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$(\tau_1, \tau_2)^*$ -Q^*CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In the present paper, we introduced $(\tau_1, \tau_2)^*$-Q^*closed sets in bitopological spaces and studied its some of their bitopological properties. Also some relations are established with known generalized closed sets.

Keywords : *(τ_1, τ_2) *-Q^* closed sets, (τ_1, τ_2)*-Q^* open sets .*

2000 Mathematics Subject Classification: *54E55.*

1. Introduction

A triple (X,τ_1, τ_2) where X is a non-empty set and τ_1, τ_2 are topologies on X is called a bitopological space and Kelly initiated the study of such spaces. Maheswari and prasad introduced semi open sets in bitopological spaces in 1977.

Levine [8] introduced the concept of generalized closed sets in topological spaces. Also he introduced the notion of semi open sets in topological spaces .Bhattacharyya and Lahiri [1] introduced a class of sets called semi generalized closed sets by means of semi open sets of Levine and obtained various topological properties.

In 1985, Fukutake [5] introduced the concepts of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces.

The notion of Q^* - closed sets in a topological space was introduced by Murugalingam and Lalitha [9] in 2010.

Recently, P. Padma and S .Udayakumar [11] introduced the concept of $\tau_1\tau_2$ - Q^*continuous maps in bitopological spaces.

In the present paper, we introduced $(\tau_1, \tau_2)^*$ - Q^* closed sets in bitopological spaces and studied its some of their bitopologicalproperties. Also some relations are established with known generalized closed sets.

2.1 PRELIMINARIES

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2). For a subset A of X, τ_i - cl (A), τ_i - Q^*cl (A) (resp.τ_i - int (A) .τ_i - Q^*int (A)) represents closure of A and Q^*closure of A (resp. interior of A , Q^* - interior of A) with respect to the topology τ_i. We shall now require the following known definitions.

Definition 2.1: A subset S of X is called $\tau_1\tau_2$ - open if $S \in \tau_1 \cup \tau_2$ and the complement of $\tau_1\tau_2$ - open set is $\tau_1\tau_2$ - closed.

Example 2.1: Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}} and τ_2 = {ϕ, X, {b}}. Then τ_1 - open sets on X are ϕ, X, {a}, {a, b} and τ_2 - open sets on X are ϕ, X, {b}. Therefore, $\tau_1 \tau_2$ - open sets on X are ϕ, X, {a}, {b}, {a, b} and $\tau_1 \tau_2$ - closed sets are X, ϕ , {b, c}, {c, a}, {c}.

Definition 2.2: A set A of a bitopological space (X, τ_1, τ_2) is called

a) $\tau_1 \tau_2$ - semi open if there exists an τ_1 - open set U such that $U \subseteq A \subseteq \tau_2$ - cl (A). Equivalently, a set A is $\tau_1 \tau_2$ - semi open if $A \subset \tau_2$ - cl (τ_1-int (A)).

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b) $\tau_1 \tau_2$ - semi closed if X – A is $\tau_1 \tau_2$ - semi open.
c) $\tau_1 \tau_2$ - generalized open ($\tau_1 \tau_2$ - g open) if X – A is $\tau_1 \tau_2$ - generalized closed.
d) $\tau_1 \tau_2$ - generalized closed ($\tau_1 \tau_2$ - g closed) if τ_2 - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 - open in X.
e) $\tau_1 \tau_2$ - generalized open ($\tau_1 \tau_2$ - g open) if X – A is $\tau_1 \tau_2$ - g closed.
f) $\tau_1 \tau_2$ - semi generalized closed ($\tau_1 \tau_2$ - sg closed) if τ_2 -scl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 - semi open in X
g) $\tau_1 \tau_2$ - semi generalized open ($\tau_1 \tau_2$ - sg open) if X – A is $\tau_1 \tau_2$ -sg closed.
h) $\tau_1 \tau_2$ - generalized semi closed ($\tau_1 \tau_2$ - gs closed) if τ_2 - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 - open in X.
i) $\tau_1 \tau_2$ - generalized semi open ($\tau_1 \tau_2$ - gs open) if X – A is $\tau_1 \tau_2$ -gs closed.
j) $\tau_1 \tau_2$ - regular open if A = τ_1 - int [τ_2 - cl (A)].
k) $\tau_1 \tau_2$ - regular closed if A = τ_1 - cl [τ_2 - int (A)].

Definition 2.3: A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)* -semi generalized closed (briefly (τ_1, τ_2)* -sg closed) set if and only if $\tau_1\tau_2$ -scl (S) $\subseteq$ F whenever S $\subseteq$ F and F is $\tau_1\tau_2$ -semi open set. The complement of (τ_1, τ_2)* - semi generalized closed set is $\tau_1\tau_2$ - semi generalized open.

Definition 2.4: A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)* -generalized closed (briefly (τ_1, τ_2)* -g closed) set if and only if $\tau_1\tau_2$ -cl (S) $\subseteq$ F whenever S$\subseteq$F and F is $\tau_1\tau_2$ -open set. The complement of (τ_1, τ_2)*-generalized closed set is (τ_1, τ_2)* - generalized open.

3. (τ_1, τ_2) * - Q* - CLOSED SETS

The family of all (τ_1,τ_2)* - Q* closed subsets of a bitopological space (X, τ_1, τ_2) is denoted by (τ_1, τ_2)* - Q*.

Definition 3.1: A subset A of a bitopological spaces (X, τ_1, τ_2) is called

i) (τ_1,τ_2)* - Q* closed if $\tau_1\tau_2$ -int (A) = ϕ and A is $\tau_1 \tau_2$- closed.
ii) (τ_1, τ_2)*- Q* - open if X – A is (τ_1,τ_2)* - Q* closed in X .

Example 3.1: Let X = {a, b, c}, τ_1 ={ϕ, X, {c}}, τ_2= {ϕ, X, {b, c}, {c}}. Then $\tau_1 \tau_2$ - open sets on X are ϕ, X, {b, c}, {c} and $\tau_1 \tau_2$ - closed sets on X are ϕ , X , {a}, {a, b}.Clearly ϕ , {a, b} and {a} are (τ_1,τ_2)* - Q*closed.

Remark 3.1: Since every (τ_1, τ_2)* - Q* closed is $\tau_1\tau_2$- closed and $\tau_1\tau_2$- closed set is (τ_1, τ_2)* -g closed, (τ_1,τ_2)* -sg closed, we have (τ_1, τ_2)* - Q* closed is (τ_1, τ_2)* -g closed, (τ_1, τ_2)* -sg closed. But the converse is not true in general.

The following example supports our claim.

Example 3.2: Let X ={a, b, c},τ_1 = {ϕ, X, {a}, {b}, {a, b}}, τ_2= {ϕ, X}. Then $\tau_1 \tau_2$ - open sets on X are ϕ, X, {a},{b}, {a, b} and $\tau_1 \tau_2$ - closed sets on X are ϕ, X, {b, c}, {c, a}, {c}. Clearly {b, c} is (τ_1, τ_2)* -sg closed but it is not (τ_1, τ_2)* - Q* closed since $\tau_1\tau_2$ - int ({b, c}) = b $\neq\phi$.

Example 3.3: Let X = {a, b, c},τ_1 = {ϕ , X, {a}}, τ_2= {ϕ, X}. Then $\tau_1 \tau_2$ - open sets on X are ϕ , X,{a} and $\tau_1 \tau_2$ - closed sets on X are ϕ, X, {b, c}. Clearly {c} is (τ_1, τ_2)* -g closed but not (τ_1,τ_2)* - Q* closed .

Remark 3.2: Since every (τ_1,τ_2)* - Q* closed is τ_2- closed and τ_2- closed set is $\tau_1 \tau_2$ - g closed, $\tau_1 \tau_2$ - gs closed $\tau_1 \tau_2$ - sg closed, s*g closed, we have every (τ_1,τ_2)* - Q* closed is $\tau_1 \tau_2$ - g closed, $\tau_1 \tau_2$ - gs closed $\tau_1 \tau_2$ - sg closed $\tau_1 \tau_2$ - s*g closed. But the converse is not true in general. The following example supports our claim.

Example 3.4: In example 3.1,{c} $\tau_1 \tau_2$ - g closed, $\tau_1 \tau_2$ - sg closed and $\tau_1 \tau_2$ -gs closed but not (τ_1,τ_2)* - Q* closed .

Remark 3.3: Since every (τ_1, τ_2)* - Q* closed set is $\tau_1 \tau_2$ - Q* closed. But the converse is not true in general. The following example supports our claim.

Example 3.5: Let X = {a, b, c}, τ_1 = {ϕ, X, {a}}, τ_2= {ϕ, X, {a}, {b, c}}. Then ϕ, X, {a}, {b, c} are $\tau_1 \tau_2$ - open and X, ϕ, {b, c}, {a} are $\tau_1 \tau_2$ - closed. Clearly {b, c} is $\tau_1 \tau_2$ - Q* closed but it is not (τ_1,τ_2)* - Q* closed since $\tau_1 \tau_2$ -int ({b, c}) = {b, c} $\neq \phi$.

Remark 3.4: Every (τ_1, τ_2)* -Q* closed set is (τ_1 ,τ_2)* - semi closed. But the converse need not be true. The following example supports our claim.

Example 3.5: In example 3.2, {a} is (τ_1 ,τ_2)* - semi closed but not (τ_1 , τ_2)* - Q*closed .

Proposition 3.1: If A, B $\in(\tau_1, \tau_2)^*$ - Q^* then $A \cup B \in(\tau_1, \tau_2)^*$ - Q^* .

Proof: Let A and B be $(\tau_1, \tau_2)^*$- Q* closed sets in (X, τ_1,τ_2).

Claim: $A\cup B$ be a $(\tau_1, \tau_2)^*$- Q* closed sets in (X, τ_1, τ_2).

i.e) to prove $\tau_1 \tau_2$- int $(A \cup B) = \phi$ and A is $\tau_1\ \tau_2$ - closed.

Since, A and B be $(\tau_1, \tau_2)^*$- Q* closed sets in (X, τ_1, τ_2) we have

$\tau_1\tau_2$- int (A) = ϕ and A is $\tau_1\tau_2$ - closed and $\tau_1 \tau_2$- int (B) = ϕ and B is $\tau_1 \tau_2$ - closed.

Since (X, τ_1, τ_2) be a bitopological space, we have finite union of $\tau_1 \tau_2$ - closed sets are $\tau_1 \tau_2$ - closed.

$\Rightarrow \tau_1 \tau_2$- int $(A \cup B) = \phi$ and A is $\tau_1\ \tau_2$ - closed .

$\Rightarrow A \cup B$ is $(\tau_1, \tau_2)^*$- Q* closed sets in (X, τ_1, τ_2).

$\Rightarrow A \cup B \in(\tau_1, \tau_2)^*$ - Q*.

Proposition 3.2: Every $(\tau_1, \tau_2)^*$ - Q^* closed set is τ_2 - closed.

Proof: Let A be a $(\tau_1 , \tau_2)^*$ - Q^* closed set in X .

Then X – A is $(\tau_1, \tau_2)^*$ Q^* - open.

We have to show that

A is $(\tau_1, \tau_2)^*$ - Q^*closed.

Since every $(\tau_1 , \tau_2)^*$ - Q^*open set is τ_2- open, we have X – A is τ_2- open.

Thus,

A is τ_2 - closed.

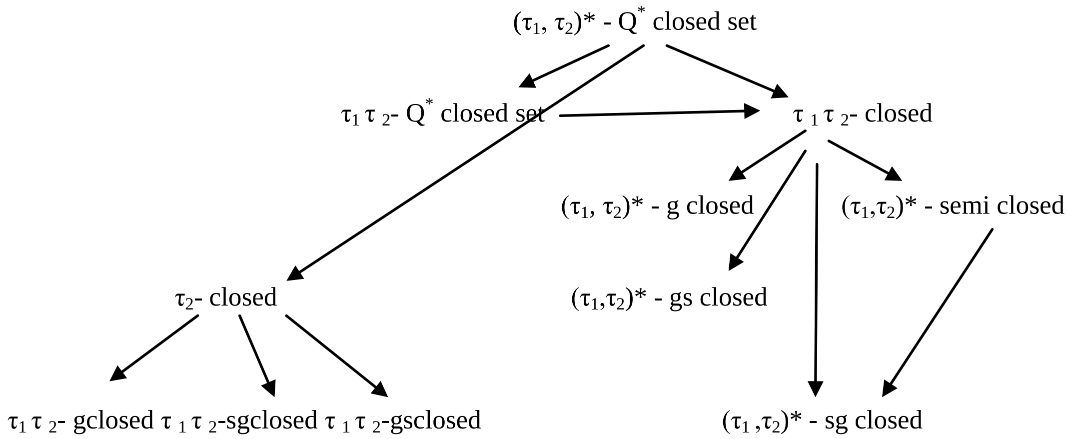
Remark 3.5: The converse of the above proposition is not true in general
ie) τ_2 - closed is not A is $(\tau_1 , \tau_2)^*$ - Q^*closed. The next example supports our claim.

Example 3.7: In example 3.1, X is not A is $(\tau_1, \tau_2)^*$ - Q^*closed.

Remark 3.6: $\tau_1\tau_2$ - regular closed sets and $(\tau_1, \tau_2)^*$ - Q^* closed sets are independent of each other in general. It is proved in the following example.

Example 3.8: X = {a, b, c}, τ_1 = {ϕ, X, {b, c}}, τ_2= {ϕ, X, {a}, {b, c}}. Then {a} is $\tau_1\tau_2$ - regular closed but not $(\tau_1, \tau_2)^*$ - Q^* closed set.

Result 3.1: From the above results we conclude the following



Theorem 3.1: If A is $(\tau_1, \tau_2)^*$ - Q^* closed then A is nowhere dense.

Proof: Since A is $(\tau_1, \tau_2)^*$ - Q^* closed, we have $\tau_1\tau_2$ - int (A) = ϕ and A is $\tau_1\tau_2$- closed.

Therefore,

$$\tau_1\tau_2 \text{ - cl } [\tau_1\tau_2 \text{ - int } (A)] = \phi .$$

Hence A is nowhere dense.

Theorem 3.2: Every $(\tau_1, \tau_2)^*$ - Q^* closed set is $(\tau_1, \tau_2)^*$ - δ set.

Proof: Let A be $(\tau_1, \tau_2)^*$ - Q^* closed.

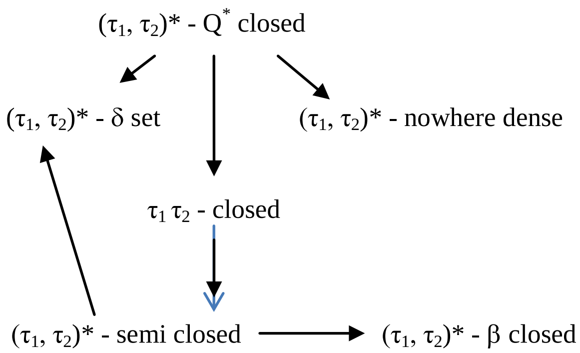
Then $\tau_1\tau_2$ -int (A) = ϕ and A is $\tau_1\tau_2$ - closed.

Consequently,

$$\begin{aligned} \tau_1\tau_2 \text{ - int } \{\tau_1\tau_2 \text{ - cl } [\tau_1\tau_2 \text{ - int } (A)]\} &= \tau_1\tau_2 \text{ - int } \{\tau_1\tau_2 \text{ - cl } (\phi)\} \\ &= \tau_1\tau_2 \text{ -int } (\phi) \\ &= \phi . \end{aligned}$$

Therefore, A is $(\tau_1, \tau_2)^*$ - δ set .

Result 3.2: The relationship between $(\tau_1, \tau_2)^*$ - Q^* closed sets and other generalizations is given by the following figure



4. $(\tau_1, \tau_2)^*$ - Q^* OPEN SETS

Definition 4.1: A subset A of a bitopological spaces (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$- Q^* open if X – A is $(\tau_1,\tau_2)^*$ - Q^* closed in X .

Example 4.1: In example 3.1, X, {c}, {b, c} are $(\tau_1, \tau_2)^*$- Q^* open.

Remark 4.1: Since every $(\tau_1, \tau_2)^*$ - Q^* open set is τ_2 - open and every τ_2 - open set is $\tau_1\tau_2$-g open, $\tau_1\tau_2$ - sg open, $\tau_1\tau_2$ - gs we have every $(\tau_1, \tau_2)^*$ - Q^* open set is $\tau_1\tau_2$ - g open, $\tau_1\tau_2$ - sg open and $\tau_1\tau_2$ - gs open . But the converse need not be true in general. The following example supports our claim.

Example 4.2: In example 3.1, {a, b} $\tau_1\tau_2$ - is g open, $\tau_1\tau_2$ - sg open and $\tau_1\tau_2$ -gs open but not $(\tau_1, \tau_2)^*$ - Q^* open.

Remark 4.2: Since every $(\tau_1,\tau_2)^*$ - Q^*open is $\tau_1\tau_2$- open and $\tau_1\tau_2$- open set is $(\tau_1, \tau_2)^*$ -gopen, $(\tau_1, \tau_2)^*$ -s g open, we have $(\tau_1,\tau_2)^*$ - Q^*open is $(\tau_1, \tau_2)^*$ -gopen, $(\tau_1, \tau_2)^*$ -s g open. But the converse is not true in general. The following example supports our claim.

Example 4.3: In example 3.2, {a} is $(\tau_1, \tau_2)^*$ -sg open but not $(\tau_1, \tau_2)^*$ - Q^* open.

Example 4.4: In example 3.3, {a, b} is $(\tau_1, \tau_2)^*$ -g open but not $(\tau_1,\tau_2)^*$ - Q^* open.

Theorem 4.1: A set A of a bitopological space (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*$ -Q^* open if and only if $\tau_1\tau_2$ - cl (A) = X and A is $\tau_1\tau_2$ - open .

Proof: Necessity: Suppose that A is $(\tau_1, \tau_2)^*$ - Q^* open.

Then A^c is $(\tau_1, \tau_2)^*$ - Q^* closed.

Therefore,

$\tau_1\tau_2$ - int (A^c) = $[\tau_1\tau_2$ - cl $(A)]^c$ = ϕ and A^c is $\tau_1\tau_2$- closed.

Consequently,

$\tau_1\tau_2$ - cl (A) = X and A is $\tau_1\tau_2$ - open.

Sufficiency: Suppose that $\tau_1\tau_2$ - cl (A) = X and A is $\tau_1\tau_2$ - open.

Then $[\tau_1\tau_2$ - cl $(A)]^c$ = $\tau_1\tau_2$ - int (A^c) = ϕ and A^c is $\tau_1\tau_2$- closed.

Consequently,

A^c is $(\tau_1, \tau_2)^*$ - Q^* closed.

This completes the proof.

Corollary 4.1: A set A of a bitopological space (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*$ - Q^* open if and only if A is $\tau_1\tau_2$ - dense and $\tau_1\tau_2$ - open.

Theorem 4.2: If A and B are $(\tau_1, \tau_2)^*$ - Q^* open sets then so is $A \cap B$.

Proof: Suppose that A and B are $(\tau_1, \tau_2)^*$ - Q^* open sets.

Then A^c and B^c are $(\tau_1, \tau_2)^*$ - Q^* closed sets.

Therefore,

$A^c \cup B^c$ is $(\tau_1, \tau_2)^*$ - Q^* closed sets.

But $A^c \cup B^c = (A \cap B)^c$.

Hence $A \cap B$ $(\tau_1, \tau_2)^*$ - Q^* open.

Theorem 4.3:
i) X is not $(\tau_1, \tau_2)^*$ - Q^* - closed.
ii) ϕ is $(\tau_1, \tau_2)^*$ - Q^* - closed.
iii) X is $(\tau_1, \tau_2)^*$ - Q^* - open
iv) X is not $(\tau_1, \tau_2)^*$ - Q^* - open.

Remark 4.3: It is obvious that every $(\tau_1, \tau_2)^*$ - Q^* - open set is τ_2 - open, but the converse is not true in general. The following example supports our claim.

Example 4.5: In example 3.1, ϕ is τ_2 - open but not $(\tau_1, \tau_2)^*$ - Q^* - open set.

Remark 4.4: Every $(\tau_1,\tau_2)^*$ - Q^*open is $(\tau_1,\tau_2)^*$ - semi open. But the converse need not be true.

The following example supports our claim.

Example 4.6: In example 3.2, {b. c} is $(\tau_1, \tau_2)^*$ - semi open but not $(\tau_1, \tau_2)^*$ - Q^* open.

Theorem 4.4:- If $B \subset A \subset X$, where A is $(\tau_1,\tau_2)^*$ - Q^*open and B is $(\tau_1,\tau_2)^*$ - Q^*open in A then B is $(\tau_1,\tau_2)^*$ - Q^*open in X .

Proof: Since B is $\tau_1\tau_2$- open in A, A is $\tau_1\tau_2$- open in X and B is $\tau_1\tau_2$- open in X.

We claim that $\tau_1\tau_2$- cl (B) = X.

Let U be any $\tau_1\tau_2$- openset.

Since $\tau_1\tau_2$- cl (B) is A, $(U \cap A) \cap B \neq \phi$.

Then

$(U \cap A) \cap B \neq \phi$.

Hence

$\tau_1\tau_2$- cl (B) = X.

Therefore,

B is (τ_1,τ_2) * - Q^*open in X .

Theorem 4.5: If A and B are $\tau_1\tau_2$- open sets with $A \cap B = \phi$ then A and B are not (τ_1,τ_2) * - Q^*open.

Proof: Since $A \cap B = \phi$, the points of B cannot be limit points of A.

Then $\tau_1\tau_2$- cl (A) $\neq$ X.

Hence A is not (τ_1,τ_2) * - Q^*open.

Similarly,

B is not (τ_1,τ_2) * - Q^*open.

Theorem 4.6 - Let (X, τ_1, τ_2) be a hyper connected bitopological space. Let $A \subset X$. If A is $\tau_1\tau_2$- open then A is (τ_1,τ_2) * - Q^*open in X .

Proof: It is enough to prove that A is $\tau_1\tau_2$- dense.

Suppose that $\tau_1\tau_2$- cl (A) $\neq$ X.

Then $[\tau_1\tau_2\text{- cl} (A)]^c \neq \phi$.

Consequently,

$A \cap [\tau_1\tau_2\text{- cl} (A)]^c \neq \phi$.

This is a contradiction to the fact that (X, τ_1, τ_2) is a hyper connected bitopological space.

Hence A is $\tau_1\tau_2$- dense.

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