International Journal of Mathematical Archive-3(8), 2012, 2869-2874
IJMA Available online through www.ijma.info ISSN 2229 – 5046

CSP-1 FOR A SYSTEMATICALLY VARYING PROCESS

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(Received on: 17-07-12; Revised & Accepted on: 05-08-12)

ABSTRACT

Dodge(1943) introduced a single level attribute continuous sampling plan designated as CSP-1 for the application of continuous production process in which infinite sequence of articles is considered with probability of each unit being defective is constant. . In this paper the deviation of the assumption of CSP-1constant p model is considered with systematic variation in p. Expressions for the performance measures of CSP-1 for an oscillating sequence (p_1,p_2) are derived using Markov chain approach for even and odd values of i. Tables are also presented to numerically analyze the behavior of the performance measures with reference to process quality.

Subject classification: *62P99, 6207.*

Keywords: *Markov chain, Transition matrix, Average Outgoing Quality (AOQ).*

1. INTRODUCTION

The present scenario of global competition and the increasing awareness of the customer demand the application of sampling plans in assessing the quality of products in manufacturing industries. Continuous sampling plans introduced by Dodge (1943) were widely used to monitor continuous production processes where the formation of lots for sampling inspection is impracticable or artificial. Cars coming off an assembly line, soft drink bottles from a continuous glass ribbon machine, welded leads emanating from a welding operation etc are some of the examples of production processes where continuous sampling plans can be applied.

Dodge's initial plan is applied to monitor processes under the assumptions that the probability of each article being created a defective, p is constant, the number of articles arrives for inspection in the order of production is infinite and qualities of successive items of production are independent. CSP-1 plan with the assumptions that the probability, p of producing a defective article as constant and the process produces infinite sequence of article are however not fully realized in real life situations. This motivated to consider a model for a process with non-constant p for an infinite sequence of articles. The probability of producing a defective article is not constant but alternates systematically between two values of p indefinitely as p_1,p_2,p_1,p_2,…is considered. Performance measures such as the Average outgoing quality (AOQ), the average fraction of units inspected (AFI) and the probability of acceptance (p_a) for CSP-1 oscillating model are derived using Markov chain approach. Tables are constructed for the numerical analysis of performance measures.

2. BASIC PLAN AND PARAMETER

A continuous sampling plan CSP-1, probability of producing a defective article, alternates between two values of p (p_1, p_2) involves 100% screening and sampling inspections. The plans screening inspection is characterized by the clearance number, i and sampling inspection is indexed by constant sampling rate f. Defective article found is replaced in the inspection by a good article. Further, the system will be adjusted to $p = p_1$ when a defective article is found at the start of 100% inspection period. This situation can be described by one step transition matrix i of order m (=i+7) given in Table 1. The CSP-1 for systematically varying production process is defined with parameters i, f and p_1, p_2. When $p_1 = p_2$ the plan reduces to CSP-1 constant p model.

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Markov chain formulation of CSP-1 for a systematically varying process.

Let $[X_n]$ (n=1, 2, 3…) denote a discrete parameter Markov chain with finite state space, S_k (k=1, 2…i+7). The states of the process are defined, as

S_k = H (k-1) (k=1, 2,…, i+1)

= 100% inspection is being conducted and including the latest article inspected, the last K-1)articles were non-defectives.

$S_{i+2+3(r-1)}$ = SDr (r=1, 2)

=sampling inspection is in effect, the last article submitted was inspected and found to be defective, and $p=p_r$

$S_{i+3+3(r-1)}$ = SNDr (r=1, 2)

=sampling inspection is in effect, the last article submitted was inspected and found to be non- defective, and $p=p_r$

$S_{i+4+3(r-1)}$ =SNr (r=1, 2)

=sampling inspection is in effect, the last article submitted was not inspected and found to be defective, and $p=p_r$

If the CSP-1 plan used has an even value of i, the last article inspected in every 100% inspection segment will have $p = p_2$ as its probability of being defective. If the value of i is odd, this probability will be $p = p_1$.

Table 1.Transition matrix

	H0	H1	H2	.	.	.	Hi-1	Hi	SD1	SND1	SN1	SD2	SND2	SN2
H0	p_1	q_1	0	.	.	.	0	0	0	0	0	0	0	0
H1	p_2	0	q_2	.	.	.	0	0	0	0	0	0	0	0
H2	p_1	0	0	.	.	.	0	0	0	0	0	0	0	0
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Hi-1	p_2	0	0	.	.	.	0	q_2	0	0	0	0	0	0
Hi	0	0	0	.	.	.	0	0	fp_1	fq_1	1-f	0	0	0
SD1	p_1	q_1	0	.	.	.	0	0	0	0	0	0	0	0
SND1	0	0	0	.	.	.	0	0	0	0	0	fp_2	fq_2	1-f
SN1	0	0	0	.	.	.	0	0	0	0	0	fp_2	fq_2	1-f
SD2	p_1	q_1	0	.	.	.	0	0	0	0	0	0	0	0
SND2	0	0	0	.	.	.	0	0	fp_1	fq_1	1-f	0	0	0
SN2	0	0	0	.	.	.	0	0	fp_1	fq_1	1-f	0	0	0

The transition matrix shows that it is a DFRIA Markov chain. An obvious modification occurs when i is an odd integer. Vector of limiting probabilities for even values of i:

$\pi_0= p_1 \pi_0+p_2 \pi_1+\ldots+p_2 \pi_{i-1}+p_1 \pi_{D1}+p_1 \pi_{D2}$
$\pi_1= q_1 \pi_0+q_1 \pi_{D1}+q_1 \pi_{D2}$
$\pi_2= q_2 \pi_1$
$\pi_3= q_1 \pi_2 \ldots$
$\pi_i= q_2 \pi_{i-1}$

$$\left.\begin{aligned} \pi_{D1} &= fp_1\pi_i + fp_1\pi_{ND2} + fp_1\pi_{N2} \\ \pi_{ND1} &= fq_1\pi_i + fq_1\pi_{ND2} + fq_1\pi_{N2} \\ \pi_{N1} &= (1-f)\pi_i + (1-f)\pi_{ND2} + (1-f)\pi_{N2} \end{aligned}\right\} \tag{3.1}$$

$$\left.\begin{aligned} \pi_{D2} &= fp_2\pi_{ND1} + fp_2\pi_{N1} \\ \pi_{ND2} &= fq_2\pi_{ND1} + fq_2\pi_{N1} \\ \pi_{N1} &= (1-f)\pi_{ND1} + (1-f)\pi_{N1} \end{aligned}\right\} \tag{3.2}$$

From (3.1) and (3.2)

$$\pi_{D1} + \pi_{ND1} + \pi_{N1} = \pi_i + \pi_{ND2} + \pi_{N2}$$
$$\pi_{D2} + \pi_{ND2} + \pi_{N2} = \pi_{ND1} + \pi_{N1}$$
$$\pi_{D1} + \pi_{D2} = \pi_i$$

Using the same notion, the limiting probabilities for odd value of i can also be evaluated as above. solving the above equations we get the values,

For even values of i,

$$\pi_0 = \frac{\left\{(1-(q_1q_2)^{i/2})\, fD\right\}}{D*e}$$

$$\pi_i = \frac{\left\{q_1^{[i+1/2)]} q_2^{[i/2]}\, fD\right\}}{D*e}$$

$$\pi_{D1} = \frac{\left\{fp_1(q_1q_2)^{i/2}\right\}}{D*e}$$

$$\pi_{D2} = \frac{\left\{f(1-fp_1)(q_1q_2)^{i/2}\right\}}{D*e}$$

$$\pi_{ND2} = \frac{\left\{fq_2(1-fp_1)(q_1q_2)^{i/2}\right\}}{D*e}$$

$$\pi_{N2} = \frac{\left\{(1-f)(1-fp_1)(q_1q_2)^{i/2}\right\}}{D*e}$$

$$\pi_{N1} = \frac{\left\{(1-f)(q_1q_2)^{i/2}\right\}}{D*e}$$

$$\pi_{ND1} = \frac{\left\{fq_1(q_1q_2)^{i/2}\right\}}{D*e}$$

For odd values of i

$$\pi_0 = \frac{\left\{(1-q_1(q_1q_2)^{(i-1)/2})\, fD\right\}}{D*o}$$

$$\pi_i = \frac{\left\{(q_1^{[i+1]/2} q_2^{[i/2]}\, fD\right\}}{D*o}$$

$$\pi_{D1} = \frac{\left\{(1-fp_2)\, fp_1q_1(q_1q_2)^{(i-1)/2}\right\}}{D*o}$$

$$\pi_{D2} = \frac{\left\{fp_2q_1(q_1q_2)^{(i-1)/2}\right\}}{D*o}$$

$$\pi_{ND2} = \frac{\left\{f(q_1q_2)^{(i+1)/2}\right\}}{D*o}$$

$$\pi_{N2} = \frac{\left\{(1-f)q_1(q_1q_2)^{(i-1)/2}\right\}}{D*o}$$

$$\pi_{N1} = \frac{\left\{(1-f)(1-fp_2)q_1(q_1q_2)^{(i-1)/2}\right\}}{D*o}$$

$$\pi_{ND1} = \frac{\left\{(1-fp_2)\, fq_1^{\,2}(q_1q_2)^{(i-1)/2}\right\}}{D*o}$$

where

$$D = p_1 + p_2 - fp_1p_2$$
$$D*e = \frac{\left\{(1+q_1)(1-(q_1q_2)^{i/2}\, fD + (2-fp_1)(1-q_1q_2)(q_1q_2)^{i/2}\right\}}{(1-q_1q_2)},$$
$$D*o = \frac{\left\{(1+q_1)-(1+q_2)q_1(q_1q_2)^{(i-1)/2}\, fD + (2-fp_2)(1-q_1q_2)q_1(q_1q_2)^{(i-1)/2}\right\}}{(1-q_1q_2)}$$

3. PERFORMANCE MEASURES CORRESPONDING TO i EVEN AND ODD

(i) The average number of units inspected under the screening inspection

$$u = \frac{1-(q_1q_2)^{i/2}}{(q_1q_2)^{i/2}(1-(q_1q_2)^{1/2}} \qquad \text{(i even)}$$

$$u = \frac{1-q_1(q_1q_2)^{(i-1)/2}}{q_1(q_1q_2)^{(i-1)/2}(1-q_1)} \qquad \text{(i odd)}$$

(ii) The average number of units passed under the sampling procedure

$$v = \frac{2 - fp_1}{f(p_1 + p_2 - fp_1 p_2)} \qquad \text{(i even)}$$

$$v = \frac{2 - fp_2}{f(p_1 + p_2 - fp_1 p_2)} \qquad \text{(i odd)}$$

(iii) The average fraction of total produced units inspected in the long run,

$$F = \frac{f(2 - fp_1)}{D^* e} \qquad \text{(i even)}$$

$$F = \frac{f(2 - fp_2)}{D^* o} \qquad \text{(i odd)}$$

(iv) The average fraction of total produced units accepted on a sampling basis ,p_a

$$p_a = \frac{(2 - fp_1)(q_1 q_2)^{i/2}}{D^* e} \qquad \text{(i even)}$$

$$p_a = \frac{(2 - fp_2) q_1 (q_1 q_2)^{(i-1)/2}}{D^* o} \qquad \text{(i odd)}$$

(v) The average outgoing quality,

$$AOQ = \frac{(p_1 + p_2 - fp_1 p_2)(1 - f)(q_1 q_2)^{i/2}}{D^* e} \qquad \text{(i even)}$$

$$AOQ = \frac{(p_1 + p_2 - fp_1 p_2)(1 - f) q_1 (q_1 q_2)^{(i-1)/2}}{D^* o} \qquad \text{(i odd)}$$

4. NUMERICAL ANALYSIS

Numerical values of performance measures are computed for various p_1 and p_2 , even and odd values of i with fixed f and presented in tables 2 & 3.

Tables 2 & 3 reveal the following features:

a. when p1>p2, with simultaneous increase of p1& p2, the OC values decreases, AFI values increases and AOQ increases to a certain level and then decreases for systematically varying process model of CSP-1.
b. when p1=p2, with simultaneous increase of p1& p2, the OC values decreases ,AFI values increases and AOQ increases to a certain level and then decreases for systematically varying process model of CSP-1.
c. when p1<p2, with simultaneous increase of p1& p2, the OC values decreases , AFI values increases and AOQ increases to a certain level and then decreases for systematically varying process model of CSP-1.

Simultaneous increase of p_1 & p_2 with $p_1>p_2$,

- The rate of increase of OC value for systematically varying process model of CSP-1 is more when compared to CSP-1 model of constant probability.
- The rate of decrease of AFI is greater for systematically varying process model of CSP-1 than CSP-1 model of constant probability.
- AOQ value is greater for systematically varying process model of CSP-1 than CSP-1 model of constant probability.

Simultaneous increase of p_1 & p_2 with $p_1<p_2$,

- Systematically varying process model of CSP-1 has lower OC value than the CSP-1 model of constant probability.
- Systematically varying process model of CSP-1 has higher AFI value than the CSP-1 model of constant probability.

- Systematically varying process model of CSP-1 has lower AOQ value than the CSP-1 model of constant probability.

Table 2: Comparison of performance measures for fixed even value of i and fixed f

		$p_1 > p_2$		
p_1	p_2	OC	AFI	AOQ
0.001	0.0002	0.9524	0.1905	0.0005
0.003	0.0005	0.8295	0.2949	0.0009
0.005	0.0008	0.6744	0.4268	0.0016
0.007	0.0011	0.5058	0.5701	0.0017
0.009	0.0014	0.3496	0.7028	0.0015
		$p_1 = p_2$		
p_1	p_2	OC	AFI	AOQ
0.001	0.001	0.9113	0.2254	0.0008
0.003	0.003	0.6563	0.4422	0.0017
0.005	0.005	0.3719	0.6869	0.0016
0.007	0.007	0.7101	0.8554	0.0010
0.009	0.009	0.0684	0.9419	0.000
		$p_1 < p_2$		
p_1	p_2	OC	AFI	AOQ
0.001`	0.0015	0.8874	0.2457	0.0009
0.003	0.0044	0.5569	0.5266	0.0017
0.005	0.0073	0.2445	0.7922	0.0013
0.007	0.0102	0.0834	0.9291	0.0006
0.009	0.0131	0.0255	0.9783	0.0002

Table 3: Comparison of performance measures for fixed odd value of i and fixed f

		$p_1 > p_2$		
p_1	p_2	OC	AFI	AOQ
0.001	0.0002	0.9692	0.0793	0.0006
0.003	0.0005	0.8554	0.1873	0.0010
0.005	0.0008	0.6504	0.3821	0.0017
0.007	0.0011	0.3954	0.6244	0.0015
0.009	0.0014	0.1923	0.8173	0.0009
		$p_1 = p_2$		
p_1	p_2	OC	AFI	AOQ
0.001	0.001	0.9367	0.1102	0.0009
0.003	0.003	0.6240	0.4072	0.0018
0.005	0.005	0.2182	0.7927	0.0010
0.007	0.007	0.0471	0.9553	0.0003
0.009	0.009	0.0087	0.9917	0.0001
		$p_1 < p_2$		
p_1	p_2	OC	AFI	AOQ
0.001`	0.0013	0.9252	0.1211	0.0010
0.003	0.0038	0.5405	0.2653	0.0013
0.005	0.0063	0.1396	0.8674	0.0004
0.007	0.0088	0.0227	0.9784	0.0001
0.009	0.0113	0.0033	0.9968	0.0000

5. CONCLUSION

In this paper CSP-1 model for a non-constant p is evaluated by considering a model under which p varies systematically. Performance measures have been derived using Markov chain approach and evaluated numerically for certain selected values.

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Source of support: Nil, Conflict of interest: None Declared