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SOME LRS BIANCHI TYPE II STRING COSMOLOGICAL MODELS FOR VISCOUS FLUID DISTRIBUTION IN GENERAL RELATIVITY

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ABSTRACT:

Some LRS Bianchi type II string cosmological models are discussed. We have assumed the condition $\rho = k\lambda$, here ρ is the energy density, λ is the string tension density and k is a constant. We have also used a condition that, The scalar expansion is proportional to the shear, to get determinate solution in terms of cosmic time t. The physical and geometrical aspects of the models are also discussed.

Keywords and Phrases: *LRS Bianchi type II, massive string, viscous fluid.*

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1. INTRODUCTION

Bianchi type II models play an important role in current modern cosmology for simplification and description of the large scale behavior of the actual universe. Banerjee *et. al.* [7] investigated a spatially homogeneous and locally rotationally symmetric Bianchi type II cosmological model under the influence of both shear and bulk viscosity.

Singh and Agrawal [12] have studied Bianchi type II, VIII and IX models in scalar tensor theory under the assumption of a relationship between the cosmological constant (Λ) and the scalar field (ψ).

In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble [9]). Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature decreased below some critical temperature as predicted by grand unified theories (Zel' dovich *et al.* [22]; Kibble [8]; Vilenkin [16]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies. These cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formation of the energy-momentum tensor for classical-massive strings was done by Letelier who considered the massive strings to be formed by geometric strings with particle attached along its extension and Letelier [10] first used this idea in obtaining cosmological solutions in Bianchi I and Kantowski-Sachs space-times. Stachel [13] has studied massive string. Bali *et al.* [1-6] have obtained Bianchi type I, III, V and IX string cosmological models with magnetic field and bulk viscosity in general relativity. Yadav *et al.*[21] have studied some Bianchi type I viscous fluid string cosmological models with magnetic field. Wang [17-19] has also discussed LRS Bianchi type I, Kantowski-Sachs and Bianchi type III cosmological models for a cloud of string with bulk viscosity. Recently Yadav *et al.* [20] have obtained the integrability of cosmic string in Bianchi type III space-time in presence of bulk viscous fluid by applying a new technique. Pradhan *et al.* [11] have dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings.).

Recently, Tyagi and Sharma [14] have studied Locally Rotationally Symmetric Bianchi Type-II Magnetized String Cosmological Model with Bulk Viscous Fluid in General Relativity. Tyagi et al. [15] have also studied Bianchi Type-II Bulk Viscous String Cosmological Models in General Relativity.

In this paper, we have investigated some Bianchi type II bulk viscous string cosmological model in general relativity. To get a deterministic model, it is assumed that $\rho = K\lambda$, here ρ is the rest energy density, λ is the string tension density.

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We have also assumed that The scalar expansion is proportional to the shear , this gives a a relation between metric potential $B = lA^n$, where A and B are function of time alone. The physical and geometrical implications of the model are discussed.

2. THE METRIC AND FIELD EQUATIONS

We consider LRS Bianchi type II metric in the form

$$ds^2 = -dt^2 + A^2\left(dx^2 + dz^2\right) + B^2\left(dy - xdz\right)^2 \tag{1}$$

The Einstein field equation is given by

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{2}$$

(in geometrical unit $8\Lambda G = 1, C = i$)

The energy momentum tensor T_i^j for a cloud of massive string for viscous fluid distribution is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi\theta\left(v_i v^j + g_i^j\right) \tag{3}$$

Here ρ is the rest energy density for a cloud of massive strings with particle attached along its extension given by

$$\rho = \rho_\rho + \lambda \tag{4}$$

ρ_ρ being the particle energy density, λ the string tension density, v^i the four velocity for the cloud of particles and x^i the four vector which represents the strings direction. Thus we have.

$$v_i v^i = -1 = -x_i x^i \tag{5}$$

and $$v_i x^i = 0 \tag{6}$$

In commoving co-ordinates, we have

$$v^i = (0,0,0,1) \tag{7}$$

$$x^i = \left(\frac{1}{A}, 0, 0, 0\right) \tag{8}$$

The Einstein's field equations (2) for metric (1) lead to

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{4A^4} = \lambda + \xi\theta \tag{9}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = \xi\theta \tag{10}$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{B^2}{4A^4} = \rho \tag{11}$$

For the complete determination of the set, we assume that

$$\rho = K\lambda \tag{12}$$

From (9), (11) and (12) we have

$$K\frac{\ddot{A}}{A} - K\frac{\ddot{B}}{B} + (2-K)\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2}(K+1) = \frac{B^2}{4A^4}(4K+1) \tag{13}$$

Using (13) and $B = lA^n$ we obtain

$$\frac{\ddot{A}}{A} + \gamma \frac{\dot{A}^2}{A^2} = \frac{M}{A^{4-2n}} \tag{14}$$

Where $\gamma = \left\{ \frac{K(1-n^2) + (2n+1)}{K(1-n)} \right\}$ (15)

$$M = \frac{l^2(4K+1)}{4K(1-n)} \tag{16}$$

(14) reduces to

$$\frac{d}{dA}\left[A^{2\gamma}\,\dot{A}^2\right] = \frac{2M}{A^{3-2n-2\gamma}} \tag{17}$$

On Integrating (17) we obtain

$$dt = \left[\frac{l^2(4K+1)}{4A^{2-2n}\left[2nK(1-n)+(2n+1)\right]} + \frac{L}{A^{2\left\{(1+n)+\frac{(2n+1)}{K(1-n)}\right\}}} \right]^{-1/2} dA \tag{18}$$

Here L, the constant of integration.

Using proper transformation and equation (18) the above metric (1) reduces to.

$$ds^2 = \left[\frac{l^2(4K+1)}{4T^{2-2n}\left[2nK(1-n)+(2n+1)\right]} + \frac{L}{A^{2\left\{(1+n)+\frac{(2n+1)}{K(1-n)}\right\}}} \right] dT^2 + T^2\left(dx^2+dz^2\right) + l^2T^{2n}(dy-xdz)^2 \tag{19}$$

3. SOME PHYSICAL AND GEOMETRICAL FEATURES

The rest energy density (ρ), the string tension density (λ), the particle density (ρ_p) for model (19) are given by

$$\rho = \frac{l^2}{4T^{4-2n}}\left[\frac{(2n+1)(4K+1)}{\left\{2nK(1-n)+(2n+1)^2\right\}} \right] + \frac{L}{T^{2\left\{\frac{(n+2)}{K(1-n)}\right\}}} \tag{20}$$

$$\lambda = \rho/K \tag{21}$$

$$\rho_p = \frac{(k-1)}{k}\rho \tag{22}$$

Where ρ, is given by equation (20)

The scalar of expansion (θ), coefficient of bulk viscosity (ξ), the spatial volume R^3 and the shear (σ) for the model (19) are given by

$$\theta = (n+2)\left\{ \frac{l^2(4K+1)}{4\left\{2nK(1-n)+(2n+1)\right\}}\frac{1}{T^{4-2n}} + \frac{L}{T^{2\left\{(n+2)+\frac{(2n+1)}{K(1-n)}\right\}}} \right\}^{1/2} \tag{23}$$

$$\sigma = \frac{(1-n)}{\sqrt{3}}\left\{ \frac{l^2(4K+1)}{4\left\{2nK(1-n)+(2n+1)\right\}T^{4-2n}} + \frac{L}{T^{2\left\{(n+2)+\frac{(2n+1)}{K(1-n)}\right\}}} \right\}^{1/2} \tag{24}$$

The spatial volume R^3 is ξ are given by

$$R^3 = lT^{n+2} \tag{25}$$

$$\xi = \left[\frac{l^2\left(K(5+3n)+2\right)}{4T^{4-2n}} + \left\{ \frac{2\left(n^2-2n-1\right)-K(n+1)}{K(1-n)} \right\} \left\{ \frac{l^2(4K+1)}{4\left\{2nk(1-n)+(2n+1)\right\}T^{4-2n}} \right\} + \frac{L}{T^{2\left\{(n+2)+\frac{(2n+1)}{K(1-n)}\right\}}} \right] + \frac{1}{(n+2)} \left[\frac{l^2(4K+1)}{4\left\{2nK(1-n)+(2n+1)\right\}T^{4-2n}} + \frac{L}{T^{2\left\{(n+2)+\frac{(2n+1)}{K(1-n)}\right\}}} \right]^{-1/2} \tag{26}$$

4. DISCUSSION

The energy conditions $\rho \geq 0$ leads to

$$T^{\left\{\frac{(2n+1)}{K(1-n)}-4n\right\}} \geq \frac{l^2}{L}\left[1 - \frac{(2n+1)(4K+1)}{\left\{2nK(1-n)+(2n+1)\right\}}\right] \tag{27}$$

The model starts expanding with a big bang at $T = 0$.

The expansion in the model decrease slowly and it stops when $n = -2$ or when $T \to \infty$.

When $T \to 0$ then $\rho \to \infty$ which shows that there is massive mass at $T \to 0$

The spatial volume (R^3) increase as T increase provided $(n+2) > 0$.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ (Provide $n = 1$)

Hence model does not isotropize for large value of T. When $n \to 1$, $\sigma = 0$, this shows, that the model (19) is quasi isotropic when $n \to 1$.

Eqs. (20) and (22) show that when k $\geq$ 1, the particle density $\rho_p \geq 0$ and the string tension density $\lambda \geq 0$, however, $\rho_p > 0$ and $\lambda < 0$ when k < 0. Further, when k > 2 or k < 0, we have $\rho_p > |\lambda|$,therefore, in this case the massive strings dominate the universe in the process of evolution. However, when 1 < k < 2, we have $\rho_p < |\lambda|$, and in this case, the strings dominate over the particles.

5. SPECIAL CASE

When k = 1 then $\rho = \lambda$,then the Line element (1) becomes

$$ds^2 = -\left[\frac{5l^2}{4\left(4n-2n^2+1\right)} + \frac{L}{T^{2(2+2n-n2)}}\right]dT^2 + T^2\left(dx^2+dz^2\right) + l^2T^{2n}(dy - xdz)^2 \tag{28}$$

The rest energy density (ρ), the scalar of expansion (θ), the shear (σ), the particle density (ρ_p) and the coefficient of bulk viscosity (ξ), for the model (28) are given by

$$\rho = \frac{l^2}{4T^{4-2n}}\left[\frac{2n^2+3n+2}{\left(1+4n-2n^2\right)}\right] + \left[\frac{L}{T^{2\frac{\left(3+n-n^2\right)}{1-n}}}\right] \tag{29}$$

$$\theta = (n+2)\left[\frac{5l^2}{4(4n-2n^2+1)T^{4-2n}} + \frac{L}{T^{2\frac{(3+n-n^2)}{1-n}}}\right]^{½} \tag{30}$$

$$\sigma = \frac{(1-n)}{\sqrt{3}}\left[\frac{5l^2}{4(4n-2n^2+1)T^{4-2n}} + \frac{L}{T^{2\frac{(3+n-n^2)}{(1-n)}}}\right]^{½} \tag{31}$$

$$\rho_\rho = 0 \tag{32}$$

$$\xi = \left[\frac{l^2(7+3n)}{4T^{4-2n}} + \frac{(2n^2-5n-3)}{(1-n)}\left\{\frac{5l^2}{4(1+4n-2n^2)T^{4-2n}} + \frac{L}{T^{2\frac{(3+n-n^2)}{1-n}}}\right\}\right]$$

$$\frac{x^1}{(n+2)}\left[\frac{5l^2}{4(1+4n-2n^2)T^{4-2n}} + \frac{L}{T^{2\frac{(3+n-n^2)}{(1-n)}}}\right]^{½} \tag{33}$$

The energy condition for model (28) leads to

$$T^{\frac{2(2n^2-4n-1)}{(1-n)}} \geq \frac{l^2(n^2+3n+2)}{2L(2n^2-4n)} \tag{34}$$

From equation (32) and (29)

$$\frac{\rho_p}{|\lambda|} = 0$$

Hence, in this case the strings dominate over the particles.

When $T \to 0$, then the energy density $\rho \to \infty$ and the scalar of expansion $\theta \to \infty$.

When $T \to \infty$, then $\rho \to 0$ and $\theta \to 0$

The cosmological model (34) represents shearing & nonrotating universe starts with a big bang.

$\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$ so model does not isotropize for large value of T.

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