parts (2), (3), (4i) and the others follow directly from the definitions and theorems.
(2) $I\text{-}ext(A) = I\text{-}int(X\backslash A) = X\backslash I\text{-}cl(A)$,
(3) $I\text{-}ext(I\text{-}ext(A)) = I\text{-}ext(X\backslash I\text{-}cl(A)) = X\backslash I\text{-}int(X\backslash I\text{-}cl(A)) = I\text{-}int(I\text{-}cl(A))$,
(4i) $I\text{-}ext(A \cup B) = I\text{-}int(X\backslash(A \cup B)) = I\text{-}int((X\backslash A) \cap (X\backslash B)) \subset I\text{-}int(X\backslash A) \cap I\text{-}int(X\backslash B) = I\text{-}ext(A) \cap I\text{-}ext(B)$.

The reverse inclusion in theorem 2.4, part (4ii) is not true as shown by the following example.

Example: 2.5 Consider the bitopological space (X, τ_1, τ_2) defined in example 1.13. Take $A = \{a, b\}$ and $B = \{a, c\}$, then $I\text{-}ext(A) = \emptyset$, $I\text{-}ext(B) = \emptyset$ and $I\text{-}ext(A \cap B) = \{b, c, d\}$. Therefore $I\text{-}ext(A \cap B) \not\subset I\text{-}ext(A) \cup I\text{-}ext(B)$.

CONCLUSION:

The class of open sets initiated in this paper has an importance in both theoretical and application domains. In the theory of bitopological spaces, the class can be used in constructing new concepts related to bitopological concepts, for example compactness, separation axioms and connectedness among others. In the application fields the concepts of lower and upper approximations in rough set theory can be obtained in a different manner from that used in [5] and [7], which in turn help in looking to the process of decision making from a different point of view. In our future work we will study in detail the effect of applying this class in rough set data analysis and its generalizations.

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