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EFFECT OF MHD VISCO-ELASTIC FLUID (OLDROYD) AND POROUS MEDIUM THROUGH A CIRCULAR CYLINDER BOUNDED BY A PERMEABLE BED

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ABSTRACT

The analysis of the unsteady laminar visco-elastic (Oldroyd) conducting fluid through a circular cylinder bounded by permeable bed under the influence of exponentially decreasing pressure gradient and porous medium is presented. A uniform consent magnetic field is applied in azimuthal direction (i.e. in the direction of θ). The expressions for velocity are obtained and discussed with the help of Tables and Graphs.

Keywords: *Visco-elastic (Oldroyd) fluid, MHD flow, Porous medium, A circular cylinder, A Permeable Bed.*

INTRODUCTION

MHD flow of visco-elastic fluid through pipes plays significant role in different areas of science and technology such as Petroleum industry, Biomechanics, Drainage and Irrigation engineering and so on. The steady flow of a viscous incompressible fluid through various cross-sections have been studied extensively by many authors. Yuan and Finkelstein[19] have obtained the solution of laminar pipe flow with injection and suction through a porous wall. Drake[3] and Khamrui[5] have studied steady viscous incompressible flow through circular and elliptic tubes under the influence of periodic pressure. Laminar flow of a steady viscous incompressible fluid through a circular pipe under the influence of aligned magnetic field was discussed by Hughes and Young.[4] Sinha and Chowdary[14] have studied viscous incompressible flow through the cylinder.

Batchelor[2] studied the startling flow in a circular pipe which is initially at rest, and set in motion by pressure gradient suddenly imposed and maintained by external means. Patabhirama[7] made a study on the flow through a circular pipe completely filled with porous material. The laminar flow of a unsteady viscous liquid through a circular cylinder under the influence of exponentially decreasing pressure gradient was studied by Sambasiva Rao and Rama Murthy[11]. Bardar[1] studied numerically the development with time of the two dimensional flow of viscous incompressible fluid around a circualr cylinder which suddenly starts rotating about its axis with constant speed. Kumari and Bansal[6] used Oseen's approximation to study the slow motion of a viscous incompressible electrically conduction fluid past a circular cylinder in the presence of aligned magnetic filed and obtained the numerical values for the tangential drag for different values of magnetic interaction parameter and viscous Reynolds number. Sharma et al.[13] obtained the numerical solution of steady motion of second order fluid past a circular cylinder with suction or injection. Soundalgekar[15] obtained the finite difference solution for the laminar developing flow in an annulus between two rotating cylinders. Steady two dimensional flow of viscous incompressible fluid at low Reynolds number past a circular cylinder was studied by Raghava Rao[8] and Swachuk and Zamir[12] presented quasi-laminar solutions for the boundary layer on a circular cylinder in axial flow using a Kellerbox numerical scheme for velocity components rather than a stream function. The solutions extend earlier results considerably and cover a wide range of cylinder radii from very small to very large. Recently, Subramanyam et. al.[17] have studied the unsteady laminar viscous conducting fluid through a circular cylinder boundary by permeable bed under the influence of exponentially decreasing pressure gradient.

In this section, we study the problem of Subramanyam[17] with visco-elastic fluid (Oldroyd) through porous medium. Purpose of this study is to obtain the velocity distribution to compare with already existing paper Subramanyam et al.[17] .

NOMENCLATURE

μ-coefficient of visosity, J- current density vector, (r, θ, z) - cylinderical polar co-ordinates, Q-Darcy velocity, α-dimensionless parameter, p-dimensional pressure parameter, J_1 - first order Bessel function of first kind, p- fluid density, P- fluid pressure, K-permeability of the bed, $\sigma_1 = \frac{a}{\sqrt{K}}$ permeability parameter, M-magnetic paramter

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density, P- fluid pressure, K-permeability of the bed, $\sigma_1 = \frac{a}{\sqrt{K}}$ permeability parameter, M-magnetic paramter (Hartmann number), H= (0, H_0, 0) - magnetic field vector, μ_e - magnetic permeability, 1_1 - modified Bessel function of first order, I_0 - modified Bessel function of zeroth order, α_I - slip parameter, V_B - slip velocity, t-time, q=(V_r, V_θ, V_z) - velocity vector, J_0 - zeroth order Bessel function of first kind, λ_0, λ_1 - visco-elastic coefficients, K_0 - permeability of the porous medium, v- kinematic viscosity, K_1 - permeability parameter of the medium, β and β_1 - visco-elastic parameters.

FORMULATION OF THE PROBLEM

Consider the laminar flow of unsteady visco-elastic (Oldroyd) electricity conducting fluid through a circular cylinder boundary by permeable bed under the influence of exponentially decreasing pressure gradient and porous medium. The axis of the cylinder is taken as z-axis. Cylindrical polar coordinates (r, θ, z) are used. The velocity of the fluid is taken in z direction. A uniform magnetic field H_0 is applied in θ direction. We make the following assumptions:

1. Flow is laminar and fully developed.
2. The direction of the flow is in the z direction i.e. in the direction of the axis of the cylinder. So $V_r = V_\theta = 0$ and V_z is function of r and t only.
3. Electric field is neglected Rossow[9]. Induced magnetic field is also neglected Sparrow and Cess[16].

Under the above assumptions, the governing equations are:

$$-\frac{\partial p}{\partial r} = 0 \tag{1}$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} = 0 \tag{2}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial V_z}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 V_z}{\partial r^2}+\frac{1}{r}\frac{\partial V_z}{\partial r}\right) - \left(\frac{\sigma}{\rho}\mu_e^2 H_0^2 + \frac{\nu}{K_0}\right)\left(1+\lambda_1\frac{\partial}{\partial t}\right)V_z \tag{3}$$

Boundary conditions are

V_z is finite at r = 0, $V_z = V_B$, $\frac{dV_z}{dr} = \frac{\alpha_1 a}{\sqrt{K}}(V_B - Q)$ at r = a (4)

where Q = $-\frac{K}{\mu}\left(\frac{\partial p}{\partial z} + \sigma\mu_e^2 H_0^2 Q\right)$

NON-DIMENSIONAL PARAMETERS:

We define the following non-dimensional quantities

$$V_z{*} = \frac{V_z}{V_0} \qquad r{*} = \frac{r}{a} \qquad z{*} = \frac{z}{a} \qquad p{*} = \frac{pa}{\mu V_0}$$

$$t{*} = \frac{t\nu}{a^2} \qquad Q{*} = \frac{Q}{V_0} \qquad V_B{*} = \frac{V_B}{V_0}$$

Using the above quantities and after removing the asterisks, the equation (1) and (4) become

$$-\frac{\partial p}{\partial r} = 0 \tag{5}$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} = 0 \tag{6}$$

$$\left(1+\frac{\nu\lambda_1}{a^2}\frac{\partial}{\partial t}\right)\frac{\partial V_z}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(1+\frac{\nu\lambda_0}{a^2}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 V_z}{\partial r^2}+\frac{1}{r}\frac{\partial V_z}{\partial r}\right) - \left(M^2 + \frac{1}{K_1}\right)\left(1+\frac{\nu\lambda_1}{a^2}\frac{\partial}{\partial t}\right)V_z \tag{7}$$

where $M^2 = \dfrac{\sigma a^2 \mu_e^2 H_0^2}{\rho \nu}$ (Magnetic Parameter)

$K_1 = \dfrac{K_0}{a^2}$ (Permeability Parameter)

Boundary conditions are V_z is finite at r = 0,

$$V_z = V_B, \frac{dV_z}{dr} = \alpha_1 a(V_B - Q)\, at\, r = 1 \tag{8}$$

where $Q = -\dfrac{K}{\mu}\left(\dfrac{\mu}{a^2}\dfrac{\partial p}{\partial z} + \sigma \mu_e^2 H_0^2 Q\right)$

SOLUTION OF THE PROBLEM:

From equation (5) and (6), it is observed that the pressure is constant in r and θ directions. Taking the pressure to be an exponential functions of t given by

$$-\frac{\partial p}{\partial z} = Pe^{-\alpha t} \tag{9}$$

Here P is constant and equals pressure gradient in fully developed motion when t = 0. Let us consider

$$V_z = f(r)e^{-\alpha t} \tag{10}$$

With the help of equations (9) and (10), the equation becomes (7)

$$\frac{d^2 f(r)}{dr^2} + \frac{1}{r}\frac{df(r)}{dr} + S^2 f(r) = -\frac{P}{\beta} \tag{11}$$

where

$$\beta = 1 - \frac{\alpha \nu \lambda_0}{a^2}$$

$$\beta_1 = 1 - \frac{\alpha \nu \lambda_1}{a^2}$$

$$S^2 = \frac{\beta_1}{\beta}\left\{\alpha - \left(M^2 + \frac{1}{K^1}\right)\right\}$$

Now, Boundary conditions become V_z is finite at r = 0,

$$f(r) = V_B e^{-\alpha t}, \frac{df(r)}{dr} = \alpha_1 a(V_B - Q)e^{\alpha t} \text{ at r=1} \tag{12}$$

Case-I: Let $\alpha - \left(M^2 + \dfrac{1}{K_1}\right) > 0$

On solving equation (11) using boundary condition (12), we get

$$f(r) = \frac{P}{\beta S^2}\left[\frac{J_0(Sr)}{J_0(S)} - 1\right] + V_B\left[\frac{J_0(Sr)}{J_0(S)}\right]e^{\alpha t} \tag{13}$$

The velocity distribution is

$$V_z = f(r)e^{-\alpha t} = \frac{P}{\beta S^2}\left[\frac{J_0(Sr)}{J_0(S)} - 1\right]e^{-\alpha t} + V_B\left[\frac{J_0(Sr)}{J_0(S)}\right] \tag{14}$$

Case-II: Let $\alpha - \left(M^2 + \frac{1}{K_1} \right) = 0$

The equation (11) becomes

$$\frac{d^2 f(r)}{dr^2} + \frac{1}{r}\frac{df(r)}{dr} + \frac{P}{\beta} = 0 \tag{15}$$

On solving equation (15) using boundary condition (12), we get

$$f(r) = \frac{P}{4\beta}(1 - r^2) + V_B e^{\alpha t} \tag{16}$$

In this case, the velocity distribution is

$$V_z = f(r)e^{-\alpha t} = \frac{P}{4\beta}(1 - r_2)ee^{-\alpha t} + V_B \tag{17}$$

Case-III: Let $\alpha - \left(M^2 + \frac{1}{K_1} \right) < 0$

The equation (11) becomes

$$\frac{d^2 f(r)}{dr^2} + \frac{1}{r}\frac{df(r)}{dr} - S_1^2 f(r) = -\frac{P}{\beta} \tag{18}$$

where

$$f(r) = \frac{P}{\beta S_1^2}\left[\frac{I_0(S_1 r)}{I_0(S_1)} - 1\right] + V_B\left[\frac{I_0(S_1 r)}{I_0(S_1)}\right]e^{\alpha t} \tag{19}$$

The velocity distribution is

$$V_z = f(r)e^{-\alpha t} = \frac{P}{\beta S_1^2}\left[\frac{I_0(S_1 r)}{I_0(S_1)} - 1\right]e^{-\alpha t} + V_B\left[\frac{I_0(S_1 r)}{I_0(S_1)}\right] \tag{20}$$

PARTICULAR CASE

When β, β_1 (Visco-elastic parameters) are one and K_1 one and K_1 tends to infinite, this problem reduces the problem of Subramanyam et at [17].

RESULT AND DISCUSSION

The Velocity Profile is tabulated in Table-I and plotted in Fig.-I having Graph-I to V at α=0.2, t=0.2, t=0.5, P=0.077, V_B = 0.001 and following different values of M,β, β_1 and K_1.

	M	β	β_1	K_1
For Graph-I	0.2	1	1	10
For Graph-II	0.3	1	1	10
For Graph-III	0.2	4	1	10
For Graph-IV	0.2	1	4	10
For Graph-V	0.2	1	1	100

From the Graph-I to V of Fig. I it is observed that the velocity decreases with the increase in r. It is also noticed that the velocity increases with the increase in β_1 and K_1, but it decreases with the increase in M and β.

CONCLUSION

The velocity of fluid increases with the increase in β, β_1 (Visco-elastic Parameters).

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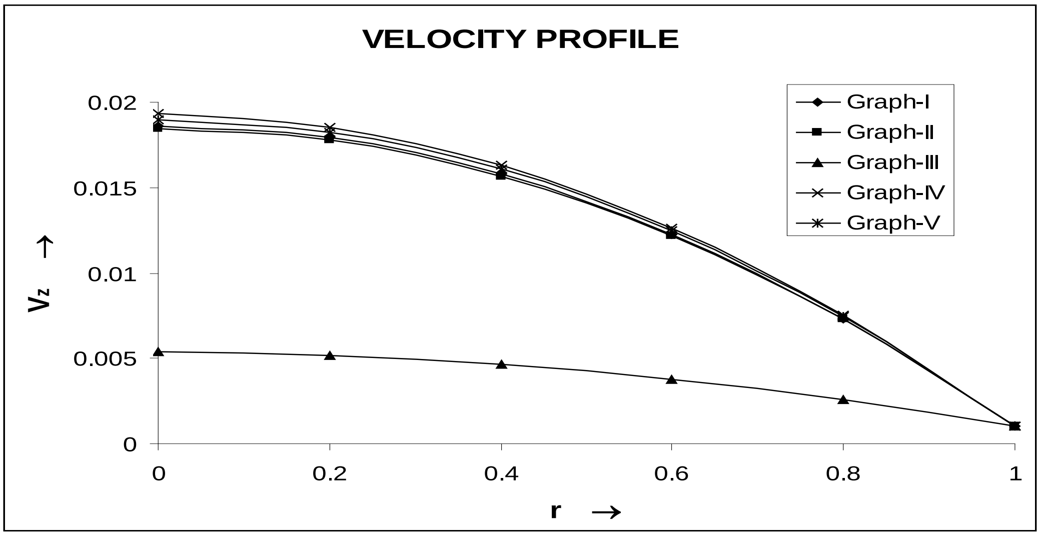

Fig.-I

Table-I: Values of velocity at $\alpha = 0.2$, $t = 0.5$, $P = 0.077$, $V_B = 0.001$ and different values of M, β, β_1 and K_1.

r	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V
0	0.018631	0.018453	0.005371	0.019299	0.018960
0.2	0.017924	0.017755	0.005196	0.018557	0.018235
0.4	0.015801	0.015659	0.004671	0.016334	0.016064
0.6	0.012269	0.012168	0.003796	0.012648	0.012455
0.8	0.007332	0.007281	0.002572	0.007524	0.007427
1	0.001000	0.001000	0.001000	0.001000	0.001000

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