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On Fuzzy measure of Symmetry Breaking of Conditions, Similarity and Comparision: Non Statistical information for the Single Patient

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ABSTRACT

In this article, we intend to show that the results which were obtained on the basis of the fact that fuzzy sets donot satisfy excluded midle laws can no longer hold. This in turn hepls us to look into the matter seriously because otherwise, if these are used in other fields without proper thinking, we would have to be satisfied with a result having insufficient information. Here we would like to state about measures of breaking symmetry of conditions, similarity and comparisions for different patients' state which were derived by some authors becomes inappropriate from our standpoints. The shortcomings that arise in dealing with complementation of fuzzy sets should not be sidestepped for future works to get a fruitful outcome. Again a new proposal for defining entropy of fuzzy sets has been put forward.

Keywords: *Membership value, membership finction, C-fuzzy set theory, Glivenko-Cantelli's theorem.*

1. INTRODUCTION

Fuzzy Set Theory was formalised by Professor Lofti Zadeh at the University of California in 1965. Since fuzzy set theory proposed by Zadeh, it has been developed in theory and applications in the past 45 years. What Zadeh proposed is very much a paradigm shift that first gained acceptance in the Far East and its successful application has ensured its adoption around the world. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition-an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in a set, this is described with the aid of a membership function valued in the real unit interval [0, 1]. Fuzzy sets generalize classical sets, since the indicator functions of classical sets are special cases of the membership functions of fuzzy sets, if the later only take values 0 or 1.

Helgason and Jobe [1], found measures which account for different and changing conditions of individual patients. Measures of breaking symmetry of conditions, similarity and comparisions for different patients' state are measured by fuzzy subsethood, measured in fuzzy cardinality. Fuzzy entropy measures for similarity and symmetry are discovered through the fuzzy entropy theorem. Literature dealing with measure of fuzziness is quite extensive. Several approaches to measuring fuzziness have been proposed and studied. The view that degree of fuzziness of a fuzzy set, can be best expressed interms of lack of distinction between the set and its complement predominant. It is important to mention here that the authors in their research used particularly Kosko's entropy formula. Here we would like to say that Kosko's entropy was derived in accordance with the existing definition of complementation of fuzzy sets .This was derived in the following way:

$$E(A) = \frac{M(A \cap A^c)}{M(A \cup A^c)}$$

where M (A) indicates the size or cardinality of fuzzy sets. The above definition of entropy is not surprising, since fuzzy sers violate by definition, the two basic properties of complements of crisp set, the law of contradiction and the law of excluded middle.

It was also mentioned that the dynamic of change in one patient's state over time can be captured by fuzzy entropy. Likewise the fuzzy Subsethood theorem was derived both algebraically and geometrically by Kosko who designated it the fundamental unifying structure in fuzzy set theory and later on it was associated with the entropy formula. Given two fuzzy sets A and B, it defines the degree to which one set belongs to another and it is expressed by the formula:

$$S(A,B) = \frac{M(A \cap B)}{M(A)}$$

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which is read as "degree to which A is a subset of B" and M (A) represents the fuzzy cardinality of fuzzy set A, which is defined by

$$M(A) = \sum \mu_A (x), x\epsilon\Omega$$

It is important to mention here that both entropy and subsethood are the result of the Kosko's concept of geometrical representation of fuzzy sets. But, it was found that these representations have no logical foundations and as such the results which were obtained on the basis of the geometrical representations of fuzzy sets can never be considered to be effective, Dhar. et. al [7]. Another thing for which we would like to discard the subsethood theorem is that by using the susethood theorem, the degree of subsethood of $A \cup A^c$ in $A \cap A^c$ was derived. This is not desirable. In that study, patients were compared in their known measured elements as well as rheir different conditions. They defined an entropy measure for this and called them respectively fuzzy entropy of similarity and fuzzy entropy of symmetry. The expression for fuzzy entropy of similarity and fuzzy entropy of symmetry which were derived in the process can be summarised in the following way:

The fuzzy entropy of similarity:

E (Fsim (A, B), Fsim (A, B)-(K-1)) $= \frac{\{(\text{F Sim }(A,B),\text{F Sim }(A,B)-(K-1)\}\cap\{(\text{F Sim }(A,B),\text{F Sim }(A,B)-(K-1)\}^c\}}{\{\text{F Sim }(A,B),\text{F Sim }(A,B)-(K-1)\}\cup\{(\text{F Sim }(A,B),\text{F Sim }(A,B)-(K-1))\}^c}$

The fuzzy entropy of symmetry is defined as:

$$E\left(K(A,B),\frac{1}{K}(A,B)\right) = \frac{M\left(\left\{K,\frac{1}{K}\right\} \cap \left\{K,\frac{1}{K}\right\}^c\right)}{M\left(\left\{K,\frac{1}{K}\right\} \cup \left\{K,\frac{1}{K}\right\}^c\right)}$$

Where, the measure K stands for the fuzzy measure of breaking of symmetry of conditions. It is defined by using fuzzy subsethood and cardinality and was expressed in the manner written below:

$$K = \frac{\sqrt{\frac{M(A\cap B)}{M(A)}} \times \sqrt{\frac{M(A\cap B)}{M(B)}}}{(\frac{M(A\cap B)}{M(A)})^2 \times (\frac{M(A\cap B)}{M(B)})^2}$$

Fuzzy similarity (F Sim(A,B)) =n/Z(S(A,B)+S(B,A)),where S(A,B) and S(B,A) are fuzzy subsethood measure of A in B and B in A. The variable 'n' refers to the number of dimensions of the unit hypercube and it is same as the number of elements in each fuzzy set. The variable 'Z', on the other hand, represents the number of fuzzy sets to be compared.

It was mentioned that the measure K breaks the unit interval and $\frac{1}{K}$ restores it. An entropy measure for the tendency to break and restore conditions is defined using fuzzy entropy theorem. It is to be mentioned here that while finding the fuzzy entropy of similarity and fuzzy entropy of symmetry, Kosko's entropy formula occupies a very important place, which may cause serious problem if proper attention is not given.

In other words, it can be said that the aforesaid expressions were based on the initial conceptions that fuzzy sets donot satisfy excluded middle laws. That is to say,

If A be a fuzzy set and A^c its complement then it was accepted that

$$A \cap A^c \neq \text{the null set } \emptyset \quad \text{and}$$
$$A \cup A^c \neq \text{the universal set } \Omega$$

In this article, our main intention is to show that the expressions for the fuzzy entropy of similarity and fuzzy entropy of symmetry which were used to derive how much the fuzzy sets A nad B are the same or how much these are different and also to find the degree to which symmetry of conditions breakage equals to that of restoration are based on a result which itself was found on the beliefs which have no logical bearings and hence it becomes obvious that these types of results which were based on some misconceptions cannnot produce a fruitful results. So here, in this context we would like to mention the definition of complementation of fuzzy sets as proposed by Baruah [4], the use of which nullifies all the results obtained above. It is this definition which plays a key role in pointing out the shortcomings in the existing theory. Before proceeding with the proposed definition of complementation, we would like to discuss some of the works of other researchers who had some doubts about Zadeh's fuzzy set theory. Here we would like to draw special attention to the complement of a fuzzy set because both entropy and subsethood involves the complement directly or indirectly. It is for this reason, the work which specially discarded the existing definition of complemention is highlighted in this article.

2. SOME PAPERS ABOUT FUZZY SET THEORY

Qing-Shi Gao, Xiao-Yu Gao and Yue Hu [8] went a step further and found that there is some mistakes Zadeh's fuzzy sets and found that it is incorrect to define the set complement as:

$$\mu_{A^c}(\mathbf{x}) = 1 - \mu_A(\mathbf{x}),$$

because it can be proved that set complement may not exist in Zadeh's fuzzy set. And it also leads to logical confusion, and is seriously mistaken to believe that logics of fuzzy sets necessarily go against classical and normal thinking, logic, and concepts. Hence they went a step forward and found some shortcomings in the Zadeh's fuzzy set theory and consequently they wanted to remove the shortcomings which seemed to them debarred fuzzy sets to satisfy all the properties of classical sets.They introduced a new fuzzy set theory, called C-fuzzy set theory which satisfies all the formulas of the classical set theory. The *C*-fuzzy set theory proposed by them was shown to overcome all of the errors and shortcomings, and more reasonably reflects fuzzy phenomenon in the natural world. It satisfies all relations, formulas, and operations of the classical set theory.

There are many such cases where the researchers found some sort of problems in the Zadeh's initial conception of fuzzy set theory; a few of these are discussed above.

Here in this article, we would like to question some of the results obtained by the researchers which involve particularly the complementation of fuzzy sets because from the standponts of the new definition of complementation, there is some lackings in the existing definition of complementation. Efforts have been made to get rid of any kind of shortcomings by considering a new definition of complementation which is rooted in the concept of reference function. It is seen that without the use of reference function, the definition of complementation is rather incomplete. This activity seems more mathematical or formal in character.

After observing the differences in opinion regarding the existing definition of complementation of a fuzzy set, it seems that alternatives are required to fill the gap. But when we have a situation, where there are several alternatives available, there may be disagreement in choosing amomg several alternatives. In such a situation, we would like to adopt one which we think can bridge the gap that exists. One such alternative can be had from the definition of complementation provided to us by Baruah. Let us have a brief visit to definition of complementation of fuzzy sets as proposed by Baruah [2] which claims to satisfy all the properties of classical set theory. Here we would not like to discuss in details about this definitions because these were written in details in our previous works. In the next section we shall discuss in brief about the new definition of complementation rooted in reference function. This concept of defining the complementation played a vital role in establishing our claim that the definition of similarity and symmetry are not appropiate.

3. BARUAH'S DEFINITION OF COMPLEMENTATION OF FUZZY SETS

Baruah [2& 3] has defined a fuzzy number N with the help of two functions : a fuzzy membership function $\mu_2(x)$ and a reference function $\mu_1(x)$ such that $0 \leq \mu_1(x) \leq \mu_2(x) \leq 1$ Then for a fuzzy number denoted by {x,μ_1(x) ,$\mu_2(x)$} we would call {$\mu_2(x) - \mu_1(x)$ }as the fuzzy membership value ,which is different from fuzzy membership function .It is to be noted here that in the definition of complement of a fuzzy set, fuzzy membership value and the fuzzy membership function have to be two different things.

This concept of reference function provides us with a method of obtaining the complementation of a fuzzy set in an efficient manner. It is emerged from the need to bridge the gap which is increasingly disturbing the applications. The significance of the suggested definition os that it provides us with an efficient procedure for calculating the complement. This seems to have a great potential applicability. According to the above concept, we would get a result which contradicts the existing beliefs that excluded middle laws are violated in case of fuzzy sets.

In other words, the above definition using a reference function leads to the assertion that for any fuzzy set A, we must have

$$A \cap A^c = \text{the null set } \varphi \text{ and}$$
$$A \cup A^c = \text{the universal set } \Omega$$

It has to be worth mentioning here that this new definition was derived after finding some shortcoming in the existing definition. In this article, our intention is to say that the Fuzzy Entropy theorem as defined by Kosko has no meaning from our standpoints as can be found in Dhar. et. al [5,6&7] and consequently the Subsethood theorem . Hence it can be said that the results which were basically dependent on such theorems cannot yield a logical result. As a consequence of the aforesaid discussions, the adoption of the definition of complementation suggested in this article is recommended, if we wish to have a logical result.

Again it is to be mentioned here that the existence of two laws of randomness is required to define a law of fuzziness, Baruah [3]. In other words, not one but two distributions with reference to two laws of randomness defined on two disjoint spaces can construct a fuzzy membership function, for which however one needs to look into the matter through application of Glivenko-Cantelli's theorem of order statistics on superimposed uniformly fuzzy intervals.Now as for every law of fuzziness, there are two laws of randomness , for a fuzzy number we shall therefore have two laws of randomness which would lead to two Shannon's entropies(Dhar. et. al [7]). The above mentioned concepts will provide us with a board framewok within which it is convenient to reach the desired result.

4. CONCLUSIONS

In this article, efforts have been made to show that the expressions for the fuzzy entropy of similarity and fuzzy entropy of symmetry which were used to derive how much the fuzy sets A nad B are the same or how much these are different and also used to find the degree to which symmetry of conditions breakage equals to that of restoration are not acceptable.The rationale behind such a statement is that these formulas are based on a result which was found only on the beliefs which have no logical bearings . Hence it is unexpected to get a logical outcome from the results which were derived and based on some misconceptions. So the time has come to look into the matter very carefully so that we can remove those shortcomings to get a logical result .The basic aim was to show that there are some problems in the exsisting techmiques if it is seen from our angle. One important thing to be mentioned here that the proposal for new definition of fuzzy entropy is expected to work towards replacing all other existing definitions for the same because it is based on some logical standpoints.

So it is concluded that that some further research is required to bring some logic in the aforesaid expressions. Here one thing is worth mentioning that while dealing with a situation in which the life of people are involved, care should be taken so that it should be free from any error terms as can be found in the works finding symmetry of breaking conditions, similarity and comparisions. Finally, we would like to say that the suggested definition of complementation sould be followed to reduce the gap in the existing definition.

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