International Journal of Mathematical Archive-3(7), 2012, 2589-2593
IJMA Available online through www.ijma.info ISSN 2229 – 5046

SEMI GLOBAL DOMINATION

S. V. Siva Rama Raju
Department of Mathematics, M. V. G. R. College of Engineering, Vizianagaram, India
&
S. S. V. R. Kumar Addagarla
Department of Computer Science & Engineering, VITS Group of Institutions, Visakhapatnam, India

(Received on: 25-06-12; Accepted on: 15-07-12)

ABSTRACT

A subset D of vertices of a graph connected graph G is called a semi global dominating set(sgd - set) iff D is a dominating set for both G and G^{sc}, where G^{sc} is the semi complementary graph of G. The semi global domination number (sgd - number) is the minimum cardinality of a semi global dominating set of G and is denoted by γ_{sg} (G). In this paper sharp bounds for γ_{sg}, are supplied for graphs whose girth is greater than three. Exact values of this number for paths and cycles are presented as well. The characterization result for a subset of the vertex set of G to be a semi global dominating set for G is given and also characterized the graphs of order n having sgd - numbers 2, n – 1, n.

Subject Classification: *05C69.*

Keywords: *semi global neighbourhood domination, semi global domination number, global domination, restrained domination, connected domination.*

1. INTRODUCTION & PRELIMINARIES

Domination is an active subject in graph theory, and has numerous applications to distributed computing, the web graph and adhoc networks. For a comprehensive introduction to theoretical and applied facets of domination in graphs the reader is directed to the book [2].

A set *D* of vertices is called a *dominating set* of *G* if each vertex not in *D* is joined to some vertex in *D*. The *domination number* $\gamma(G)$ is the minimum cardinality of the dominating set of *G*[2].

Many variants of the domination number have been studied. For instance a dominating set *D* is called a *global dominating set* of *G* if *D* is a dominating set of both *G* and its complement G^c. The *global domination number* of *G*, denoted by $\gamma_g(G)$ is the smallest cardinality of the global dominating set of *G*[5]. A dominating set *D* of connected graph *G* is called a *connected dominating set* of *G* if the induced sub graph $< D >$ is connected. The *connected domination number* of *G*, denoted by γ_c (*G*) is the smallest cardinality of the connected dominating set of *G*[6]. A dominating set *D* of connected graph *G* is called a *independent dominating set* of *G* if the induced sub graph $< D >$ is a null graph[2].

G be a connected graph, then the *Semi Complementary Graph* of *G* is denoted by G^{sc} and it has the same vertex set as that of *G* and edge set being $\{uv/u, v \in V(G), uv \notin E(G), \text{there is } w \in V(G) \text{ such that } uw, wv \in E(G)\}$[4].

Recently we have introduced a new type of graph known as *semi complete graph*. Let *G* be a connected graph, then *G* is said to be *semi complete* if any pair of vertices in *G* have a common neighbour. The necessary and sufficient condition for a connected graph to be semi complete is any pair of vertices lie on the same triangle or lie on two different triangles having a common vertex [3].

In the present paper, we introduce a new graph parameter, the *semi global domination number*, for a connected graph *G*. We call $D \subseteq V(G)$ a semi global dominating set (*sgd - set*) of *G* if *D* is a dominating set for both *G*, G^{sc}. The semi global domination number is the minimum cardinality of a semi global dominating set of *G* and is denoted by γ_{sg} (*G*).

Corresponding author: ****S. V. Siva Rama Raju****
Department of Mathematics, M. V. G. R. College of Engineering, Vizianagaram, India

All graphs considered in this paper are simple, finite, undirected and connected. For all graph theoretic terminology not defined here, the reader is referred to [1].

In this paper, sharp bounds for γ_{sg} are supplied for the graphs whose girth is greater than three. Also, we have given a characterization result for a proper subset of the vertex set of G to be a s*gd - set* of G and characterized the graphs whose s*gd - numbers* are $2, n, n-1$.

Note: Unless mentioned by G we mean a connected graph.

2. MAIN RESULTS

Here, we obtain some bounds for the *sgd - numbers* of graphs whose girth is greater than three.

Theorem 2.1: If G is a triangle free graph, then

$$\frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G) \leq n - \Delta(G) + 1.$$

Proof: Suppose that D be a minimum *sgd - set* of G. By our supposition each vertex in $V - D$ is non adjacent with atleast one vertex in D. Otherwise we get a contradiction to that D is a *sgd - set* for G.

$$\Rightarrow e \leq \frac{n(n-1)}{2} - [n - \gamma_{sg}(G)]$$

$$\Rightarrow \frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G)] \qquad (1)$$

Suppose that $d_G(v) = \Delta(G)$ for some v in V(G).

Let $v_1, v_2, \ldots, v_{\Delta(G)}$ be the neighbours of v in G. Since G is triangle free, $[V - \{v_1, v_2, \ldots v_{\Delta(G)}\}] \cup \{v_i : i \text{ is one of } 1, 2, \ldots, \Delta(G)\}$ is a *sgd – set* of G and its cardinality is $n - \Delta(G) + 1$.

$$\Rightarrow \gamma_{sg}(G) \leq n - \Delta(G) + 1 \qquad (2)$$

From (1) and (2)

$$\frac{2e - n(n-3)}{2} \leq \gamma_{sg}(G) \leq n - \Delta(G) + 1$$

Furthermore the lower bound is attained in the case of C_4 and upper bound is attained in the case of P_3. Hence the bounds are sharp.

Note: The upper bound holds good for any graph G.

Proposition 2.2:

1. $\gamma_{sg}(K_n) = n, n \geq 3$
2. $\gamma_{sg}(S_n) = 2, n \geq 3$
3. $\gamma_{sg}(K_{m,n}) = 2, m + n \geq 3$
4. $\gamma_{sg}(P_n) = [n/3]$, n =3m+1
   = [n/3]+2, n =3m, 3m+2

Here $n \geq 4$.

5. $\gamma_{sg}(C_n)$ = [n/3] , n =3m
   = [n/3]+1, n=3m+1, 3m+2
6. $\gamma_{sg}(C_n O K_2)$= n.

Proposition 2.3: $G = P_n (n \geq 4)$. Then there is an independent sgd – set for G iff $n = 3m+1$.

Proposition 2.4: $G = C_n (n \geq 4)$. Then there is an independent sgd – set for G iff $n = 3m$.

Proposition 2.5: $G = P_n (n \geq 3)$. Then $\gamma_{sg}(G) = n - 2$ iff $n = 4, 5$.

Proposition 2.6: $G = C_n (n \geq 4)$. Then $\gamma_{sg}(G) = n - 2$ iff $n = 4, 5$.

Proposition 2.7: If T is a tree of order $n \geq 3$, then $\gamma_{sg}(T) = 2$ iff T is obtained from P_3 or P_4 by adding zero or more leaves to the stems of the path.

Note: $2 \leq \gamma_{sg}(G) \leq n$.

Theorem 2.8: $\gamma_{sg}(G) = n$ if and only if $G \cong K_n$.

Theorem 2.9: $\gamma_{sg}(G) = n - 1$ if and only if $G \cong K_n - \{e\}$, where e is any edge in K_n.

Proof: Assume that $\gamma_{sg}(G) = n - 1$. Suppose diam(G) = l, $l \geq 3$. W.l.g. assume that $d_G(u, v) = l$ for some u, v in G. Clearly u *or* v is not a cut vertex in G. Hence $D - \{u, v\}$ is a connected dominating set in G. Follows that $D - \{u, v\}$ is a *sgd − set* in G of cardinality $n - 2$, which is a contradiction to our assumption. So diam(G) ≤ 2. If *diam* (G) = 1, then $G = K_n$.

This implies $\gamma_{sg}(G) = n$, a contrary to our assumption. Hence *diam (G)* = 2. This implies G has atleast one pair of non adjacent vertices. If G has a pendant vertex, then $\gamma_{sg}(G) = 2$.

Clearly $n \geq 4$. Hence $\gamma_{sg}(G) < n - 1$, a contrary to our assumptio n. Let $u_1 v_1$, $u_2 v_2$, ..., $u_s v_s$ be distinct pairs of non adjacent vertices in G. Since diam (G) = 2, $< u_1 w_1 v_1 >$, $< u_2 w_2 v_2 >$, ..., $< u_s w_s v_s >$ are paths in G for some w_1, w_2, ..., w_s in G. Clearly $V - \{u_1, u_2 ... u_s\}$ or $V - \{v_1, v_2 ... v_s\}$ is a *sgd − set* in G. If $|s| \geq 2$, then we get a contradiction to our assumption. So $|s| = 1$. This implies there is exactly one pair of non adjacent vertices in G.

Hence $G \cong K_n - \{e\}$.

The converse part is clear.

Corollary 2.10: If G is a tree, then $\gamma_{sg}(G) = n - 1$ if and only if $G \cong P3$.

Note: By Theorem.2.9
(i) $\gamma_{sg}(C_n) \neq n - 1$ for any n.
(ii) $\gamma_{sg}(P_n) = n - 1$ if and only if $n = 3$.

Theorem 2.11: $\gamma_{sg}(G) = 2$ if and only if
(i) There is an edge uv in G such that each vertex in $V - \{u, v\}$ is adjacent to u or v but not both.
or
(ii) There is a path P_4, each vertex in $V - V(P_4)$ lies on an edge whose end vertices are totally dominated by end vertices of P_4.

Proof: Suppose that $\gamma_{sg}(G) = 2$. W.l.g assume that $D = \{u, v\}$ be γ_{sg} – *set* in G.

Case: 1 $< D >$ is connected in G.

Clearly uv is an edge in G. If any vertex w in $V - \{u, v\}$ is adjacent to both u and v, then D is not a dominating set for G^{sc}. Hence (i) holds.

Case: 2 $< D >$ is not connected in G.

Clearly any vertex in $V - D$ cannot be adjacent to both u and v. Hence there is a path P_4 from u to v in G, say $< u v_1 v_2 v >$. Let $v_3 \in V - V(P_4)$. Since D is a *sgd − set* in G, v_3 is adjacent to *u or v(in G)* but not both. W.l.g assume that $v_3 v_1$ is in G. For v_3 to be dominated by a vertex in D, v_3, v are to be connected by a path of length two in G,say $< v_3 v_4 v >$.

Hence v_3 lies on an edge $v_3 v_4$ and v_3, v_4 are totally dominated by u, v(end vertices in P_4) respectively. Hence (ii) holds.

The converse part is clear.

Result 2.12: A sgd – set *for* G is a global dominating set for G.

Note: $\gamma_g(G) \leq \gamma_{sg}(G)$.

Result 2.13: If diam(G) = 2, then D is a sgd − set in G if and only if D is a global dominating set in G.

Corollary 2.14: G be a semi complete graph $D \subset V$. Then D is a sgd-set in G if and only if D is a global dominating set in G.

Proof: By hypothesis, *diam (G) = 2*. Hence proof follows from the above result.

Now, we give the characterization result for a non empty subset of *V* to be *sgd − set* in G

Theorem 2.15: $D \subset V$ is a sgd − set in G if and only if each vertex in V-D lies on an edge whose end points are totally dominated by distinct vertices in D.

Proof: Assume that D is a *sgd−set* in G. Let $v_1 \in V - D$. By our assumption, there exists v_2, v_3 *in D* $(v_2 \neq v_3)$ such that $v_1 v_2$ is in E (G) and $v_1 v_3$ is in E (G^{sc}). Since $v_1 v_3$ is in E (G^{sc}), there is v_4 in V such that $< v_1 v_4 v_3 >$ is a path in G.

Now, we have the following cases:

Case: 1 $v_4 = v_2$.

Then $< v_1 v_2 v_3 >$ is a path in G, which implies v_1 lies on the edge $v_1 v_2$ and v_1 , v_2 are dominated by v_2 , v_3 respectively from $D - \{v_1\}$, $D - \{v_1 , v_2\}$.

Case: 2 $v_4 \neq v_2$.

Then $< v_2 v_1 v_4 v_3 >$ is a path in G which implies v_1 lies on the edge $v_1 v_4$ and v_1 , v_4 are dominated by v_2 , v_3 respectively from $D - \{v_1 , v_4\}$.

Hence in either case the claimant holds.

Conversely assume that $v_1 \in V - D$. By our assumption there is an edge $v_1 v_2$ in G such that $v_1 v_3$, $v_2 v_4$ are in G and v_3, v_4 are in D $(v_3 \neq v_4)$.

If $v_3 = v_2$, then $< v_1 v_2 v_4 >$ is a path in G and $v_1 v_2$ is in G, $v_1 v_4$ is in G^{sc}.

If $v_2 \neq v_3$, then $< v_3 v_1 v_2 v_4 >$ is a path in G, which implies $v_1 v_3$ is in G and $v_1 v_4$ is in G^{sc}.

Hence, in either case for v_1 in D, there are v_3, v_4 in D such that $v_1 v_3$ is in G and $v_1 v_4$ is in G^{sc}. Hence D is a *sgd − set* in G.

Theorem 2.16: G be a connected graph and D be a γ_c − set in G. Then $d_{<D \cup \{v\}>}(v) < n$ for each v in V − D if and only if D is a sgd − set in G.

Proof: Assume that $d_{<D \cup \{v\}>}(v) < n$ for each v in $V - D$. Let $v \in V - D$.

Then by our assumption $d_{<D \cup \{v\}>}(v) < n$. This implies there is v_1 in D such that $d(v, v_1) \neq 1$. Since $< D \cup \{v\} >$ is connected, this implies there is a $v - v_1$ path in $D \cup \{v\}$ (say) $P = < v v_2 v_3 v_4 \ldots v_1 >$, where $v_2, v_3 \ldots \in D$. Since $d_{<D \cup \{v\}>}(v) < n$, there is a $v_i \in D$ such that $d_G(v, v_i) = 2$.This implies $vv_i \in E(G^{sc})$. *D is a sgd − set in G.*

Conversely assume that $v \in V - D$. By our assumption, there is v_1 in D such that $d_G(v, v_1) = 2$. This implies $vv_1 \in G$.

Hence $d_{<D \cup \{v\}>}(v) < n$.

Theorem 2.17: G be a connected graph such that $\delta(G) \geq 2$ and D is an independent sgd −set for G. If D^c is independent, then D^c is a sgd – set in G.

Proof: Assume that D^c is independent. Let $v \in V - D^c = D$. This implies there is v_1 in D^c such that vv_1 is in *G* (since $\delta(G) \geq 2$). Since v_1 is in D^c and *D* is independent *sgd − set* in *G*, there is v_2 in *D*, v_3 in V such that $< v_1 v_3 v_2 >$ is a path in *G*. Clearly $v_3 \in D^c$. Since D^c is independent, $< vv_1 v_3 v_2 >$ is a path in *G* and vv_3 is not an edge in *G*. For $v \in V - D^c$, there is $v_1 \in D^c$ such that vv_1 is in *G* and vv_3 is in G^{sc} . Since *v* is arbitrary, D^c is a *sgd − set* in *G*.

Note: The converse is not true in view of *P7*.

Result 2.18: For a semi complete graph G, $\gamma_{sg}(G) \geq 3$.

Proof: Suppose claimant does not hold. Since $\gamma_{sg}(G) \neq 1$, $\gamma_{sg}(G) = 2$. Let $D = \{v_1, v_2\}$ be a *sgd − set* in *G*.

Case: 1 $< D >$ is connected in *G*.

Then v_1 v_2 is an edge in G. By the nature of semi complete graph there is a v_3 in G such that $< v_1\ v_2\ v_3 >$ is a triangle in G. This implies D is not a dominating set in G^{sc}, which is a contradiction to D is a $sgd - set$ in G.

Case: 2 $< D >$ is disconnected in G.

Since G is semi complete there is v3 in G such that $< v_1\ v_3\ v_2 >$ is a path in G. Then in G^{sc} , v_3 is not dominated by vertex in D, a contradiction to D is a $sgd - set$ in G.

Hence in either case, we get a contradiction to D is a $sgd - set$ in G.

So, Our supposition is false. This implies γ_{sg} (G) $\geq$ 3.

ACKNOWLEDGEMENTS

Thanks are due to Prof. I. H. Nagaraja Rao for his valuable suggestions and encouragement given by him throughout the preparation of the paper.

REFERENCES

[1] Bondy J.A. and Murthy, U.S.R., *Graph theory with Applications*, The Macmillan Press Ltd, 1976.

[2] T. W. Haynes, S. T. Hedetneimi, P.J. Slater, *Fundamentals of Dominations in Graphs*, Marcel Dekker, New York, 1988.

[3] I. H. Naga Raja Rao, S. V. Siva Rama Raju, On Semi-Complete Graphs, *International Journal Of Computational Cognition*, **Vol.7 (3)**, (2009), 50-54.

[4] I. H. Naga Raja Rao, S. V. Siva Rama Raju, Semi Complementary Graphs, *Communicated to Thai Journal Of Mathematics*.

[5] D. F. Rall, *Congr. Numer.*, **80** (1991), 89 - 95.

[6] E. Sampathkumar, H. B. Walikar, The connected Domination Number of a Graph, *J. Math. Phy. Sci*, **13** (1979), 607 - 613.

Source of support: Nil, Conflict of interest: None Declared