International Journal of Mathematical Archive-3(8), 2012, 2891-2898
IJMA Available online through www.ijma.info ISSN 2229 – 5046

$g^{**}I-closed$ and $g^{**s}I-closed$ sets

Sr. Pauline Mary Helen*
Associate Professor, Nirmala College, Coimbatore, India

Mrs. Ponnuthai Selvarani
Professor, Nirmala College, Coimbatore, India

Mrs. Veronica Vijayan
Aassociate Professor, Nirmala College, Coimbatore, India

Mrs. Punitha Tharani
Professor, St. Mary's College, Tuticorin, India

(Received on: 06-07-12; Revised & Accepted on: 25-07-12)

ABSTRACT

In this paper g^{**}I-closed sets and g^{**s}I-closed sets are defined and their properties are investigated.

Key words: *$g^{**}I-closed$, $g^{**s}I-closed$ sets, $*-$ countably additive, $*-$ additive, $*s-$ additive, $*s-$ countably additive $*s-$finitely additive ideal spaces, locally finite and locally countable family of sets.*

1. INTRODUCTION

Ideal topological spaces have been first introduced by K. Kuratowski **[2]** in 1930.Vaidyanathaswamy **[6]** introduced local function in 1945 and defined a topology τ. M.E. Abd El Monsef, E. F. Lashien and A. a Nasef **[1]** introduced semi local function in 1992 and defined a topology τ^{*s}.

In this paper g **I-closed sets and g^{**s}I-closed sets are defined and their properties are investigated.

2. PRELIMINARIES

Definition 2.1: An ideal**[2]** I on a non empty set X is a collection of subsets of X which satisfies the following properties.(i) $A \in I$, $B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I$, $B \subset A \Rightarrow B \in I$ .A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I) .Let Y be a subset of X . $I_Y = \{I \cap Y / I \in I\}$ is an ideal on Y and by $(Y,\tau/Y,I_Y)$ we denote the ideal topological subspace.

Definition 2.2: Let $P(X)$ be the power set of X , then a set operator ( )*: $P(X) \to P(X)$ called the local function**[6]** of A with respect to τ and I is defined as follows: For $A \subset X$, $A^*(I,\tau) = \{x \in X / U \cap A \notin I$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(I,\tau)$ in case there is no confusion. A Kuratowski closure operator $cl^*(\ )$ for a topology $\tau^*(I,\tau)$, called the τ^* - topology is defined by $Cl^*(A) = A \cup A^*$

For A, B in (X,τ,I) we have

(i) If $A \subset B$ then $A^* \subset B^*$

(ii) $(A^*)^* \subseteq A^*$

(iii) $A^* \cup B^* = (A \cup B)^*$

(iv) $(A \cap B)^* \subseteq A^* \cap B^*$ (v) If $I = \{\phi\}$, $A^* = cl(A)$ and $cl^*(A) = cl(A)$

Corresponding author: *Sr. Pauline Mary Helen*, Associate Professor, Nirmala College, Coimbatore, India*

(vi) If $I = P(X)$ then $A^* = \phi$ and $cl^*(A) = A$(vii) $A^* = cl(A^*) \subset cl(A)$ and A^* is a closed subset of cl(A).

Definition 2.3: A subset A of a space (X,τ) is said to be semi-open **[3]** if $A \subset cl(\text{int}(A))$

Definition 2.4: A set operator **[1]** $(\ )^{*S} : P(X) \to P(X)$ called a semi local function and $cl^{*s}(\ )$ of A with respect to τ and I are defined as follows: For $A \subset X$, $A^{*S}(I,\tau) = \{x \in X / U \cap A \notin I$ for every semi open set U containing $x\}$. and $Cl^{*S}(A) = A \cup A^{*S}$.

For a subset A of X, $cl(A)$ (resp. $scl(A)$) denotes the closure (resp. semi closure) of A in (X,τ). Similarly $cl^*(A)$ and $\text{int}^*(A)$ denote the closure of A and interior of A in (X,τ^*).

Definition 2.5: A subset A of X is called * closed **[6]** (resp.$*s$ closed **[1]**) if $A^* \subseteq A$(resp. $A^{*S} \subseteq A$).

Definition 2.6: A subset A of X is called * - dense **[6]** in itself (resp.$*s$ - dense **[1]**) if $A \subset A^*$ (resp. $A \subset A^{*S}$).

Definition 2.7: A subset A of X is called * - perfect **[6]** (resp.$*s$ - perfect **[1]**) if $A = A^*$ (resp. $A = A^{*S}$)

LEMMA 2.5: **[1]** For A, B in (X,τ,I) we have

(i) If $A \subset B$ then $A^{*S} \subset B^{*S}$
(ii) $(A^{*S})^{*S} \subseteq A^{*S}$
(iii) $A^{*S} \cup B^{*S} \supseteq (A \cup B)^{*S}$
(iv) $(A \cap B)^{*S} \subseteq A^{*S} \cap B^{*S}$
(v) If $I = \{\phi\}$, $A^{*S} = scl(A)$ and $cl^{*S}(A) = scl(A)$
(vi) If $I = P(X)$ then $A^{*S} = \phi$ and $cl^{*S}(A) = A$
(vii) $A^{*S} = scl(A^{*S}) \subset scl(A)$ and A^{*S} is semi closed.

A subset A of an ideal space (X,τ,I) is said to be g- closed **[4]**, if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $open$ in X. The complement of $g-closed$ set is said to be $g-open.$

A subset A of an ideal space (X,τ,I) is said to be g^*- closed **[7]**, if $cl\ (A) \subseteq U$ whenever $A \subseteq U$ and U is $g-open$ in X. The complement of $g^*-closed$ set is said to be g^*-open.

A subset A of an ideal space (X,τ,I) is said to be $g^{**}-$ closed **[5]**, if $cl\ (A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in X. The complement of $g^{**}-closed$ set is said to be $g^{**}-open$

3. $g^{**}I-closed$ and $g^{**s}I-closed$ sets

Definition 3.1: subset A of an ideal space (X,τ,I) is said to be $g^{**}I$ closed, if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in X. The complement of $g^{**}I-closed$ set is said to be $g^{**}I--open$.A subset A of an ideal space (X,τ,I) is said to be $g^{**S}I$ closed, if $cl^{*S}(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in X. The complement of $g^{**s}I-closed$ set is said to be $g^{**s}I--open$.The collection of all $g^{**}I-closed$ sets (resp. $g^{**S}I-closed$ sets) is denoted by $G^{**}IC(X)$ (resp. $G^{**S}IC(X)$) Similarly the collection of all $g^{**}I-open$ sets (resp. $g^{**S}I-open$ sets) is denoted by $G^{**}IO(X)$ (resp. $G^{**S}IO(X)$)

Remarks: 3.2:

1. If $I = P(X)$ then $cl^{*S}(A) = cl^{*}(A) = A\ forall\ A \subseteq X$ and hence every subset of X is $g^{**}I-closed$. and $g^{**S}I-closed$

2. Since $A^{*S} = \{\phi\} = A^{*}$ for every $A \in I$, every member of I is $g^{**}I-closed$ .and $g^{**S}I-closed$

3. Every closed set A is $g^{**}I-closed$ since $cl^{*}(A) \subseteq cl(A) = A$ and every semi closed set A is $g^{**s}I-closed$ since $cl^{*s}(A) \subseteq scl(A) = A$

4. Every $*-closed$ set is $g^{**}I-closed$ since $cl^{*}(A) = A$

5. Finite union of $g^{**}I-closed$ sets is $g^{**}I-closed$ since $cl^{*}\left[\bigcup_{i=1}^{n} A_i\right] = \bigcup_{i=1}^{n} cl^{*}(A_i)$

In general, $(A \cup B)^{*S} \neq A^{*S} \cup B^{*S}$ for subsets A and B in X as seen from the following example.

Example 3.3: Let $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$, $I = \{\phi\}$. Then $\{a\}^{*S} = \{a\}$, $\{b\}^{*S} = \{b\}$ and $\{a,b\}^{*S} = X$ .This shows $(A \cup B)^{*S} \neq A^{*S} \cup B^{*S}$ in general.

Moreover in general $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*} \neq \bigcup_{i=1}^{\infty}(A_i)^{*}\ and \left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} \neq \bigcup_{i=1}^{\infty}(A_i)^{*S}$ For arbitrary indexing seΩ , $\left[\bigcup_{\alpha\in\Omega} A_\alpha\right]^{*} \neq \bigcup_{\alpha\in\Omega}(A_\alpha)^{*} and \left[\bigcup_{\alpha\in\Omega} A_\alpha\right]^{*S} \neq \bigcup_{\alpha\in\Omega}(A_\alpha)^{*S}$ as seen from the following examples.

Example 3.4: Let X=Z, τ be the cofinite topology in X and $I = \{\phi\}$.Then $A^{*} = A = A^{*s}$ if A is finite and $A^{*} = Z = A^{*s}$ if A is infinite. $G^{**}IC(X) = \{X, \varphi, all\ \ finite$ subsets $\}$.

Let A_n = {-n,-n+1,…..-1,1,…..n-1,n} for every positive integer n. Then $A_n^{*} = A_n^{*s} = A$ for every n.

$\left[\bigcup_{i=1}^{\infty} A_i\right]^{*}$ = Z $= \left[\bigcup_{i=1}^{\infty} A_i\right]^{*s}\ and \bigcup_{i=1}^{\infty}(A_i)^{*} = Z - \{0\} = \bigcup_{i=1}^{\infty}(A_i)^{*s}$

Therefore $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*} \neq \bigcup_{i=1}^{\infty}(A_i)^{*}\ \ and \left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} \neq \bigcup_{i=1}^{\infty}(A_i)^{*S}$

Here A_n is g^{**}I –closed and g^{**s}I- closed for every n. But $\left[\bigcup_{i=1}^{\infty} A_i\right]$ is not g^{**}I- closed and g^{**s}I -closed.

Definition 3.5: An ideal space (X, τ, I) is said to be

(i) $*-$ countably additive if $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*} = \bigcup_{i=1}^{\infty}(A_i)^{*}$

(ii) $*-$ additive if $\left[\bigcup_{\alpha\in\Omega} A_\alpha\right]^{*} = \bigcup_{\alpha\in\Omega}(A_\alpha)^{*}$ for all indexing sets Ω.

(iii) $*-$ additive if $\left[\bigcup_{i=1}^{n} A_i\right]^{*S} = \bigcup_{i=1}^{n}(A_i)^{*S}$ for every positive integer n.

(iv) $*S-$ countably additive if $\left[\bigcup_{i=1}^{\infty} A_i\right]^{*S} = \bigcup_{i=1}^{\infty}(A_i)^{*S}$

(v) $*S-$ additive if $\left[\bigcup_{\alpha\in\Omega} A_\alpha\right]^{*S} = \bigcup_{\alpha\in\Omega}(A_\alpha)^{*S}$ for all indexing sets Ω.

Remark: 3.6: In an ideal topological space (X,τ,I) which is $*S-$ finitely additive we have following results:

1. $cl^{*S}(\phi)=\phi$
2. $cl^{*S}(X)=X$
3. $A\subseteq cl^{*S}(A)$
4. $cl^{*S}(A\cup B)=cl^{*S}(A)\cup cl^{*S}(B)$
5. $cl^{*S}\left(cl^{*S}(A)\right)=cl^{*S}(A)$ for all subsets A, B in X.

Therefore $cl^{*S}(\ )$ satisfies Kuratowski Closure axioms and hence it defines a topology τ^{*S} whose closure operation is given as $cl^{*S}(A)=A\cup A^{*S}$. Note that $\tau\subseteq\tau^{*}\subseteq\tau^{*S}$. $cl^{*S}(A)$ and $\operatorname{int}^{*S}(A)$ denote the closure and interior of A in (X,τ^{*S}).

Theorem3.7 In a $*-$ countably additive (resp $*-$ additive) ideal topological space (X,τ,I) Countable union (resp arbitrary union) of $g^{**}I-closed$ sets is $g^{**}I-closed$.

Proof: It follows since in $*-$ countably additive space $cl^{*}\left[\bigcup_{i=1}^{\infty}A_i\right]=\bigcup_{i=1}^{\infty}cl^{*}(A_i)$ and in $*-$ additive space $cl^{*}\left[\bigcup_{\alpha\in\Omega}A_\alpha\right]=\bigcup_{\alpha\in\Omega}cl^{*}(A_{\alpha i})$

Theorem 3.8: In a $*s-$ finitely additive(resp *s-countably additive , *s- additive) ideal topological space (X,τ,I) finite union (resp countable union ,arbitrary union) of $g^{**}I-closed$ sets is $g^{**}I-closed$

Proof: It follows since in *s finitely additive space $cl^{*s}\left[\bigcup_{i=1}^{n}A_i\right]=\bigcup_{i=1}^{n}cl^{*s}(A_i)$ and in *s-countably additive space $cl^{*s}\left[\bigcup_{i=1}^{\infty}A_i\right]=\bigcup_{i=1}^{\infty}cl^{*s}(A_i)$ and in $*s-$ additive space $cl^{*s}\left[\bigcup_{\alpha\in\Omega}A_\alpha\right]=\bigcup_{\alpha\in\Omega}cl^{*s}(A_{\alpha i})$

Definition 3.9: $\{A_\alpha/\alpha\in\Omega\}$ is said to be a locally finite (resp locally countable) family of sets in (X,τ,I) if for every $x\in X$, there exists an open set U in X containing x that intersects only a finite (resp countable) number of members $A_{\alpha_1},\ldots\ldots,A_{\alpha_n}$ (resp $A_{\alpha_i}, i=1,\ldots\ldots\infty$) of $\{A_\alpha/\alpha\in\Omega\}$.

Theorem 3.10: Let (X,τ,I) be an ideal space, and let $\{A_\alpha/\alpha\in\Omega\}$ be a locally finite family of g^{**}I-closed sets in (X,τ,I). Then $\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)$ is also g^{**}I- closed.

Proof: $A_\alpha\subseteq\bigcup A_\alpha$ implies $A_\alpha^{*}\subseteq\left(\bigcup A_\alpha\right)^{*}$ for every α.Therefore $\bigcup_{\alpha\in\Omega}(A_\alpha)^{*}\subseteq\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)^{*}$ ------ (1)

On the otherhand, if $x\in\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)^{*}$ then there exists an open set U containing x, that intersects only finite number of members $A_{\alpha_1},\ldots\ldots,A_{\alpha_n}$ .Let V be an open set containing x. Then $U\cap V$ is an open set containing x .which implies $(U\cap V)\cap\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)\notin I$.

i.e. $\left[(U\cap V)\cap\left(\bigcup_{\alpha\neq\alpha_i}A_\alpha\right)\right]\cup\left[(U\cap V)\cap\left(\bigcup_{i=1}^{n}A_{\alpha_i}\right)\right]\notin I$ i.e. $\{\phi\}\cup\left[(U\cap V)\cap\left(\bigcup_{i=1}^{n}A_{\alpha_i}\right)\right]\notin I$ and this implies $V\cap\left(\bigcup_{i=1}^{n}A_{\alpha_i}\right)\notin I$ Therefore $x\in\left(\bigcup_{i=1}^{n}A_{\alpha_i}\right)^*=\bigcup_{i=1}^{n}(A_{\alpha_i})^*\subseteq.\bigcup_{\alpha\in\Omega}(A_\alpha)^*$

Therefore $\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)^*\subseteq\bigcup_{\alpha\in\Omega}(A_\alpha)^*$ --------(2) From (1) and(2) we get $\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)^*=\bigcup_{\alpha\in\Omega}(A_\alpha)^*$

Let $\bigcup_{\alpha\in\Omega}A_\alpha\subseteq U$ and $U-g^*-open$ in X. Then $A_\alpha\subseteq U\ \forall\,\alpha\in\Omega$ implies $cl^*(A_\alpha)\subseteq U\ \forall\,\alpha\in\Omega$.

Then $cl^*\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)=\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)\cup\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)^*=\bigcup_{\alpha\in\Omega}cl^*(A_\alpha)\subseteq U$ Therefore $\bigcup_{\alpha\in\Omega}A_\alpha$ is g^{**}I- closed.

Theorem 3.11: Let (X,τ,I) be an ideal space which is $*$ - countably additive, and let $\{A_\alpha/\alpha\in\Omega\}$ be a locally countable family of g^{**}I- closed. sets in (X,τ,I). Then $\left(\bigcup_{\alpha\in\Omega}A_\alpha\right)$ is also g^{**}I- closed.

Proof: Similar to proof of above Theorem since in $*$ - countably additive space $\left[\bigcup_{i=1}^{\infty}A_i\right]^*=\bigcup_{i=1}^{\infty}(A_i)^*$

Theorem 3.12: Let the ideal space (X,τ,I) be $*s$ - finitely additive (resp. $*s$ - countably additive), and let $\{A_\alpha/\alpha\in\Omega\}$ be a locally finite (resp locally countable) family of sets in (X,τ,I). If each A_α is $g^{**S}I-closed$ then $\bigcup_{\alpha\in\Omega}A_\alpha$ is also $g^{**S}I-closed$

Proof: Similar to proof of above Theorem, since in $*S$ - finitely additive and $*S$ - countably additive spaces, $\left[\bigcup_{i=1}^{n}A_i\right]^{*S}=\bigcup_{i=1}^{n}(A_i)^{*S}$ and $\left[\bigcup_{i=1}^{\infty}A_i\right]^{*S}=\bigcup_{i=1}^{\infty}(A_i)^{*S}$ respectively

Remark 3.13: In general intersection of two g^{**}I – closed sets need not be g^{**}I – closed as seen from the following example.

Example 3.14: Let X ={a,b,c,d} $\tau=\{\phi,X,\{a,b\}\}$ $I=\{\phi\}$, Then A ={a,c} and B ={a,d} are g^{**}I-closed and g^{**s}I – closed but $A\cap B=\{a\}$ is not – g^{**}I-closed and g^{**s}I – closed

Theorem 3.15: A subset A of an ideal space (X,τ,I) is $g^{**}I-open$ if and only if $F\subset Int^*(A)$ whenever $F\subseteq A$ and F is a $g^*-closed$ subset of X.

Proof: Let A be $g^{**}I-open$ and F be a $g^*-closed$ subset of X contained in A. Then $(X-F)$ is a g^*-open set containing $X-A$ which implies $X-Int^*(A)=cl^*(X-A)\subset X-F$. So $F\subset Int^*(A)$

Conversely, let $F\subset Int^*(A)$ whenever $F\subseteq A$ and F is a $g^*-closed$ subset of X. Let U be a g^*-open and $X-A\subset U$. Then $X-U\subset Int^*(A)=X-cl^*(X-A)$. Therefore $cl^*(X-A)\subseteq U$ which proves $X-A$ is $g^{**}I-closed$. So A is $g^{**}I-open$.

Theorem 3.17: A subset A of a $*s$ - finitely additive ideal space (X,τ,I) is $g^{**s}I-open$ if and only if $F\subset Int^{*s}(A)$ whenever $F\subseteq A$ and F is a $g^*-closed$ subset of X.

Proof: similar to the proof of the above theorem since in $*s$ - finitely additive ideal space $Int^{*s}(A)=X-cl^{*s}(X-A)$

Theorem 3.18: For each $x\in(X,\tau,I)$ either $\{x\}$ is $g^{*}-closed$ or $\{x\}^{c}$ is $g^{**}I-closed$ in X .

Proof: Suppose $\{x\}$ is not $g^{*}-closed.$ then $\{x\}^{c}$ is not $g^{*}-open$.

Therefore the only $g^{*}-open$ set containing $\{x\}^{c}$ is X and $cl^{*}(\{x\}^{c})\subseteq X$ which proves that $\{x\}^{c}$ is $g^{**}I-closed$.

Theorem 3.19: For each $x\in(X,\tau,I)$ either $\{x\}$ is $g^{*}-closed$ or $\{x\}^{c}$ is $g^{**s}I-closed$ in X .

Proof: Similar to the above proof.

Theorem 3.20: In an ideal space (X,τ,I), if U is open and A is $g^{**}I-open$, then $U\cap A$ is $g^{**}I-open$.
Proof follows from (5) of remark (3.2) since every open set is $g^{**}I-open$

Theorem 3.21: In an ideal space (X,τ,I) which is finitely $*s$ - additive, if U is semi open and A is $g^{**S}I-open$, then $U\cap A$ is $g^{**S}I-open$.

Proof follows from theorem (3.8) since every semi open set is $g^{**S}I-open$

Theorem 3.22: If B is a subset of an ideal space (X,τ,I) such that $A\subset B\subset cl^{*}(A)$ and A is $g^{**}I-closed$, then B is also $g^{**}I-closed$ in X .

Proof: Let U be $g-open$ and $B\subset U$ .Then $A\subset U$ and this implies $cl^{*}(A)\subset U$ Therefore $cl^{*}(B)\subset cl^{*}(cl^{*}(A))\subset cl^{*}(A)\subset U$ which proves B is $g^{**}I-closed$.

Theorem 3.23: If B is a subset of a finitely $*s$ - additive ideal space (X,τ,I) such that $A\subset B\subset cl^{*S}(A)$ and A is $g^{**S}I-closed$, then B is also $g^{**S}I-closed$ in X .

Proof is similar to the proof of above theorem.

Theorem 3.24: Let (X,τ,I) be an ideal space and A be a $g^{**}I-closed$ subset of X . Then
(i) $cl(A^{*})\subseteq U$ for all $g^{*}-open$ set U containing A.
ii) $cl^{*}(A\ )-A$ contains no non empty $g^{*}-closed$ set.
(ii) $cl(A^{*})-A$ contains no non empty $g^{*}-closed$ set.
(iii) $(A^{*})-A$ contains no non empty $g^{*}-closed$ set.

Proof:

(i) Let U be $g^{*}-open$ set containing A.Then $cl\,A^{*}=A^{*}\subseteq cl^{*}(A)\subseteq U$,

(ii) Suppose that there exists a non empty $g^{*}-closed$ set F such that $F\subset cl^{*}(A)-A$.,

then $A\subseteq X-F$ which is $g^{*}-open.$ So $cl^{*}(A)\subseteq X-F$ and this implies $F\subset X-cl^{*}(A\ )$ Hence $F\subseteq(X-cl^{*}(A\ ))\cap(cl^{*}(A)-A)=\{\phi\}$

(iii) It follows from (ii) since $cl(A^*) = A^* \subseteq cl^*(A)$

(iv) It follows from (ii) since $A^* - A \subseteq cl^*(A) - A$.

Theorem 3.25: Let (X,τ,I) be an ideal space and A a $g^{**S}I-closed$ subset of X. Then

(i) $scl(A^{*S}) \subseteq U$ for all g^*-open set U containing A.

(ii) $cl^{*s}(A) - A$ contains no non empty $g^*-closed$ set.

(iii) $scl(A^{*S}) - A$ contains no non empty $g^*-closed$ set.

(iv) $(A^{*s}) - A$ contains no non empty $g^*-closed$ set.

Proof is similar to the proof of above theorem.

Theorem 3.26: Let (X,τ,I) be an ideal space and $A \subseteq Y \subseteq X$. If A is $g^{**}I-closed$ in $(Y,\tau/Y,I/Y)$, Y is open and τ^*-closed in X then A is $g^{**}I-closed$ in X.

Proof: Let U be g^*-open set in X containing A. Then $A^*(I,\tau)\cap Y = A^*(I_Y,\tau_Y) \subseteq U \cap Y$. Then $Y \subseteq U \cup (X - A^*(I,\tau))$. Since Y is τ^*-closed, $A^* \subseteq Y^* \subseteq Y \subseteq U \cup (X - A^*(I,\tau))$.

Therefore $A^* \subseteq U$ and this implies $cl^*(A) = A \cup A^* \subseteq U$

Theorem 3.27: Let (X,τ,I) be an ideal space and $A \subseteq Y \subseteq X$. If A is $g^{**s}I-closed$ in $(Y,\tau/Y,I/Y)$, Y is open and $\tau^{*s}-$closed in X then A is $g^{**s}I-closed$ in X.

Proof is similar to the proof of above theorem.

Theorem 3.28: Let (X,τ,I) be an ideal space and $A \subseteq X$. If A is $g^{**}I-closed$ then $A \cup (X - A^*)$ is $g^{**}I-closed$.

Proof: Let U be g^*-open and $A \cup (X - A^*) \subset U$. Then $X - U \subset X - [A \cup (X - A^*)] = A^* - A$. Since A is $g^{**}I-closed$, $A^* - A$ contains no non empty $g^*-closed$ set. Therefore $X - U = \phi$ which implies $X = U$. Thus X is the only g^*-open set containing $A \cup (X - A^*)$ which proves $A \cup (X - A^*)$ is $g^{*S}I-closed$.

Theorem 3.29: Let (X,τ,I) be an ideal space and $A \subseteq X$. If A is $g^{**s}I-closed$ then $A \cup (X - A^{*s})$ is $g^{**s}I-closed$.

Proof is similar to the proof of above theorem.

Theorem 3.30: Let (X,τ,I) be an ideal space. If every g^*-open set is $*-closed$, then every subset of X is $g^{**}I-closed$.

Proof: Let $A \subset U$ and U a g^*-open set in X. Then $cl^*(A) \subset cl^*(U) = U$ which proves A is $g^{**}I-closed$

Theorem 3.31: Let (X,τ,I) be an ideal space. If every g^*-open set is $*s-closed$, then every subset of X is $g^{**s}I-closed$.

Proof is similar to the proof of above theorem.

REFERENCES

[1] M.E. Abd EI Mosef, E.F.Lashien and A.A. Nasef, some topological operators via ideals, kyungpook Mathematical Journal, vol.32 (1992)

[2] K. Kuratowski, Topologie, I. Warszawa, 1933

[3] N. Levine, Semiopen sets and semi continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41.

[4] N. Levine, Rend. Cire. Math. Palermo, 19 (1970), 89 – 96.

[5]Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces, IJM A, 3(5), (2012),1-15.

[6] R. Vaidyanathasamy, The localization theory in set-topology, Proc. Indian Acad. Sci., 20, (1945)51-61.

[7] M.K.R.S. Veera Kumar, Mem. Fac. Sci. Kochi Univ. (Math.), 21 (2000), 1 – 19.

Source of support: Nil, Conflict of interest: None Declared