International Journal of Mathematical Archive-3(7), 2012, 2728-2734
IJMA Available online through www.ijma.info ISSN 2229 – 5046

g** -closed sets in bitopological spaces

Mrs. Veronica Vijayan, Mrs. Ponnuthai Selvarani and Sr. Pauline Mary Helen. M*

Associate Professors, Nirmala College, Coimbatore-641018. India
Dr. Punitha Tharani, Associate Professor, St. Mary's College, Tuticorin-628001, India

(Received on: 12-06-12; Accepted on: 30-06-12)

ABSTRACT

*In this paper we introduce g**-closed sets in bitopological spaces. Properties of this sets are investigated and we introduce three new bitopological spaces namely, (i, j)-**$T_{½}$ spaces, (i, j)-$T_{½}$ **space and (i, j)-*$T_{½}$*spaces.*

Key words: *(i, j)-g**-closed sets, (i, j)-**$T_{½}$ spaces, (i, j)-$T_{½}$**spaces and (i, j)-*$T_{½}$*spaces.*

1. INTRODUCTION

A triple (X,τ_1, τ_2) where X is a non-empty set and $\tau_1$and τ_2 are topologies in X is called a bitopological space and Kelly[**5**] initiated the study of such spaces. In 1985, Fukutake [**2**] introduced the concepts of g-closed sets in bitopological spaces. M.K.R.S. Veerakumar**[11]** introduced and studied the concepts of g*-closed sets and g*-continuity in topological spaces. Sheik John. M and Sundaram. P **[8]** introduced and studied the concepts of g*-closed sets in bitopological spaces in 2002. The purpose of this paper is to introduce the concepts of g**-closed sets, (i, j)-**$T_{½}$ spaces, (i, j)-$T_{½}$**spaces and (i, j)-*$T_{½}$*spaces in bitopological spaces and investigate some of their properties.

2. PRELIMINARIES

Definition 2.1 A subset a of a topological space (X, τ) is said to be
1. a *pre-open* set **[7]** if $A \subseteq int(cl(A))$ and a *preclosed* set if $cl(int(A)) \subseteq A$.
2. a *semi-open* set **[6]** if $A \subseteq cl(int(A))$ and and *semi-closed* set if $int (cl(A)) \subseteq A$.
3. a regular open set **[9]** if $A = int (cl(A))$
4. a generalized closed set **[7]**(briefly g-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in(X, τ).
5. a *generalized star closed* set**[11]** (briefly g*-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ).
6. a *generalized star star closed* set**[10]** (briefly g**-closed set) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in (X, τ).

If A is a subset of X with topology τ, then the closure of A is denoted by $\tau – cl(A)$ or cl(A), the interior of A is denoted by $\tau – int(A)$ or int(A) and the complement of A in X is denoted by A^c.

For a subset of (X, τ_i, τ_j), τ_i-cl(A) (resp. τ_i-int(A))denote the closure (resp. interior)of A with respect to the topology τ_i. We denote the family of all g-open(resp.g*-open) subsets of X with respect to the topology τ_i by $GO(X, \tau_i)$ (resp. $G^*O(X, \tau_i)$ and the family of all τ_j- closed sets is denoted by the symbol F_j we mean the pair of topologies (τ_i, τ_j).

Definition 2.2 A subset A of a topology (X, τ_i, τ_j) is called
1. (i, j)-g-closed**[2]** if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is open in τ_i.
2. (i, j)-rg-closed**[1]** if τ_j-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.
3. (i, j)-gpr-closed **[4]** if τ_j-pcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is regular open in τ_i.
4. (i, j)-wg-closed**[3]** if τ_j-cl(τ_i-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is open in τ_i.

Definition 2.3 A bitopological space (X, τ_1, τ_2) is called
1. an (i, j)-$T_{½}$ space**[2]** if every (i, j)-g-closed set is τ_j-closed.
2. a strongly pairwise $T_{½}$ space**[2]** if it is both (1, 2) $T_{½}$ and (2, 1) $T_{½}$.
3. an (i, j)-$T_{½}$* space**[8]** if every (i, j)-g*-closed set is τ_j-closed.
4. a strongly pairwise $T_{½}$ *space**[8]** if it is both (1, 2) $T_{½}$* and (2, 1) $T_{½}$*.
5. an (i, j)-*$T_{½}$ space**[8]** if every (i, j)-g-closed set is g*-closed.
6. a strongly pairwise *$T_{½}$ space**[8]** if it is both (1, 2) -*$T_{½}$ and (2, 1) -*$T_{½}$.

Corresponding author: *Sr. Pauline Mary Helen. M**
Associate Professors, Nirmala College, Coimbatore-641018. India

3. (i, j) – g**-closed sets

In this section we introduce the concept of (i, j)-g**-closed sets in bitopological spaces.

Definition 3.1: A subset A of a topological space (X,τ_1,τ_2) is said to be an (i,j) - $g^{**}-closed$ set if $\tau_j - cl(A) \subseteq U$ whenever $A \subseteq U$ and $U \in G^*O(X,\tau_i)$. We denote the family of all (i,j) - $g^{**}-closed$ sets in (X,τ_1,τ_2) by $D^{**}(i,j)$.

Remark 3.2: By setting $\tau_1 = \tau_2$ in definition (3.1), a (i,j) - $g^{**}-closed$ set is a $g^{**}-closed$ set.

Proposition 3.3: Every $\tau_j - closed$ subset of (X,τ_1,τ_2) is (i,j) - $g^{**}-closed$.

The converse of the above propositions is not true as seen in the following example.

Example 3.4: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{c\},\{a,c\},X\}$ and $\tau_2 = \{\phi,\{a\},X\}$. Then the set $A = \{b\}$ is (1, 2) - $g^{**}-closed$ but not $\tau_2 - closed$ in (X,τ_1,τ_2).

Proposition 3.5: If A is both τ_i - g^*-open and (i,j) - $g^{**}-closed$ then A is $\tau_j - closed$.

Proposition 3.6: In a Bitopological space (X,τ_1,τ_2) every (i,j) - $g^{**}-closed$ set is
(i) (i,j) - $g-closed$
(ii) (i,j) - $rg-closed$ (iii) (i,j) - $gpr-closed$ (iv) (i,j) - $wg-closed$.

The following examples show that the converse of the above proposition are not true.

Example 3.7: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a,b\},X\}$ and $\tau_2 = \{\phi,\{a\},X\}$. Then the set $A = \{a\}$ is (1, 2) - $g-closed$, (1, 2) - $rg-closed$ and (1, 2) - $wg-closed$ but not (1, 2) - $g^{**}-closed$.

Example 3.8: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{c\},\{a,b\},X\}$ and $\tau_2 = \{\phi,\{a\},X\}$. Then the subset $A = \{c\}$ is (1, 2) - $gpr-closed$ but not (1, 2) - $g^{**}-closed$.

Proposition 3.9: Every (i,j) - $g^*-closed$ set is (i,j) - $g^{**}-closed$.

The converse of the above need not be true.

Example 3.10: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,\{a\},\{a,b\},X\}$. Then the subset $A = \{b\}$ is (1, 2) - $g^{**}-closed$ but not (1, 2) - $g^*-closed$.

Proposition 3.11: If $A, B \in D^{**}(i,j)$, then $A \cup B \in D^{**}(i,j)$.

Remark 3.12: The intersection two (i,j) - $g^{**}-closed$ set need not be (i,j) - $g^{**}-closed$ as seen from the following example.

Example 3.13: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},\{b,c\},X\}$ and $\tau_2 = \{\phi,\{b\},\{c\},\{b,c\},\{a,c\},X\}$. Let $A = \{a,b\}$ and $B = \{b,c\}$. Then A and B are (2,1) - $g^{**}-closed$ sets but $A \cap B = \{b\}$ is not a (2,1) - $g^{**}-closed$ set.

Remark 3.14: $D^{**}(1,2)$ is generally not equal to $D^{**}(2,1)$.

Example 3.15: In Example (3.13), $A = \{b\} \notin D^{**}(2,1)$ but $A \in D^{**}(1,2)$.

Proposition 3.16: If $\tau_1 \subseteq \tau_2$, in (X, τ_1, τ_2) then $D^{**}(2,1) \subseteq D^{**}(1,2)$.

The converse of the above need not be true as seen in the following example.

Example 3.17: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a,c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{a,b\}, X\}$ where $\tau_1 \not\subset \tau_2$ but $D^{**}(2,1) \subseteq D^{**}(1,2)$.

Proposition 3.18: For each element x of (X, τ_1, τ_2), $\{x\}$ is either $\tau_i - g^*\text{-}closed$ or $X - \{x\}$ is (i, j) - $g^{**} - closed$.

Proposition 3.19: If A is (i, j) - $g^{**} - closed$, then $\tau_j - cl(A) - A$ contains no non-empty $\tau_i - g^* - closed$ set.

Proof: Let A be (i, j) - $g^{**} - closed$ and let F be a $\tau_i - g^* - closed$ set such that $F \subseteq \tau_j - cl(A) - A$. Since $A \in D^{**}(i, j)$, we have $\tau_j - cl(A) \subseteq F^C$.

Therefore $F \subseteq (\tau_j - cl(A)) \bigcap (\tau_j - cl(A))^C = \phi$. Therefore $F = \phi$.

The converse of the above two propositions need not be true as it is seen in the following example.

Example 3.20: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, \{b\}, \{c\}, \{b,c\}, \{a,c\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{b,c\}, X\}$.

Let $A = \{b\}$. Then $\tau_2 - cl(A) \setminus A = \{c\}$ is not $\tau_1 - g^* - closed$. i.e. $\tau_2 - cl(A) \setminus A$ contains no non-empty $\tau_1 - g^* - closed$ set but $A = \{b\}$ is not (1, 2) - $g^{**} - closed$.

Theorem 3.22: If A is (i, j) - $g^{**} - closed$ in (X, τ_i, τ_j) then A is $\tau_j - closed$ if and only if $\tau_j - cl(A) \setminus A$ is $\tau_i - g^* - closed$.

Proof: Necessity: If A is $\tau_j - closed$ then $\tau_j - cl(A) = A$ that is $\tau_j - cl(A) \setminus A = \phi$ and hence it is $\tau_i - g^* - closed$ $g^* - closed$.

Sufficiency: If $\tau_j - cl(A) \setminus A$ is $\tau_i - g^* - closed$ then by proposition (3.19), $\tau_j - cl(A) \setminus A = \phi$. Therefore A is $\tau_i - g^* - closed$.

Theorem 3.23: If A is an (i, j) - $g^{**} - closed$ set of (X, τ_i, τ_j) such that $A \subseteq B \subseteq \tau_j - cl(A)$ then B is also an (i, j) - $g^{**} - closed$ set of (X, τ_i, τ_j).

Proof: Let $B \subseteq U$ and U be $\tau_i - g^* - open$ Then $A \subseteq U$ and $\tau_j - cl(A) \subseteq U$. since A is $(i, j) - g^{**} - closed$. $B \subseteq \tau_j - cl(A)$ implies $\tau_j - cl(B) \subseteq \tau_j - cl(A)$ and hence $\tau_j - cl(B) \subseteq U$.

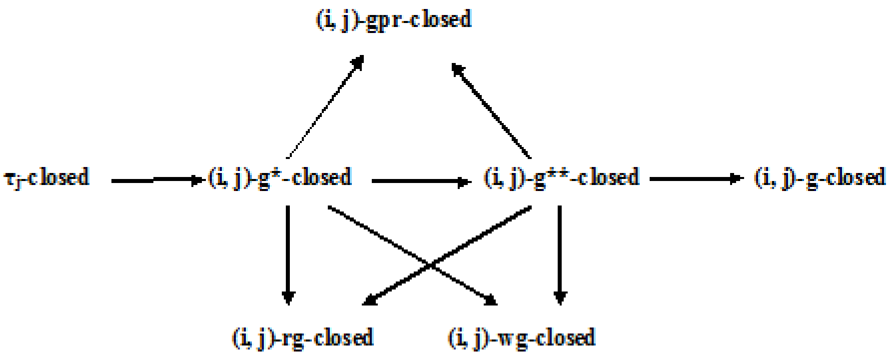
Therefore B is (i, j) - $g^{**} - closed$.

Proposition 3.24: In a Bitopological space (X, τ_i, τ_j), $G^*O(X, \tau_i) \subseteq F_j$ if and only if every subset of X is an (i, j) - $g^{**} - closed$ set.

Proof: Suppose $G^*O(X, \tau_i) \subseteq F_j$. Let A be a subset of X such that $A \subseteq U$ where $U \in G^*O(X, \tau_i)$. Then $\tau_j - cl(A) \subseteq \tau_j - cl(U) = U$ and hence A is (i, j) - $g^{**} - closed$. Conversely, suppose that every subset of X is (i, j)- $g^{**}-closed$. Let $U \in G^*O(X,\tau_i)$. Since U is (i, j)- $g^{**}-closed$, we have $\tau_j - cl(U) \subseteq U$. Therefore $U = \tau_j - cl(U)$ and hence $U \in F_j$.

Therefore $G^*O(X,\tau_i) \subseteq F_j$.

The following figure illustrates the relationships with the other closed sets:



Where A$\rightarrow$ B represents A implies B but not conversely.

4. (i, j)- $T^{**}_{1/2} - spaces$ $(i, j) - {}^{**}T_{1/2} - spaces$ $(i, j) - and\ {}^*T_{1/2}{}^* - spaces$

In this section we introduce three new bitopological spaces $(i, j) - T^{**}_{1/2} - spaces$, $(i, j) - {}^{**}T_{1/2} - spaces$ and (i,j) ${}^*T^*_{1/2} - spaces$.

Definition 4.1: A bitopological space (X,τ_1,τ_2) is said to be an $(i, j) - {}^{**}T_{1/2} - space$ if every (i, j) -set is $(i, j) - g^{**} - closed \quad g^* - closed$.

Proposition 4.2: Every (i, j)- $T_{1/2} - space$ is a (i, j)- ${}^{**}T_{1/2} - space$ but not conversely.

Example 4.3: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},\{a,c\},X\}$ and $\tau_2 = \{\phi,\{a,b\},X\}$. Then (X,τ_1,τ_2) is a (1, 2) - ${}^{**}T_{1/2} - space$ but not a (1, 2) - $T_{1/2} - space$ since $A = \{b\}$ is (1, 2) - $g - closed$ but not $\tau_2 - closed$.

Remark 4.4: A (1, 2) - $T^*_{1/2} - space$ need not be a (1, 2) - ${}^{**}T_{1/2} - space$ true as it is seen in the following example.

Example 4.5: Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. Then (X,τ_1,τ_2) is a (1, 2) - $T^*_{1/2} - space$ but not a (1, 2) - ${}^{**}T_{1/2} - space$ since $A = \{b\}$ is (1, 2) - $g^{**} - closed$ but not (1, 2) - $g^* - closed$.

Definition 4.6: A bitopological space (X,τ_1,τ_2) is said to be a (i, j) - $T^{**}_{1/2} - space$ if every (i, j) - $g^{**} - closed$ set is $\tau_j - closed$.

Proposition 4.7: If (X,τ_1,τ_2) is a (i, j)- $T^{**}_{1/2} - space$ then it is a (i, j)- $T^*_{1/2} - space$.

The converse of the above is not be true as seen in the following example.

Example 4.8: In example (4.5), (X,τ_1,τ_2) is (1, 2) - $T^*_{1/2} - space$ but not a (1, 2) - $T^{**}_{1/2} - space$. Since $A = \{b\}$ is (1, 2) - $g^{**} - closed$ but not $\tau_2 - closed$.

© 2012, IJMA. All Rights Reserved

Proposition 4.9: If a bitopological space (X,τ_1,τ_2) is a (1, 2) - $T_{1/2}-space$ then it is both (1, 2) - ${}^{**}T_{1/2}-space$ and (1, 2) - $T_{1/2}^{**}-space$.

Proof follows from propositions (4.2) and (4.7).

Proposition 4.10: Every (i, j) - $T_{1/2}^{**}-space$ is (i, j) - ${}^{**}T_{1/2}-space$ but not conversely.

Example 4.11: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,\{a\},\{a,c\},X\}$ and $\tau_2=\{\phi,\{a,b\},X\}$. Then (X,τ_1,τ_2) is a (i, j) - ${}^{**}T_{1/2}-space$ but not a (i, j) - $T_{1/2}^{**}-space$ since $A=\{a,b\}$ is (1, 2) - $g{**}-closed$ but not $\tau_2-closed$.

Definition 4.12: A bitopological space (X,τ_1,τ_2) is said to be a strongly pairwise ${}^{**}T_{1/2}-space$ if it is both (1, 2) - ${}^{**}T_{1/2}-space$ and (2, 1) - ${}^{**}T_{1/2}-space$.

Definition 4.13: A bitopological space (X,τ_1,τ_2) is said to be a strongly pairwise $T_{1/2}^{**}-space$ If it is both (1, 2) - $T_{1/2}^{**}-space$ and (2, 1) - $T_{1/2}^{**}-space$.

Proposition 4.14: If (X,τ_1,τ_2) is a strongly pairwise $T_{1/2}-space$ then it is a strongly pairwise ${}^{**}T_{1/2}-space$ but not conversely.

Example 4.15: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,\{a\},\{b,c\},X\}$ and $\tau_2=\{\phi,\{b\},\{c\},\{a,c\},\{b,c\},X\}$.

Then (X,τ_1,τ_2) is a strongly pairwise ${}^{**}T_{1/2}-space$ but not a strongly pairwise $T_{1/2}-space$

since $A=\{c\}$ is (1, 2) - $g-closed$ but not $\tau_2-closed$.

Proposition 4.16: If (X,τ_1,τ_2) is a strongly pairwise $T_{1/2}^{**}-space$ then it is a strongly pairwise $T_{1/2}^{*}-space$ but not conversely.

Example 4.17: Let $X=\{a,b,c\}$, $\tau_1=\{\phi,\{a\},\{a,b\},X\}$ and $\tau_2=\{\phi,\{a\},\{b\},\{a,b\},X\}$.

Then (X,τ_1,τ_2) is a strongly pairwise $T_{1/2}^{*}-space$ but not a strongly pairwise $T_{1/2}^{**}-space$ since $A=\{a,c\}$ is (2,1) - $g{**}-closed$ but not $\tau_1-closed$. Therefore (X,τ_1,τ_2) is not a (2, 1) - $T_{1/2}^{**}-space$ and hence it is not a strongly pairwise $T_{1/2}^{**}-space$.

Proposition 4.18: The following conditions are equivalent in a bitopological space (X,τ_1,τ_2)

(i) (X,τ_1,τ_2) is a (i, j) - $T_{1/2}^{**}-space$.

(ii) Every singleton of X is either $\tau_i-g{*}closed$ or τ_j-open.

Proof: (i) → (ii), Let (X,τ_1,τ_2) be an (i, j) - $T_{1/2}^{**}-space$. Let $x\in X$ and suppose $\{x\}$ is not $\tau_i-g{*}closed$. Then $X-\{x\}$ is not $\tau_i-g{*}open$. Therefore $X-\{x\}$ is a (i, j) - $g{**}-closed$ set of (X,τ_1,τ_2) since (X,τ_1,τ_2) is a (i, j) - $T_{1/2}^{**}-space$, $X-\{x\}$ is $\tau_j-closed$. Therefore $\{x\}$ is τ_j-open.

(ii) → (i), Let A be a (i, j) - $g{**}-closed$ set of (X,τ_1,τ_2) . $A\subseteq\tau_j-cl(A)$. Let $x\in\tau_j-cl(A)$. By (ii), $\{x\}$ is either $\tau_i-g{*}closed$ or τ_j-open ..

Case (i): Let $\{x\}$ be $\tau_i - g*closed$, Suppose $x \notin A$, then $\tau_j - cl(A) - A$ contains a non-empty $\tau_j - g*closed$ set $\{x\}$, which is a contradiction to propositions (3.22). Therefore $x \in A$.

Case (ii), Suppose $\{x\}$ is $\tau_j - open$. Since $x \in \tau_j - cl(A)$, $\{x\} \bigcap A \neq 0$. Therefore we have $x \in A$. This in both cases, we conclude that A is $\tau_j - closed$. Hence (X, τ_1, τ_2) is an (i, j) - $T_{1/2}^{**} - closed$.

Definition 4.19: A space (X, τ_1, τ_2) is called a (i, j) - ${}^*T_{1/2}^* - space$ if every (i, j) - $g - closed$ set of (X, τ_1, τ_2) is (i, j) - $g** - closed$.

Definition 4.20: A bitopological space (X, τ_1, τ_2) is said to be a strongly pairwise ${}^*T_{1/2}^* - space$ if it is both (1, 2) - ${}^*T_{1/2}^* - spaces$ and (2, 1) - ${}^*T_{1/2}^* - spaces$.

Proposition 4.21: Every (i, j) - $T_{1/2} - space$ is an (i, j) - ${}^*T_{1/2}^* - space$ but not conversely.

Example 4.22: In example (4.3), (X, τ_1, τ_2) is (1, 2) - ${}^*T_{1/2}^* - space$ but not a (1, 2) - $T_{1/2} - space$ since $A = \{b\}$ is (1, 2) - $g - closed$ but not $\tau_2 - closed$.

Remark 4.23: (i, j) - $T_{1/2}^{**} - space$ and (i, j) - ${}^*T_{1/2}^* - spaces$ are independent as seen in the following example.

Example 4.24: In example (4.5), (X, τ_1, τ_2) is (i, j) - ${}^*T_{1/2}^* - spaces$ but not a (i, j) - $T_{1/2}^{**} - space$ since $A = \{b\}$ is (1, 2) - $g** - closed$ but not $\tau_2 - closed$.

Example 4.25: Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\phi, \{a\}, \{a, b\}, X\}$. Then (X, τ_1, τ_2) is (1, 2) - ${}^*T_{1/2}^* - space$ but not a $T_{1/2}^{**} - space$ since $A = \{a, c\}$ is (1, 2) - $g** - closed$ but not $\tau_2 - closed$.

Proposition 4.26: A space (X, τ_1, τ_2) is a (i, j) - ${}^*T_{1/2} - space$ if and only if it is both (i, j) - ${}^*T_{1/2}^* - space$ and (i, j) - ${}^{**}T_{1/2} - space$.

Proof: Let A be an (i, j) - $g - closed$ set in X. Since (X, τ_1, τ_2) is a (i, j) - ${}^*T_{1/2}^* - spaces$, A is an (i, j) - $g** - closed$ set in X. Again since (X, τ_1, τ_2) is a (i, j) ${}^{**}T_{1/2} - space$, A is (i, j) $g* - closed$.

Therefore every (i, j) - $g - closed$ set of (X, τ_1, τ_2) is (i, j) - $g* - closed$. Hence (X, τ_1, τ_2) is an (i, j) - ${}^*T_{1/2} - space$.

Conversely suppose (X, τ_1, τ_2) be an (i, j) - ${}^*T_{1/2} - space$. Let A be an (i, j) - $g** - closed$ set of (X, τ_1, τ_2). Then from (i) of proposition (3.6), A is (i, j) - $g - closed$. Since (X, τ_1, τ_2) is ${}^*T_{1/2} - space$, A is (i, j) $g* - closed$ and hence (X, τ_1, τ_2) is (i, j) - ${}^{**}T_{1/2} - space$.Let A be an (i, j) - $g - closed$ set. Since (X, τ_1, τ_2) is ${}^*T_{1/2} - space$, A is (i, j) - $g* - closed$. Then by

Proposition (3.9), A is (i, j) - $g** - closed$. Therefore (X, τ_1, τ_2) is (i, j) ${}^*T_{1/2}^* - space$.

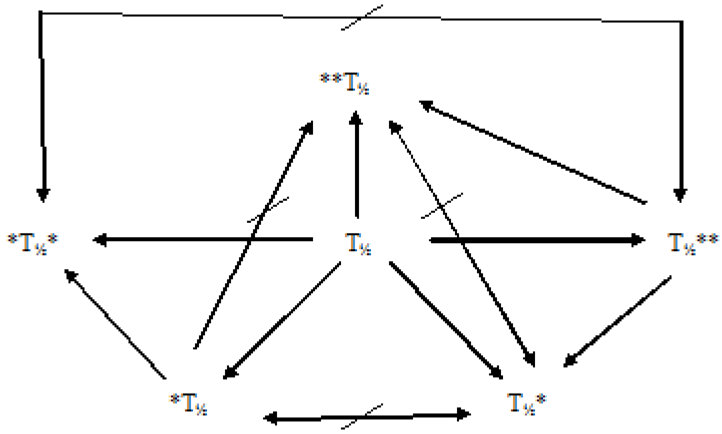
Proposition 4.27: A space (X, τ_1, τ_2) is strongly pairwise ${}^*T_{1/2} - space$ if and only if it is both strongly pairwise ${}^*T_{1/2}^* - space$ and strongly pairwise ${}^{**}T_{1/2} - space$. Proof follows from proposition 4.26.

© 2012, IJMA. All Rights Reserved

Proposition 4.28: Every strongly pairwise $T_{1/2}-space$ is strongly pairwise ${}^{*}T_{1/2}^{*}-spaces$ but not conversely. Proof follows from proposition 4.21.

Example 4.29: In example (4.22), (X,τ_1,τ_2) is strongly pairwise ${}^{*}T_{1/2}^{*}-space$ but not a strongly pairwise $T_{1/2}-space$ since $A=\{b\}$ is (1, 2) - $g-closed$ but not $\tau_2-closed$.

The results in this section can be represented in the following figure:



Where A ⟶ B represents A implies B but not conversely and A ⟷ B represents A and B are independent.

REFERENCES

[1] I. Arockiarani, Studies on generalizations of generalized closed sets and maps in topological spaces, Ph.D. Thesis, Bharathiar Univ., Coimbatore, 1997.

[2] T. Fukutake, Bull, Fukuoka Univ. Ed. Part III, 35(1985), 19 – 28.

[3] T. Fukutake, P. Sundaram and N. Nagaveni, Bull, Fukuoka Univ. Ed. Part III, 48(1999), 33 – 40.

[4] T. Fukutake, P. Sundaram, M. Sheik John, Bull, Fukuoka Univ. Ed. Part III, 51(2002), 1 – 9.

[5] J. C.Kelley, Proc., London Math. Sci. 13(1963), 71 – 89.

[6] N. Levine, Amer. Math. Monthly, 70 (1963), 36 – 41.

[7] N. Levine, Rend. Cire. Math. Palermo, 19 (1970), 89 – 96.

[8] M. Sheik John and P. Sundaram, Indian J. Pure Appl. Math. 35(1) (2004), 71 – 80.

[9] M. Stone, Trans. Amer. Math. Soc. 41 (1937) 374 – 481.

[10] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces (accepted).

[11] M.K.R.S. Veera Kumar, Mem. Fac. Sci. Kochi Univ. (Math.), 21 (2000), 1 – 19.

Source of support: Nil, Conflict of interest: None Declared