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PROBABILITY-POSSIBILITY TRANSFORMATIONS: AN OVERVIEW

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ABSTRACT

In this article, our main intention is to inform that the existing probability - possibility transformation principles are not logical and hence the applications of these principles in different fields may lead to unrealistic situations. In order to avoid such a situation, we in this article would like to suggest one principle which is formulated within an appropiate mathematical framewok and accordingly, we would like to discard the fusing process of threat assesement estimated on the basis of these existing transformations between probability and possibility.

Keywords*: Consistency principles, superimposition of sets, Dubois-Prade definition of normal fuzzy number.*

1. INTRODUCTION

Literature dealing with the link between probability and possibility is quite extensive. A wide variety of consistency principles between probability and possibility has been proposed and studied. A long standing debate took place in literature on the relationship between probability and possibility. In literature, we can see three most commonly used principles linking probability with possibility among the numerous methods suggested in the literature are Zadeh consistency principle, Klir consistency principle and Dubois and Prade consistency principles. After these, many other principles were developed by many other authors which can be found in literature references. Though they share some common features, they differ from one another from structure and details.

Threat assesement indicates the degree of severity with which the engagement of events will occur, this degree is in proportion with the capability of the enemy and its perceived intent. A fusion process proactively seeks to identify perceived threat and stop them before they occur. Lee and Llinas [8], focused on fusing process of threat by combining two different approaches.They tried to build a hybrid model of threat assesement because air-to-air battle space requires fast decision making for which it is essential to develop a software for the fast computations. To enable hybridization, they have employed representative transformation methods between probability and possibility as found in literature references. They applied two transformation methods of which one was developed by Geer and Klir whereas the other was developed by Dubois, Prade and Shandri. Unfortunately, these theories have not been sufficiently developed as yet because there are some controversial properties in the transformation procedures. Some of which can be summarized as follows:

Geer and Klir proposed the “information preserving transformations (IPT)”, concepts in transforming possibility and probability. They found the log interval transformation to be the most appropiate transformation because it satisfies the criterion of consistency in both directions. This IPT concept requires that the numbers expressing uncertainty in one theory be transformed into corresponding numbers in another theory by an appropiate scale and that the amount of uncertainty and information be preserved under the transformations. The IPT idea results in the following equation:

$$-\sum_{x\epsilon X} p(x)\log_2[p(x)] = \sum_{i=2}^{n}\log_2[\tfrac{i}{(i-1)}] - \sum_{i=1}^{n-1}(r_i - r_{i+1})\log_2[1 - i\sum_{j=i+1}^{n}\tfrac{r_j}{(j\,(j-1))}] \qquad (1)$$

The transformation formula for log interval from (1)was expressed in the following way:

$$p_i = \frac{r_i^{\frac{1}{\alpha}}}{\sum_{i=1}^{n} r_k^{\frac{1}{\alpha}}},\quad r_i = (\tfrac{p_i}{p_1})^{\alpha} \qquad (2)$$

Klir's assumptions are debatable. The uncertainty invariance equation *E (π) = H (p)*, along with a scaling transformation assumption$(\pi(x) = \alpha p(x) + \beta, \forall x)$, reduces the problem of computing π from p to that of solving an algebraic equation with one or two unknowns. Then, the scaling assumption leads to assume that $\pi(x)$ is a function of *p(x)* only. This pointwiseness assumption may conflict with the probability/possibility consistency principle that requires $\Pi \geq P$ for all events. See Dubois and Prade ([7], pp. 258-259) for an example of such a violation. Then, the nice link between possibility and probability, casting possibility measures in the setting of upper and lower probabilities cannot be maintained.

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The second most questionable prerequisite assumes that possibilistic and probabilistic information measures are commensurate. The basic idea is that the choice between possibility and probability is a mere matter of translation between languages "neither of which is weaker or stronger than the other" (quoting Klir and Parviz [13]). It means that entropy and imprecision capture the same facet of uncertainty, albeit in different guises.

The last point of divergence is that Klir did not try to respect the probability – possibility consistency principles which enable a nice link between possibility and probability to be maintained, casting possibility measure in the setting of upper and lower probabilities.

Baruah further contributed to this in the way that defining p_i from π_i in the way to satisfy uncertainty preservation principles defined by Klir himself is nothing but trying to define a probability space in the measure theoretic sense from the knowledge of possibilities concerned. It seems that it was done to normalize the values of π_i so that the total probability is equal to 1. Another thing which is worth mentioning here that the transformation derived here can only deal eith discrete cases whereas nothing was mentioned about continues cases.

There are two questions in the transformation, Yamada [12]. One is the validity of the principle of information preservation. It is incompatible with the idea on which maximal specificity is based. The other is the assumption that the transformation from probability p_i to possibility π_i is given by a function $\pi_i = f(p_i)$. The transformation is not limited to such function in general.

According to (1), Dubois, Prade and Shandri's transformation formula is

$$p_i = \frac{1}{i} r_i - \sum_{i=i+1}^{n} \frac{1}{j(j-1)} r_j \tag{3}$$

where $r_1 = 1 > r_2 > \cdots > r_n > r_{n+1} = 0$

Dubios, Prade and Shandri's transformation principles unlike Gir and Klir's transformations faced criticisms for many reasons. The authors argued that possibility transformation is weaker and consequently "turning probability into possibility measures come down to give up part of initial informations" while turning possibility measures into probability measures is always partially arbitrary since the conversion procedures always adds some informations. As a result of this arguments the transformations, $p \rightarrow \pi$ is guided by the principle of maximum specificity and , $\pi \rightarrow p$ is guided by the principle of insufficient reasons. However there is no gurantee that this is the only transformation satisfying them.

In [1], Alt, Yovits countered these arguments in the following way:

Although possibility theory employs weaker rules than probability theory in manipulating uncertainty, the basic structure of the two theories are not comparable. Hence even though manipulating uncertainty within possibility theory results in a greater loss of informations, than corresponding uncertainty within probability theory, it is neither necessary nor desirable to lose or gain informations solely by transforming uncertainty from one representtion to another.

There are other reasons also for which the principle becomes debatable. These can be described by the fact that the authors provided the transformations for continuos case , namely for unimodal continuos probability density function, with bounded support and finally arrived at the conclusion that further research is needed in continuos case. The authors failed to find for which class of pdf and possibility distribution, the transformation make sense. In their work the authors had pointed out that the transformations they devised are not related to each other and the converse transformations were also shown to be inadequate. Another thing for which the transformation becomes debatable can be found from the fact that there was the use of the word measure with possibility which is not acceptable. The measureof a point is zero in the classical sense but the possibility of a point is determined by a membership function. Further, since a possibility space can be bifurcated into two probability spaces, we can say that with the help of two probability spaces we can study possibility mathematically. So it becomes obvious that we cannot use these principles in all application areas.

Thus we see that that both the principles applied in assesing threat are questionable. It is important to mention here that since fusion process is a vital thing for safety and security, it should be assesed with thise concepts which are formalised within an appropiate mathematical framework otherwise it would lead to a very terrific situation. Although sufficient research has been carried out on transformation of one form of information to another, none of the techniques are free from criticisms either in their theoritical perspectives or in their computational burdens during the applications. It is not sure that different conversion procedure gives similar results. To select an appropiate transformation law under each particular situation is a difficult problem. Although some guidelines are available for some situations, we are still far from a common general solution. If this be the situation, newcomers in the field will be confused in adopting an appropiate link between probability and possibility. In such a situation, we need a principle instead of many which can deal uncertainty in an efficient way. Here we would like to cite the transformation which was introduced by Baruah [5]
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The central concept of the principle as the name suggest is to find a possible link between possibility and probability by bridging the gaps which are there in the existing literature references.

The principle was derived in accordance with the Dubois and Prade definition of a normal fuzzy number. The significance of the suggested principle is that it provides us with an efficient procedure to connect probability with possibility. When properly applied this principle of consistency guarantees that no information is wasted in the process. That is to say that the principle which is rooted in the operation of superimposition of sets can be recognized as the potential tool for enhancing our ability to deal with the problems which are often faced in the existing principles. Thus for the sake of consistency between possibility and probability, we would like to suggest the following principle which is better known as "The Randomness-Fuzziness Consistency Principles". The principle is discussed in details in our previous works and so let us have a look at this in short in the following section.

2. RANDOMNESS- FUZZINESS CONSISTENCY PRINCIPLES

Baruah [5] introduced a framework for reasoning with the link between probability – possibility. The develpoment of this principle focused mainly on the existence of two laws of randomness which are required to define a law of fuzziness. In other words , not one but two laws of fuzziness is required to define a law of randomness on two disjoint spaces which in turn can construct a fuzzy membership function. Fundamental to this approach is the idea that possibility distribution can be viwed as a combination of distributions of which one is a probability distribution and the other is a complementary probability distribution. The consistency principle introduced in the manner can be explained mathematically in the following form:

For a normal fuzzy number of the type $N = [\alpha, \beta, \gamma]$ with membership function

$$\mu_N(x) = \Psi_1(x), \text{ if } \alpha \le x \le \beta,$$
$$= \Psi_2(x), \text{ if } \beta \le x \le \gamma, \text{ and } = 0, \text{ otherwise, with } \Psi_1(\alpha) = \Psi_2(\gamma) = 0,$$

$$\Psi_1(\beta) = \Psi_2(\beta) = 1,$$

the partial presence of a value x of the variable X in the interval $[\alpha, \gamma]$ is expressible as

$$\mu_N(x) = \theta \text{ Prob } [\alpha \le X \le x] + (1 - \theta) \{1 - \text{Prob } [\beta \le X \le x]\},$$

where $\theta=1$ if $\alpha \le x \le \beta$ and $\theta=0$ if $\beta \le x \le \gamma$

The above relationship between probability and possibility is named as "The Randomness- Fuzziness Consistency Principle" which is more mathematical or formal in character. The above principle provides us with a logical link between the two. We can claim that this procedure is almost conflict free. As a consequence, the adoption of the above mentioned technique is recommended if we wish to have a logical result. The above principle of consistency between probability and possibility established within an appropiate mathematical framework leads us to conclude that the results obtained with the help of the eexisting consistency principles as found in the literature references would yield an illogical result.As a consequence of the reasons mentioned in this article, it can be said that the research which provided a hybrid model for threat assesement by exploring and adopting transformation methods between probability and possibility have to be reconstructed instead to get an effective estimation.

3. CONCLUSIONS

The paper discussed some promonent works for transformation between probability and possibility. The main intention of the paper is to convey the information that none of the procedures found the transformation. Here it is intended to inform that the concept of superimposition gives a very clear meaning to the transformation between probability and possibility and as such we would like to say that the applications of those transformations to various field of study would invariably lead to an illogical result which is unexpected from mathematical point of view. So it is clear from the above discussions that the two procedures which were considered in the process of hybridization of intent threat assesement are not acceptable for various reasons and consequently it can be said that the hybrid model is not a reliable one. Thus in order to get an appropiate result which can make the process effective, we need to look into the matter through the use of the consistency principle suggested in this article. Hence it can be concluded that some further works are needed to make the model an effective one.

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