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UNSTEADY BOUNDARY LAYER FLOW OF DUSTY FLUID OVER A VERTICAL STRETCHING SHEET IN THE PRESENCE OF NON-UNIFORM HEAT SOURCE/SINK

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ABSTRACT

This paper presents the two dimensional mixed convective flow of an electrically conducting dusty fluid over a porous stretching sheet in the presence of non-uniform heat source/sink. Where the flow is due to a stretching sheet and the fluid is assumed to be viscous and incompressible. The equations governing the flow and temperature fields are reduced to coupled non-linear ordinary differential equations by using the similarity transformation. Numerical solutions of these equations are obtained by using RKF-45 method. Numerical results are bench marked with the earlier studies and found to be in excellent agreement. Finally the pertinent parameter which are of physical and engineering interest are discussed graphically.

Keywords: *Boundary layer flow; mixed convective parameter; stretching porous surface; dusty fluid; heat source/sink; numerical solution.*

AMS Subject Classification (2000): *76T15 and 80A20;*

1. INTRODUCTION

The steady of unsteady flow of viscous, incompressible dusty fluid flow over a stretching sheet has become an important in past years due to the extensive engineering applications such as polymer sheet extrusion from a dye, glass fiber and paper production, drawing of plastic films, etc. In reality, most of the liquids are non- Newtonian in nature, which are abundantly used in many industrial applications, such as in manufacturing of plastic films and artificial fibers, aerodynamics extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, crystal growing, liquid condensation process, continuous polymer sheet extrusion, heat treated material traveling between a feed roll, wind up roll or on a conveyer belt, geothermal reservoirs and petroleum industries. The available literature of last few decades deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin. However, it is often argued by Gupta and Gupta [1] that realistically stretching of plastic sheet may not be necessarily linear. Kumaran and Ramaniah [2] have studied this situation in their work on boundary layer fluid flow where, the stretching sheet has been assumed quadratic.

In view of this, the study of boundary layer flow problem has been further channelized to non-Newtonian fluid flow. For horizontal plate, Sakiadis [3] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows. Tsou et al. [4] analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis [3]. Carragher et al. [5] investigated the heat transfer in the flow over a stretching surface in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Grubka et al. [6] have studied the temperature field in the flow over a stretching surface subjected to a uniform heat flux. Vajravelu et al. [7] have discussed hydromagnetic flow of a dusty fluid over a stretching sheet. Sharidan et al. [8] have studied similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet. Abel et.al [11] have studied the flow and heat transfer in a viscoelastic boundary layer flow over a stretching sheet with prescribed surface temperature (PST) case and prescribed heat flux (PHF) case, further they have discussed heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Andersson et.al [12] presented a new similarity solution for the temperature fields, which transforms the time dependent thermal energy equation to an ordinary differential equation. Aziz [13] obtained the numerical solution for laminar thermal boundary over a flat plate with a convective surface boundary condition using the symbolic algebra software Maple. Recently Gireesha et.al [14, 15] have studied unsteady hydromagnetic boundary layer flow and heat transfer of dusty fluid over a stretching sheet.

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Further they have studied boundary layer flow and heat transfer of a dusty fluid flow over a stretching sheet with non-uniform heat source /sink with prescribed surface temperature (PST) case and prescribed heat flux (PHF) case. Sharma et al. [17] have obtained the numerical results for the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Sparrow et al. [18] solved the laminar free convection flow on an isothermal vertical plate. The free convection problem on a vertical plate has been studied in various ways; [see Kuiken [19], Gireesha [20] and Elbashbeshy [21]. The similarity equations contain Prandtl number, Eckret number, number density and unsteadiness parameter and these parameter discussed with help of graphs.

In the above of referred literatures the effect of dusty parameter and unsteady parameter on heat transfer phenomena are excluded from the analysis. Hence in this paper, we contemplate to study the effect of variable non-uniform heat source on the mixed convective flow and heat transfer of electrically conducting dusty fluid over a porous stretching sheet. Further, heat transfer characteristics are examined for two different kinds of boundary conditions, namely (i) variable wall temperature (VWT) and (ii) variable heat flux (VHF). The solutions for the flow and heat transfer are solved numerically using RKF-45 method with the help of Maple.

2. MATHEMATICAL FORMULATION

Consider a two-dimensional unsteady boundary layer flow of dusty viscous and incompressible fluid (with electric conductivity σ) past a semi-infinite stretching sheet in the region $y > 0$. It is considered that the flow is generated by stretching of an elastic boundary sheet from a slit with the application of two equal and opposite forces in such way that velocity of boundary sheet is linear order of the flow directional coordinate x. A uniform magnetic field B_0 is imposed along the y-axis.

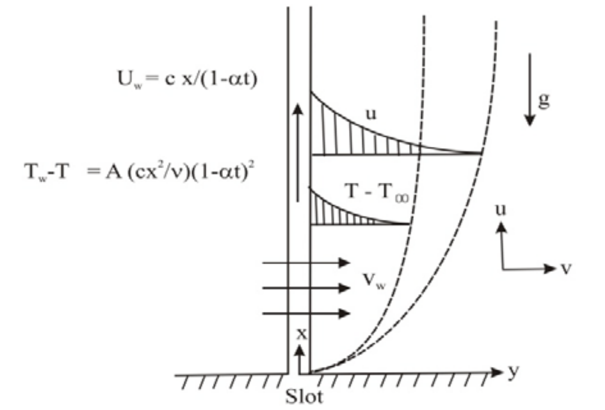


Figure 1. Physical model of the problem

The unsteady two-dimensional boundary layer equations of a dusty fluid in the usual notation are [7]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{kN}{\rho}\left(u_p - u\right) + g\,\beta^*(T - T_\infty) - \frac{\sigma\, B_0^{'2}}{\rho}u, \tag{2}$$

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = \frac{k}{m}(u - u_p) \tag{3}$$

$$\frac{\partial v_p}{\partial t} + u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y} = \frac{k}{m}(v - v_p) \tag{4}$$

$$\frac{\partial(\rho_p u_p)}{\partial x} + \frac{\partial(\rho_p v_p)}{\partial y} = 0, \tag{5}$$

where t is the time, (u, v) and (u_p, v_p) denote the velocity components of the fluid and particle phase respectively along the x-axis and y- axis. Further μ is the coefficient of viscosity of fluid, ρ and ρ_p are the density of the fluid and particle phase, B_0 is the induced magnetic field, g and β^* are acceleration due to gravity and volumetric coefficient of thermal expansion, respectively and N is the number density of the particle phase, K the Stoke's resistance co-efficient (for spherical particles of radius r is $6\pi r\mu$), m is the mass concentration of dust particles. In deriving these equations, the Stokesian drag force is considered for the interaction between the fluid and particle phase and the induced magnetic field is neglected. It also is assumed that the external electric field to be zero and the electric field as a result of polarization of charges is negligible.

The boundary conditions applicable to the above problem are defined as:

$$u = U_w(x,t),\ v = V_w(x,t) \quad \text{at} \quad y = 0,$$

$$u \to 0,\ u_p \to 0,\ v_p \to v,\ N \to K\rho \ \text{ as } \ y \to \infty, \tag{6}$$

where $U_w = \frac{cx}{1-\alpha t}$ is the velocity of sheet, $V_w(x,t) = -\frac{v_0}{\sqrt{(1-\alpha t)}}$ is the suction velocity and c is the initial stretching rate being a positive constant, α is positive constant which measures the unsteadiness. Whereas the effective stretching rate $\frac{c}{1-\alpha t}$ is increasing with time.

Equations (1) to (5) subjected to boundary condition (6), admits self-similar solution in terms of the similarity function f and the similarity variable η is defined as

$$u = \frac{cx}{1-\alpha t} f'(\eta), \qquad v = -\sqrt{\frac{c\nu}{(1-\alpha t)}} f(\eta), \qquad \eta = \sqrt{\frac{c}{\nu(1-\alpha t)}}\, y,$$

$$u_p = \frac{cx}{1-\alpha t} F(\eta), \quad v_p = \sqrt{\frac{c\nu}{(1-\alpha t)}} G(\eta), \quad \rho_r = H(\eta), \quad B'_0 = B_0(1-\alpha t)^{-\frac{1}{2}} \tag{7}$$

where a prime denotes the differentiation with respect to η. Substituting the equations (7) into equations (1) to (5) one can get

$$f'''(\eta) + f(\eta)f''(\eta) - f'(\eta)^2 - A\left[f'(\eta) + \frac{\eta}{2} f''(\eta)\right] + l\beta H(\eta)[F(\eta) - f'(\eta)] - Mf'(\eta) + \lambda\theta(\eta) = 0, \tag{8}$$

$$G(\eta)F'(\eta) + F(\eta)^2 - \beta[f'(\eta) - F(\eta)] + A\left[F(\eta) + \frac{\eta}{2} F'(\eta)\right] = 0, \tag{9}$$

$$G(\eta)G'(\eta) + \beta[f(\eta + G(\eta)] + \frac{A}{2}[G(\eta) + \eta G'(\eta)] = 0, \tag{10}$$

$$F(\eta)H(\eta) + G'(\eta)H(\eta) + H'(\eta)G(\eta) = 0, \tag{11}$$

where $\rho_r = \frac{\rho_p}{\rho}$ is the relative density, $A = \frac{\alpha}{c}$ is the parameter that measures the unsteadiness, $l = \frac{mN}{\rho_p}$ is the mass concentration, $M = \frac{\sigma B_0^2}{\rho b}$ is the magnetic field parameter, $\lambda = \frac{Gr_x}{Re_x^2}$ is the mixed convective parameter, $Gr_x = \frac{g\beta^*(T_w - T_\infty)x^2}{\nu^2}$ is the local Grashof number, $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number and $\beta = \frac{1}{\tau b}(1-\alpha t)$ is the fluid-particle interaction parameter.

The corresponding boundary conditions are transformed to:

$$\begin{aligned} &f'(\eta) = 1, \quad f(\eta) = R \quad \text{at} \quad \eta = 0, \\ &f'(\eta) = 0, \quad F(\eta) = 0, \quad G(\eta) = -f(\eta), \quad H(\eta) = E \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{12}$$

where $R = \frac{v_0}{\sqrt{\nu c}}$ is the suction parameter and E is constant density ratio.

3. HEAT TRANSFER ANALYSIS

The governing unsteady, dusty boundary layer heat transport equations in the presence of temperature-dependent internal heat generation/absorption for two-dimensional flow are [22]

$$\rho c_p \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k^* \frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{\tau_T}\left(T_p - T\right) + \frac{N}{\tau_v}\left(u_p - u\right)^2 + q''', \tag{13}$$

$$Nc_m \left[\frac{\partial T_p}{\partial t} + u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right] = -\frac{Nc_p}{\tau_T}\left(T_p - T\right), \tag{14}$$

where T and T_p are the temperature of the fluid and dust particle, c_p and c_m are the specific heat of fluid and dust particles, τ_T is the thermal equilibrium time and is time required by the dust cloud to adjust its temperature to that of fluid, k^* is the thermal conductivity, τ_v is the relaxation time of the of dust particle i.e., the time required by a dust particle to adjust its velocity relative to the fluid and q''' is the space and temperature-dependent internal heat generation/absorption (non-uniform heat source/sink) [14] and which can be expressed in the simplest form as

$$q''' = \frac{K^* u(x,t)}{xv}[\, A^*(T_w - T_\infty)f'(\eta) + (T - T_\infty)B^*], \tag{15}$$

where A^* and B^* are the coefficient of space and temperature dependent heat source/sink, respectively. Note that the case $A^* > 0$ and $B^* > 0$ corresponds to internal heat generation and $A^* < 0$, $B^* < 0$ corresponds to internal heat absorption.

The solution of equations (13)-(14) depends on the nature of the prescribed boundary condition. The two types of heating processes are discussed.

CASE-1: Variable wall temperature (VWT-Case)

For this heating process, the variable wall temperature is assumed to be a quadratic function of x and it is given by

$$\begin{aligned} &T = T_w = T_\infty + T_0\left[\frac{cx^2}{v(1-\alpha t)^2}\right] \quad \text{at} \quad y = 0,\\ &T \to T_\infty, \qquad T_p \to T_\infty \quad \text{as} \quad y \to \infty, \end{aligned} \tag{20}$$

where T_w is the surface temperature of the sheet varies with the distance x from the slot and time t, T_0 is a reference temperature such $0 \le T_0 \le T_w$ and T_∞ is the temperature far away from the stretching surface with $T_W > T_\infty$.

In order to obtain similarity solution for temperatures $\theta(\eta)$ and $\theta_p(\eta)$, define dimensionless variables as follows:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \tag{21}$$

where $T - T_\infty = T_0\left[\frac{cx^2}{v(1-\alpha t)^2}\right]\theta(\eta)$.

Using Equations (15), (19), (20) and (21) in Equations (13) and (14), we get

$$\begin{aligned} &\theta''(\eta) + \Pr[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{N}{\rho}a_1 Pr[\theta_p(\eta) - \theta(\eta)] + \frac{N}{\rho}PrEc\beta[F(\eta) - f'(\eta)]^2 - \frac{A}{2}\Pr[4\theta(\eta) + \eta\theta'(\eta)]\\ &\qquad + A^* f'(\eta) + B^*\theta(\eta) = 0, \end{aligned} \tag{22}$$

$$G(\eta)\theta_p'(\eta) + 2F(\eta)\theta_p(\eta) + \frac{A}{2}\left[4\theta_p(\eta) + \eta\theta_p'(\eta)\right] + a_1\gamma\left[\theta_p(\eta) - \theta(\eta)\right] = 0, \tag{23}$$

where $Pr = \frac{\mu c_p}{k^*}$ is Prandtl number, $Ec = \frac{v^2}{C_p T_0}$ is Eckret number, $a_1 = \frac{1}{\rho\tau_T c}(1 - \alpha t)$ is the local fluid-particle interaction parameters for temperature, $\gamma = \frac{c_p}{c_m}$.

Using Equations (20) and (21), the corresponding boundary conditions for $\theta(\eta)$ and $\theta_p(\eta)$ reduce to the following form

$$\begin{aligned} &\theta(\eta) = 1 \quad \text{at} \quad \eta = 0,\\ &\theta(\eta) = 0, \quad \theta_p(\eta) = 0 \quad \text{as} \quad \eta \to \infty. \end{aligned} \tag{24}$$

CASE-2: Variable heat flux (VHF-Case)

For this heating process, the following variable heat flux boundary condition is employed.

$$\begin{aligned} &\frac{\partial T}{\partial y} = -\frac{q_w(x,t)}{k^*} \quad \text{at} \quad y = 0,\\ &T \to T_\infty, \qquad T_p \to T_\infty \quad \text{as} \quad y \to \infty, \end{aligned} \tag{25}$$

where $q_w(x,t) = q_{w_0}\, x^2 \left(\frac{c}{\nu}\right)^{3/2} (1-\alpha t)^{-5/2}$. In order to obtain similarity solution for temperature, define dimensionless temperature variables for the VHF case as in Equation (21) where T= $T_\infty + \frac{q_{w0}}{k^*}\left[\frac{bx^2}{\nu(1-\alpha t)^2}\right]\theta(\eta)$.

Using the dimensionless variable (21), the temperature equations (13) and (14) take the form

$$\theta''(\eta) + \Pr[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + \frac{N}{\rho}a_1 Pr[\theta_p(\eta) - \theta(\eta)] + \frac{N}{\rho} PrEc\beta[F(\eta) - f'(\eta)]^2 - \frac{A}{2}\Pr[4\theta(\eta) + \eta\theta'(\eta)] + A^* f'(\eta) + B^*\theta(\eta) = 0, \tag{26}$$

$$G(\eta)\theta_p'(\eta) + 2F(\eta)\theta_p(\eta) + \frac{A}{2}\left[4\theta_p(\eta) + \eta\theta_p'(\eta)\right] + a_1\gamma\left[\theta_p(\eta) - \theta(\eta)\right] = 0, \tag{27}$$

where Eckert number $Ec = \frac{k\nu^2}{c_p q_{w0}}$.

The corresponding boundary conditions become

$$\theta'(\eta) = -1 \quad \text{at} \quad \eta = 0,$$

$$\theta(\eta) = 0, \quad \theta_p(\eta) = 0 \quad \text{as} \quad \eta \to \infty. \tag{28}$$

The physical quantities of interest are the skin friction coefficient c_f and the local Nusselt number Nu_x, which are defined as

$$c_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)},$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = k\left(\frac{\partial T}{\partial y}\right)_{y=0},$$

Using the non-dimensional variables, we obtain

$$c_f Re_x^{1/2} = f''(0), \quad Nu_x/Re_x^{1/2} = -\theta'(0) \text{ (VWT)}, \quad Nu_x/Re_x^{\frac{1}{2}} = 1/\theta(0) \text{ (VHF)}.$$

4. NUMERICAL SOLUTION

The equations (8) to (11) with the boundary conditions (12) are highly non-linear ordinary differential equations. In order to solve these non-linear equations numerically we adopted symbolic software Maple, it is very efficient in using the well known Runge-Kutta Fehlberg fourth-fifth order method (RKF45 Method) [12]. In accordance with the boundary layer analysis, the boundary condition (12) at $\eta = \infty$ were replaced by $\eta = 5$. The coupled boundary layer equations (8) to (11) and either Equations (22) and (23) or (26) and (27) were solved by RKF45 method. The accuracy of this numerical method was validated by direct comparison with the numerical results reported by Grubka and Bobba [4], Abel and Mahesha [9] with ($A = A^* = B^* = Ec = 0$). If A=0 and λ=0, the numerical solution of Equations (1) to (5) coincides with the results of Vajravelu and Nayfeh [5]. Table 1 presents results of this comparison for $-\theta'(0)$. It can be seen from this table that there is a very good agreement between the results.

Table 1: Comparison of wall temperature gradient $-\theta'(0)$ for several values of Pr with $\alpha = \beta = M = N = Ec = A^* = B^* = 0.0$.

Pr	Grubka and Bobba [4]	Abel and Mahesha [9]	Present Study $-\theta'(0)$
0.72	1.0885	1.0885	1.0885
1.0	1.3333	1.3333	1.3333
10.0	4.7969	4.7968	4.7968

5. RESULTS AND DISCUSSION

Two dimensional unsteady boundary layer flow problem for momentum and heat transfer over a stretching sheet in the presence of non-uniform heat source/sink is examined in this paper. The governing partial differential equations, which are highly non-linear and in coupled form, have been converted into a set of non-linear ordinary differential equations by applying the suitable similarity transformations. The obtained boundary layer equations of momentum and heat transfer are solved numerically. In order to have a great insight of the effects of all the physical parameter on momentum and heat transfer characteristics, we depict numerical results in the figures 1-9. From table 1, we note that there is close agreement with the results of previously published work by Grubka and Bobba [6]. And thus verify the accuracy of the method used.

Numerical values of wall temperature gradient for different values of governing parameters for VWT and VHF case are tabulated as in the table 2. From this table, it is noticed that $\theta'(0)$ (for the VWT case) and $\theta(0)$ (for the VHF case) are increases with increasing values of heat source or sink parameter and the Eckert number. Further it decreases with the increasing values of Prandtl number, unsteady parameter and convective parameter. The temperature distribution of VWT and VHF case shows that the VWT boundary conditions succeeds in keeping the liquid warmer than in the case, when the VHF boundary condition is applied. Therefore one can say that the VWT boundary condition is better than the VHF boundary condition in faster cooling of stretching sheet. From the results of VWT and VHF cases we infer that the boundary layer temperature is quantitatively higher in VWT case as compared to VHF case.

Figure 1 (a) and (b), represent horizontal velocity profile of both fluid and dust phases for various value of A and magnetic parameter M respectively, when $Pr = 0.72$, $R = 2, M = 0.1, N = 0.2, A^* = B^* = -0.05, \lambda = 0.1$ and $\beta = 1$. From the figures 1 (a) and (b), it is predicted that the velocity gradient at the surface increases (in magnitude) with A and M. Thus the magnitude of the skin friction coefficient increases as A or M increases. The figures 1(a) and (b) show that the velocity gradient at the surface $f''(0)$ is negative for all values of considered parameters. Physically, negative values of $f''(0)$ means that the solid surface exerts a drag force on the fluid. Thus the velocity is found to be decrease as the distance from the surface increases and reaches the boundary condition at infinity asymptotically. It is interesting to note that the thickness of boundary layer deceases with increasing values of A.

Figures 2(a) and 2(b) depict horizontal velocity profile of both fluid and dust phase for various value of λ for VWT and VHF case. The velocities of fluid and dust particle increases with the increasing values of mixed convective parameter. Physically $\lambda > 0$ means heating of the fluid or cooling of the boundary surface, $\lambda < 0$ means cooling of the fluid or heating of the boundary surface and $\lambda = 0$ corresponds to the absence of free convection current.

Figures 3(a) and 3(b) show that the temperature profiles of fluid and dust phase decreases with the increasing values of mixed convective parameter. It is clear that as λ increases the thermal boundary layer thickness decreases. From these figures one can observe that the temperature gradient as well as thermal boundary layer thickness is decreasing as λ increases. The temperature gradient is always negative which means that the heat is transferred from the sheet to the ambient medium. Hence, the heat transferred rate from the sheet to the ambient medium is getting large as λ increases.

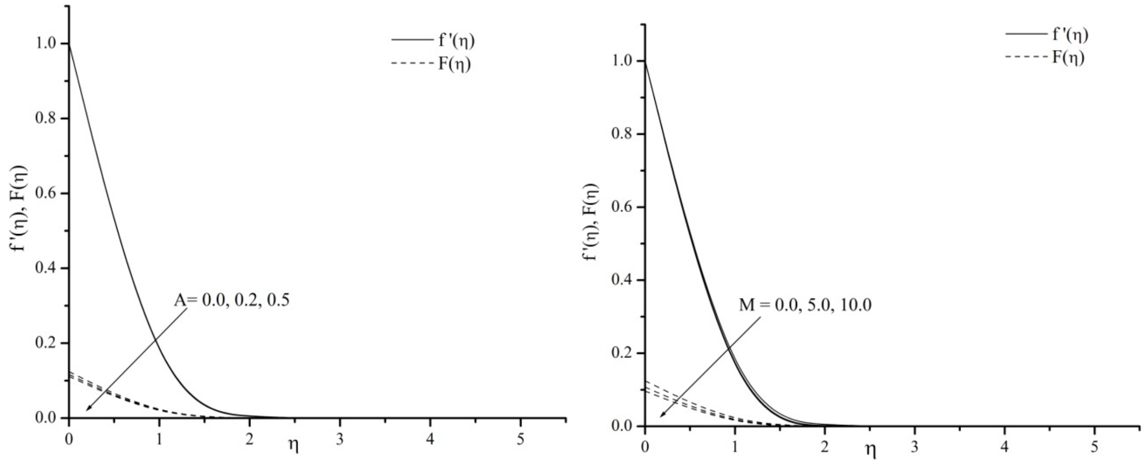
Figures 4(a) and 4(b) are the graphical representation for the temperature distribution for VWT and VHF case, for different values of unsteady parameter A versus η. It is observed that temperature of fluid and dust phase is found to be decrease with increase of unsteady parameter A. Physically, it means that the temperature gradient at the surface increases as A increases, which implies an increase of heat transfer rate at the surface $\theta'(0)$. We have used throughout our analysis the values of $a_1 = 2$.

Figures 5(a) and 5(b) depict temperature profiles $\theta(\eta)$ and $\theta_p(\eta)$ versus η, for different values of Pr. We infer from these figures that the temperature of fluid and dust phases decreases with the increase in Pr which implies that the heat transfer rate at the surface increases with increasingPr. Physically, this can be explained as higher the Prendtl number fluid has relative low thermal conductivity, reduces the conduction and hence the thermal boundary layer thickness increases the heat transfer rate at the surface. The temperature in both VWT and VHF cases asymptotically approaches to zero in the free stream region.

Figures 6(a) and 6(b) elucidate the effect of the Eckert number Ec on the temperature profile $\theta(\eta)$ and $\theta_p(\eta)$ versus η, for VWT and VHF cases respectively. It is predicted that increasing the value of Ec enhances the temperature of the fluid and dust phases at any point and that this is true for both the VWT and VHF cases. This is due to fact that the heat energy is stored in the considered liquid due to frictional heating.

Figures 7(a) and 7(b) are graphs of temperature profiles of $\theta(\eta)$ and $\theta_p(\eta)$versusη, for different values of Number density N for VWT and VHF cases respectively. From the figures, it is observed that the temperature of fluid and dust phases decreases with the increase of N. This is due to increase in number density increases dust particle which opposes the flow.

Figures 8(a) and 8(b) depict the effect of the space-dependent heat source/sink parameter A^* on the temperature profiles of both phases for the VWT and VHF cases, respectively. It is observed that the thermal boundary layer generates energy, which causes the temperature profiles of both the fluid and dust phases (for VWT and VHF cases) to increase with increasing values of $A^* > 0$ whereas in the case of $A^* < 0$, the boundary layer absorbs the energy causing the temperatures of both phases to fall considerably. Similar predictions are valid for B^* also and they are shown in Figures 9(a) and 9(b).



Figures 1(a) & 1(b): Velocity profiles for the effect of unsteadiness parameter (A) and Magnetic parameter (M) respectively.

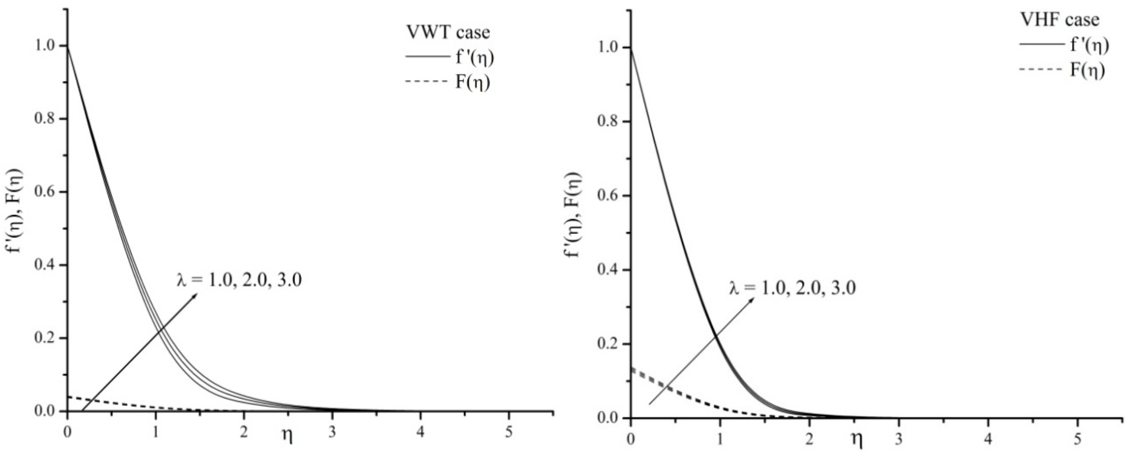


Figure 2(a) & 2(b): Effect of Convective parameter λ for fluid and dust velocity.

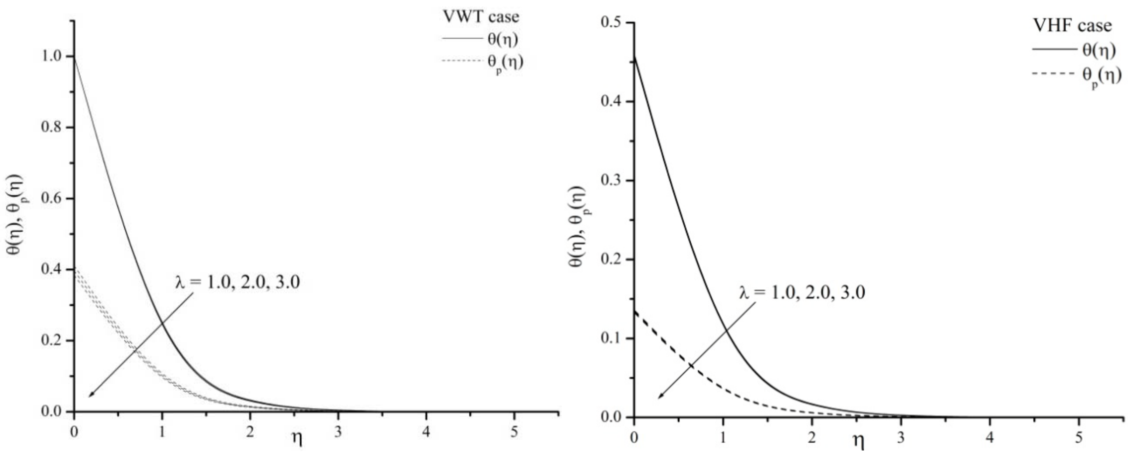


Figure 3(a) & 3(b): Effect of Convective parameter λ on temperature distribution.

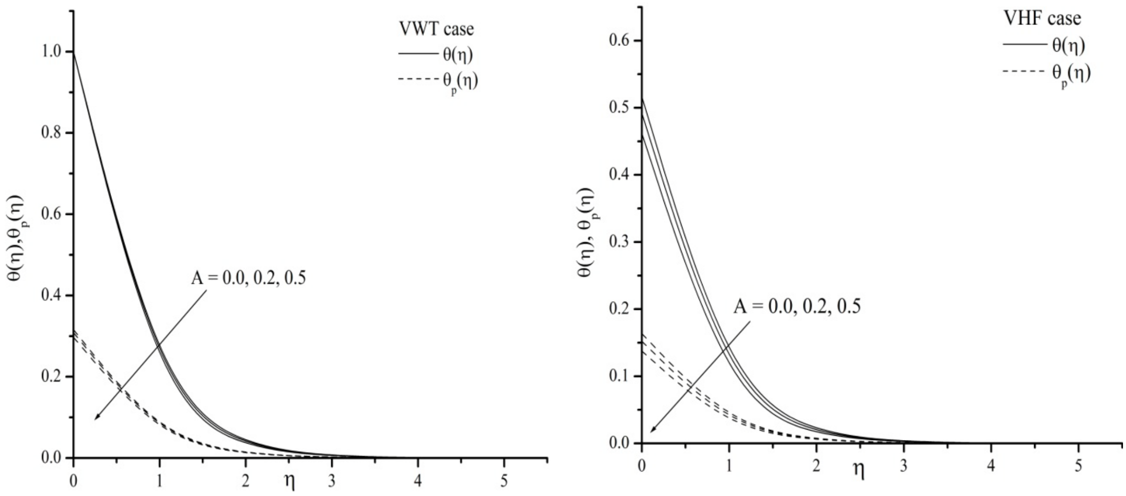


Figure 4(a) & 4(b): Effect of unsteady parameter A on temperature distribution.

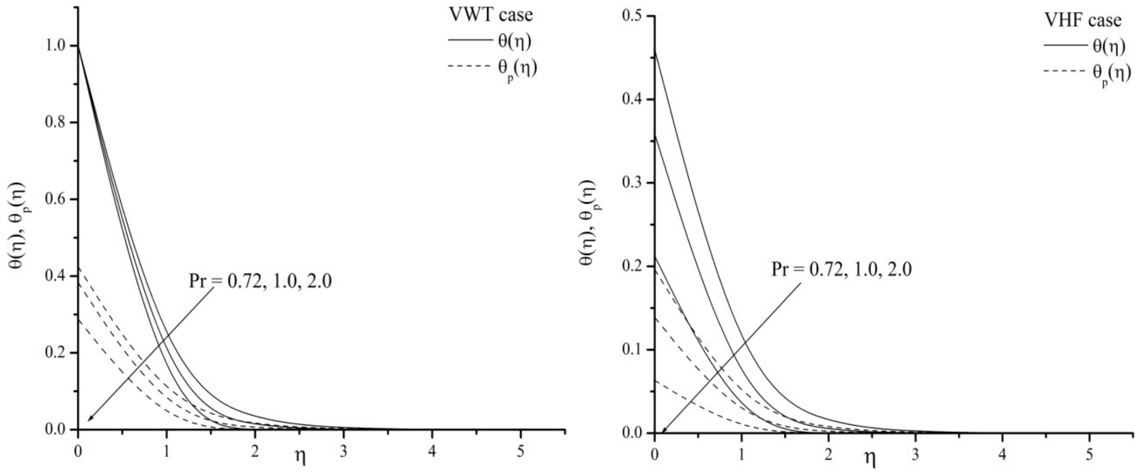


Figure 5(a) & 5(b): Effect of Prandtl number Pr on temperature distribution.

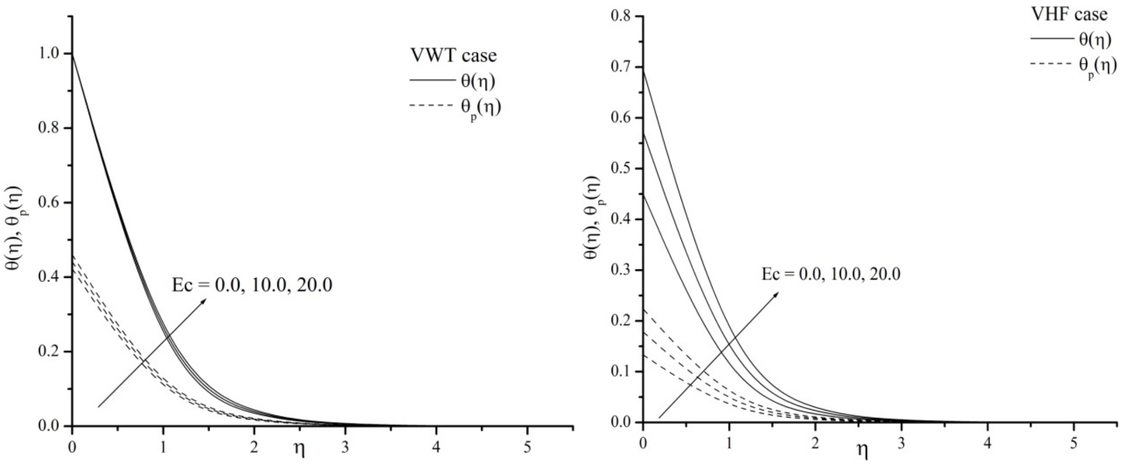


Figure 6(b) & 6(b): Effect of Eckert number Ec on temperature distribution.

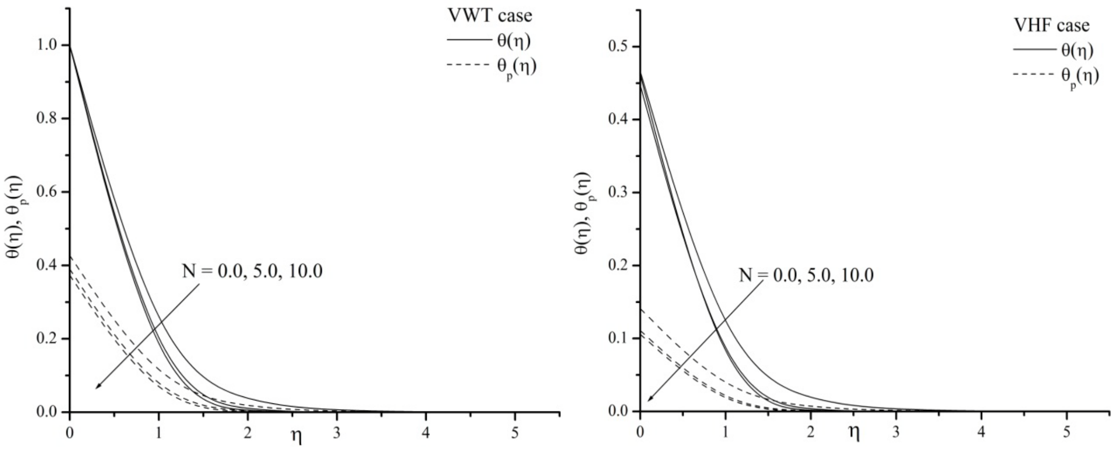


Figure 7(a) & 7(b): Effect of Number density N on temperature distribution.

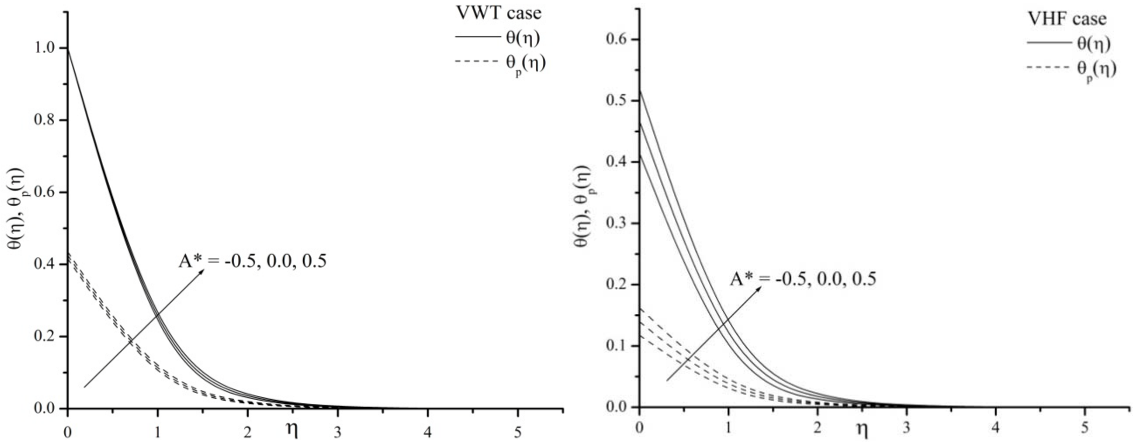


Figure 8(a) & 8(b): Effect of heat source/sink parameter A^* on temperature distribution.

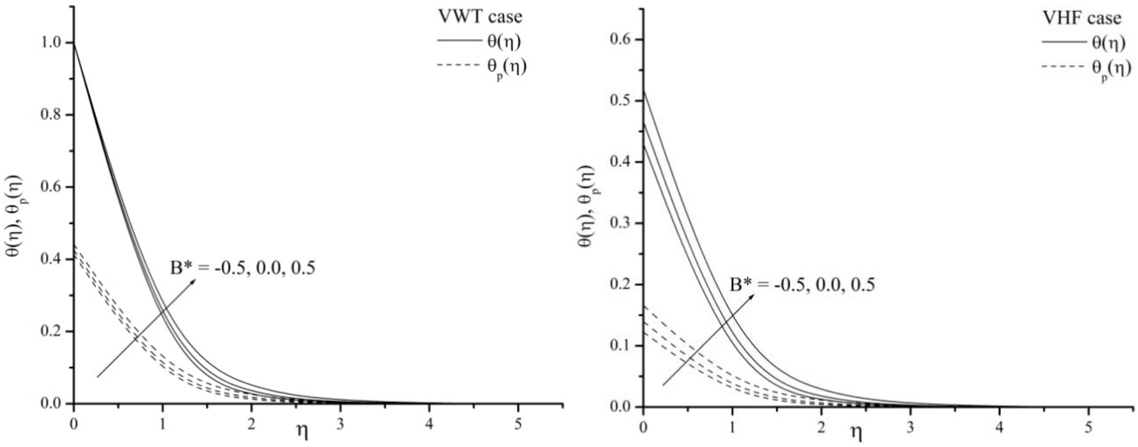


Figure 9(a) & 9(b): Effect of heat source/sink parameter B^* on temperature distribution.

Table 2: Values of wall temperature gradient $\theta'(0)$ (for VWT Case) and wall temperature function $\theta(0)$ (for VHF Case).

β	A	Pr	Ec	A^*	B^*	N	λ	$\theta'(0)$ (VWT)	$\theta(0)$ (VHF)
0.1	0.5	0.72	1	-0.05	-0.05	0.2	2.0	-2.208226	0.45974
0.5								-2.197956	0.460096
1.0								-2.184282	0.461819
0.1	0	.072	1	-0.05	-0.05	0.2	2.0	-2.011902	0.516437
	0.3							-2.141029	0.481268
	0.5							-2.208226	0.461819
0.1	0.5	0.72	1	-0.05	-0.05	0.2	2.0	-2.208226	0.461819
		1.0						-2.864033	0.357759
		20.						-5.07129	0.207063
0.1	0.5	0.72	0.0	-0.05	-0.05	0.2	2.0	-2.249975	0.449590
			0.5					-2.208226	0.455704
			2.0					-2.166468	0.474049
0.1	0.5	0.72	1	-0.05	-0.05	0.2	2.0	-2.339804	0.415409
				0.0				-2.193587	0.447875
				0.5				-2.046987	0.480367
0.1	0.5	0.72	1	-0.05	-0.05	0.2	2.0	-2.366752	0.428624
					0.0			-2.189243	0.446534
					0.05			-1.977207	0.467284
0.1	0.5	0.72	1	-0.05	-0.05	0.5	2.0	-2.234363	0.456895
						1.5		-2.305814	0.447921
						3.0		-2.383633	0.444628
0.1	0.5	0.72	1	-0.05	-0.05	0.2	1.0	-2.012118	0.460116
							2.0	-2.044332	0.458331
							3.0	-2.071614	0.456649

6. CONCLUSIONS

The problem of two dimensional boundary layer flow and heat transfer dusty fluid over a stretching sheet in the presence of non-uniform source/sink is formulated. The governing partial differential equations are reduced to ordinary differential equations by using similarity transformation. Resultant coupled ordinary differential equations (2.8) to (2.11) and (3.6) to (3.7) for VWT case and (3.10) to (3.11) for VHF case have been solved numerically by method employed by Aziz [13] i.e., RKF45 method. The effect of various physical parameter like unsteady parameter A, mixed convective parameter λ, Prandtl number Pr, Eckret number Ec, Hartmann number M, number density N, and non-uniform heat source/sink parameter A^* and B^* on various momentum and heat transfer characteristics are examined. In order to have a clear insight of the problem numerical results are displayed with the help of graphical illustrations. The important finding of our investigations are listed below,

- The effect of the magnetic field parameter is predicted to decrease the fluid- and dust-phase velocities.
- Increasing the mixed convection parameter increases the velocity profiles and decreases the temperature profiles of both the fluid and dust phases.
- The effect of the unsteadiness parameter is seen to decrease the temperature profiles of both the fluid and dust phases for both the VWT and VHF cases.
- The effect of increasing the Prandtl number is observed to decrease the thermal boundary layer thicknesses of both phases.

- Increasing the non-uniform heat source/sink parameter is predicted to increase the temperature profiles of both the fluid and dust phases for the VWT and VHF cases.
- The rate of heat transfer $\theta'(0)$ (for the VWT case) is predicted to be negative whereas $\theta(0)$ (for the VHF case) is predicted to be positive.
- The values of $\theta'(0)$ (for the VWT case) and $\theta(0)$ (for the VHF case) are predicted to increase with increases in the values of heat source or sink parameter or the Eckert number.
- If $A \to 0, A^* \to 0, \beta \to 0, \lambda \to 0$ and $N \to 0$ then our results coincides with the results of Abel et.al., [9] and Grubka et.al., [6] only when Prandtl number varies.

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