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SIMILARITY SOLUTION ON FREE CONVECTIVE HEAT TRANSFER FLOW THROUGH POROSITY MEDIUM

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ABSTRACT

Free convection heat transfer fluid flow past a vertical plate embedded in a fluid saturated porosity medium is investigated based on numerical solution. The plate with variable temperature $T = T_w = T_\infty + Ax^\lambda$ *and heat flux* $T = T_\infty + \left(\frac{q}{\kappa}\right)\sqrt{\frac{2\upsilon x}{U_0}}\,\theta(\eta)$ *on the surface is assumed. Both families submit to mathematical analysis by the conventional techniques of laminar boundary layer theory i.e. they permit the finding of similar solutions of the boundary layer equations. Expressions for the velocity, temperature, local skin-friction and rate of heat transfer are constructed on graphs.*

Keywords*: Convection flow, Heat transfer, Temperature exponent, Heat flux.*

1. INTRODUCTION

Convection flow with heat transfer has attracted considerable attention in the last several decades, owing to their applications in groundwater flow, energy storage etc. Free convection boundary layer flow assuming a power law temperature distribution was reported by [1] obtaining similarity solutions. In order to attain the flow phenomena (velocity, temperature fields) various researchers are using power law variations and heat flux distributions, among them [2, 3]. Free convection flow along a vertical plate in a porous media considering surface heat flux has obtained by [4].

The motivations for the present study is to report the free convection heat transfer flow along a vertical plate in a porosity medium (The porosity ε (<1 usually of order one) is defined as the ratio of the void space and the bulk volume of the surrounding) considering the plate with variable temperature and heat flux. The basic equations for boundary layer together with the boundary conditions are presented. The results for various parameters between the two cases are discussed.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Geometry of the problem is such that the x-axis is taken along the plate in the upward direction, and the y-axis is taken normal to it.

For the problem of free convection boundary layer flow over vertical plate in high porosity medium with boundary conditions 1) wall temperature distribution proportional to x^λ and 2) heat flux, the governing equations relevant to the problem are:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

Momentum equation

$$\frac{\rho}{\varepsilon^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\mu}{\varepsilon}\frac{\partial^2 u}{\partial y^2} + \rho g\beta\left(T - T_\infty\right) \tag{2}$$

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Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u, v are the velocity components in the x and y directions respectively, g is the acceleration due to gravity, ρ is the density, ε is the porosity, μ is the absolute viscosity, β is the coefficient of volume expansion, T, T_w and T_∞ are the temperature of the fluid inside the thermal boundary layer, the plate temperature and the fluid temperature in the free stream, respectively, α is the thermal diffusivity.

Case-I: Variable Surface temperature and the boundary conditions for the model are:

$$\begin{aligned} &u=U_0, v=0, T=T_w=T_\infty+Ax^\lambda \ \ at\ y=0 \\ &u=0,\ v=0,\ T=T_\infty\ \ as\ y\to\infty \end{aligned} \tag{4}$$

Where U_0 is the uniform velocity, λ is the temperature parameter and A is proportionality constant.

Case-II: Surface Heat flux and the boundary conditions for the model are:

$$\begin{aligned} &u=U_0, v=0, \frac{\partial T}{\partial y}=\frac{q}{\kappa}\ at\ y=0 \\ &u=0,\ v=0,\ T=T_\infty\ \ as\ y\to\infty \end{aligned} \tag{5}$$

2.1 Transformations

Let us introduce the following transformations

$$\begin{aligned} &\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\ u=U_0 f'(\eta),\ v=\sqrt{\frac{\upsilon U_0}{2x}}(\eta f'-f),\ T=T_\infty+\left(\frac{q}{\kappa}\right)\sqrt{\frac{2\upsilon x}{U_0}}\,\theta(\eta) \\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \end{aligned} \tag{6}$$

Let us transform the energy equations (3) for surface wall temperature and heat flux.

1.2 Surface wall temperature:

Using (6) into (2) and (3), we have

$$\frac{f'''}{\varepsilon}+\frac{ff''}{\varepsilon^2}+Gr\theta=0, \tag{7}$$

$$\theta''+\Pr\left(f\theta'-2\lambda f'\theta\right)=0, \tag{8}$$

Where $Gr=\dfrac{2g\beta Ax^{\lambda+1}}{U_0^2}$ is the Grashof number, $\Pr=\dfrac{\upsilon}{\alpha}$ is the Prandtl number.

The corresponding boundary conditions are

$$\left.\begin{aligned} &f=0,\ f'=1, \theta=1\ at\ \eta=0 \\ &f'=0,\ \theta=0\ \ as\ \eta\to\infty \end{aligned}\right\} \tag{9}$$

2.3 Surface Heat flux:

Using (6) into (2) and (3), we have the following equations

$$\frac{f'''}{\varepsilon}+\frac{ff''}{\varepsilon^2}+Gr\theta=0 \tag{10}$$

$$\theta''+\Pr\left(f\theta'-f'\theta\right)=0 \tag{11}$$

Where $Gr=\dfrac{g\beta q(2x)^{3/2}\upsilon^{1/2}}{\kappa U_0^{5/2}}$ is the Grashof number.

The corresponding boundary conditions are

$$\left.\begin{array}{l} f=0,\ f'=1,\theta'=-1\ at\,\eta=0 \\ f'=0,\ \theta=0\ \ as\ \eta\to\infty \end{array}\right\} \tag{12}$$

3. RESULTS AND DISCUSSIONS

A shooting method [5] is used for all the calculations in this study. Different values of the parameters have chosen arbitrarily. The values of the temperature parameter $\lambda=\left[-\frac{1}{3},1\right]$ where $\lambda=0,\ 1/3,1$ corresponding to isothermal plate, uniform heat flux independent of x but surface temperature gradient depend only and uniform heat flux independent of x respectively [1].

The flow profiles are presented for the two cases as follows.

3.1 Surface wall temperature

The result gives for equations (7) and (8) together with the boundary conditions (9).

The velocity and temperature profiles corresponding to the similarity variable are plotted in figure 1, 2 to illustrate the flow behavior of temperature exponent parameter. It is observed that the velocity and temperature both decreases with increasing temperature exponent of the thermal boundary layer. It may be also seen that velocity profiles become more over shoot when temperature exponent increasing start from $\lambda=-1/3$.

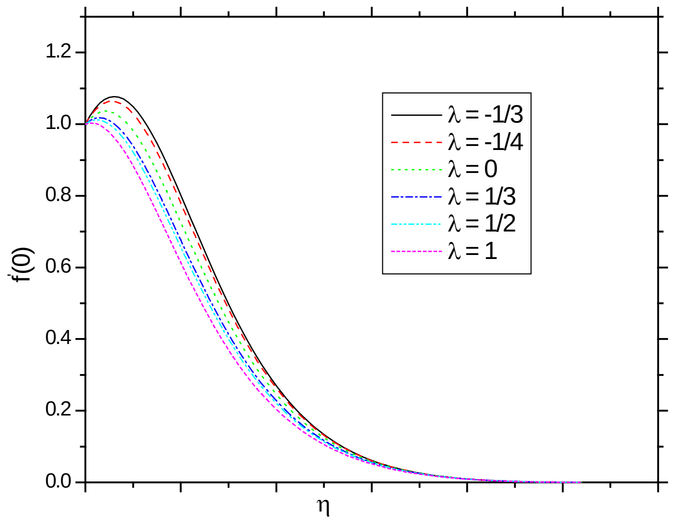

Fig.1 Variation of f'(0) with η for ε=0.9,Pr = 1 & Gr =2

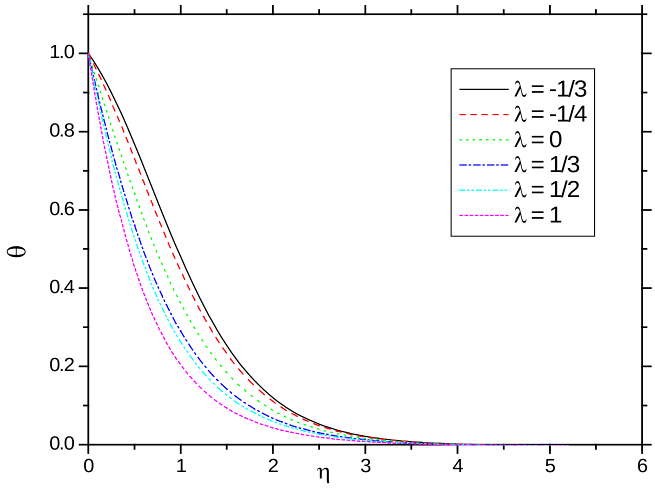

Fig.2 Variation of θ with η for ε=0.9,Pr = 1 & Gr =2

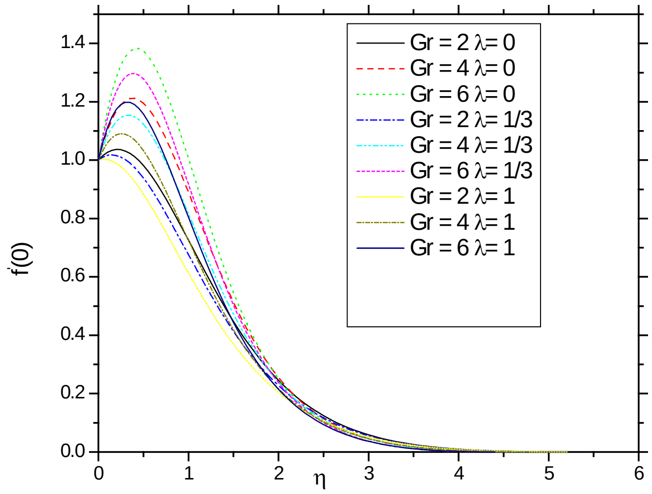

Fig.3 Variation of f'(0) with η for ε =0.9 & Pr = 1

In Fig. 5, the velocity profile is shown to see the effects on porosity. The velocity increases with porosity increasing from 0.45 to 0.9 at $\lambda = 0, \frac{1}{3}, 1$. Also velocity decreases with λ increases at porosity 0.45 to 0.9. The corresponding temperature profiles are shown in Fig.6. It is seen that the temperature decreases for both with porosity and λ increases.

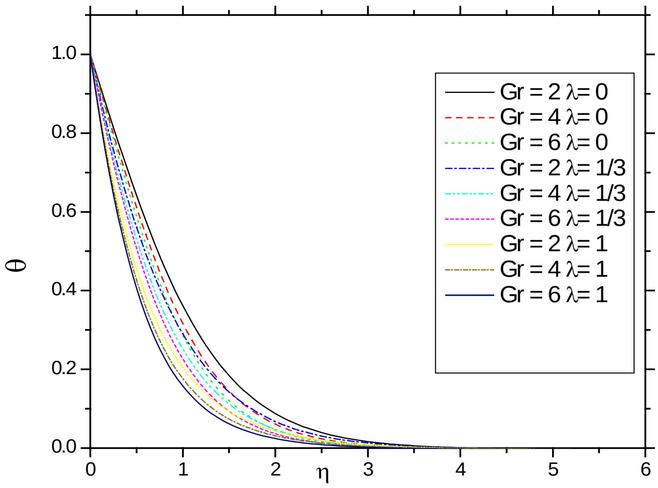

Fig.4 Variation of θ with η for ε =0.9 & Pr = 1

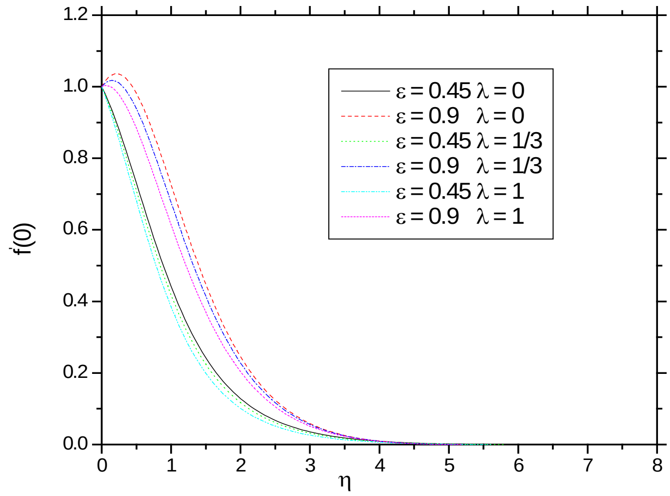

Fig.5 Variation of $f'(0)$ with η for Pr = 1 & Gr = 2

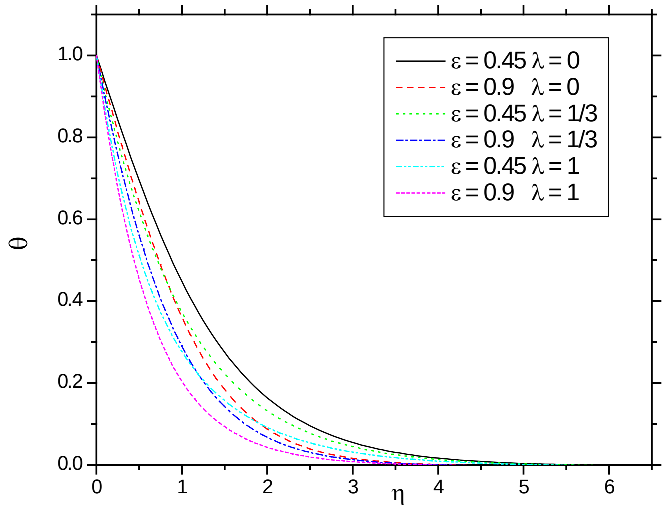

Fig.6 Variation of θ with η for Pr = 1 & Gr = 2

The effects of Prandtl number for velocity and temperature is shown in fig. 7, 8. The velocity and temperature both decreases with Pr and λ increasing.

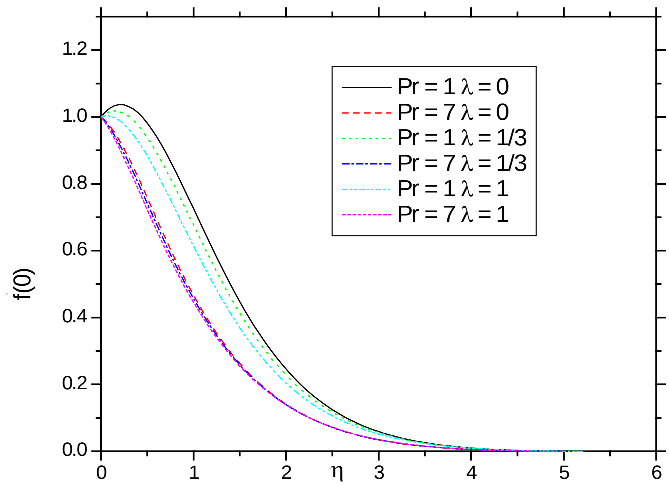

Fig.7 Variation of $f'(0)$ with η for ε = 0.9 & Gr = 2

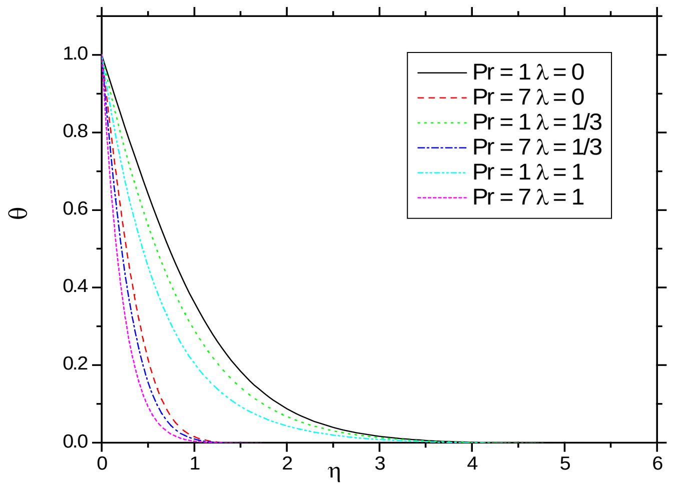

Fig.8 Variation of θ with η for $\varepsilon = 0.9$ & Gr = 2

The Nusselt number and rate of wall heat transfer (skin friction) as a function of λ is shown in fig. 9. It can be seen that increasing λ leads to a reduction in skin friction and increase in Nusselt number.

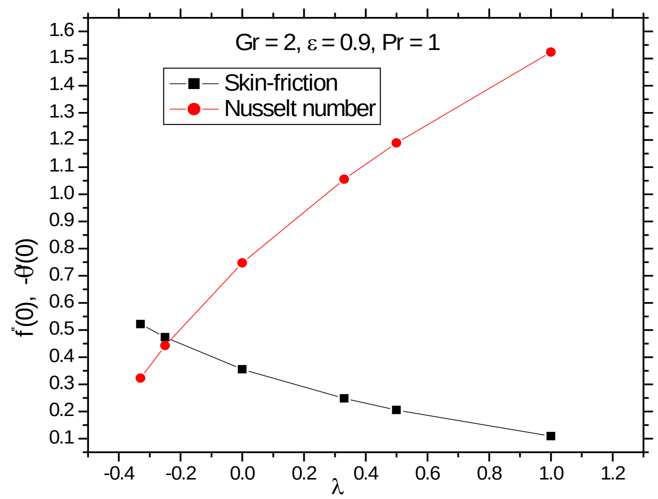

Fig.9 Variations of $f''(0)$ and $-\theta'(0)$ with λ

3.2 Surface Heat flux

The result gives for equations (10) and (11) together with the boundary conditions (12). Figure 10, 11 illustrate the influence of Pr to show the profiles of dimensionless velocity and temperature as a function of similarity variable. It is observed that velocity and temperature decreases as Pr increases. The flow leads to thermal boundary layer at higher Pr.

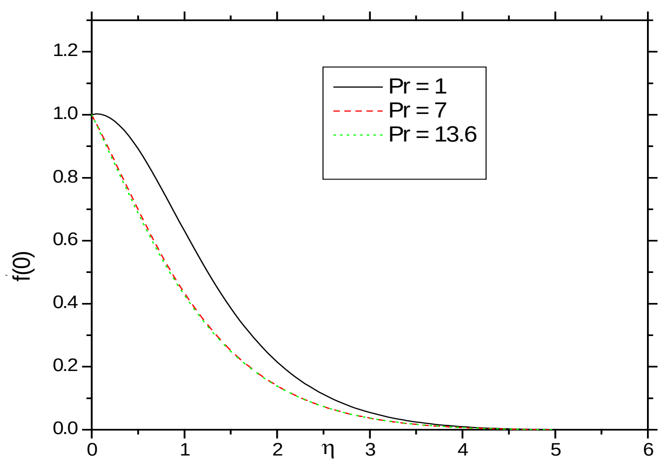

Fig.10 Variation of $f'(0)$ with variable heat fluxη for $\varepsilon = 0.9$ & Gr = 2

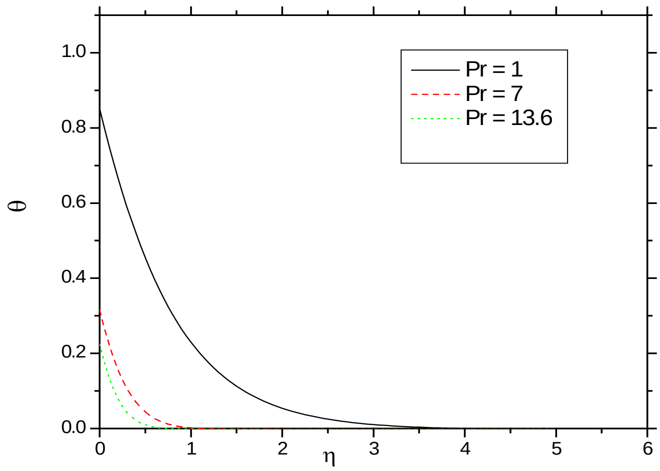

Fig.11 Variation of θ with variable heat flux η for $\varepsilon = 0.9$ & $Gr = 2$

CONCLUSIONS

In this paper we examined the effects of problem parameter on the free convection boundary layer flow with a variable plate temperature and prescribed surface heat flux in a porosity medium. The governing boundary layer equations have been solved numerically. As increase in λ tends to decrease both momentum and thermal boundary layer thickness. Also flow leads to thermal boundary layer at higher Prandtl number for the surface heat flux.

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