

ON INTUITIONISTIC Q-FUZZY IMPLICATIVE Q-IDEALS IN Q-ALGEBRA

R. Muthuraj*

Department of Mathematics, H. H. The Rajah's College, Pudukkottai-622 001, TN, India

M. S. Muthuraman, M. Sridharan and P. Vijayalakshmi

Department of Mathematics, PSNA College of Engineering and Technology, Dindigul-624 622, TN, India

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ABSTRACT

In this paper, we introduce the concept of an Intuitionistic Q-fuzzy implicative Q-ideal in Q-algebras and some related properties are investigated. We also define the level subsets of an intuitionistic Q-fuzzy implicative Q-ideals in Q-algebras and discussed some of its properties.

Keywords: Intuitionistic fuzzy set , intuitionistic Q-fuzzy set, intuitionistic fuzzy implicative Q-ideals, intuitionistic Q-fuzzy implicative Q-ideals , intuitionistic Q-fuzzy closed implicative Q-ideals.

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1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [10] in 1965. Several researchers explored on the generalization of the notion of fuzzy sets. The concept of intuitionistic fuzzy set was introduced by K.T. Atanassov [1,2] ,as a generalization of the notion of fuzzy set in 1986. Since then the literature on these concepts has been growing rapidly.

Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK-algebra and BCI-algebras. It is known that the class of BCK-algebras is proper subclass of the class of BCI- algebras. J. Neggers, S.S. Ahn and H.S. Kim [4] introduced a notion, called Q –algebras. In this paper, we introduce the notion of Intuitionistic Q-fuzzy implicative Q-ideals in Q-algebra and investigate some properties.

2. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition: A Q-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following axioms

- i. $x * x = 0$
- ii. $x * 0 = x$
- iii. $(x * y) * z = (x * z) * y$ for all $x, y, z \in X$,

In X, we can define a binary relation \leq by $x \leq y$ if and only if $x * y = 0$.

2.1 Example Let $X = \{0, 1, 2, 3\}$ be a set with the following table

*	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

$(X, *, 0)$ is a Q-algebra.

Corresponding author: R. Muthuraj*

Department of Mathematics, H. H. The Rajah's College, Pudukkottai-622 001, TN, India

2.2 Definition: Let $(X, *, 0)$ be a Q-algebra. A non empty set I of X is called an ideal of X if it satisfies

- i. $0 \in I$
- ii. $x * y \in I$ and $y \in I$ imply $x \in I$, for all $x, y \in X$.

2.3 Definition: An ideal A of a Q-algebra X is said to be closed if $0 * x \in A$ for all $x \in A$.

2.4 Definition: A fuzzy set μ in a nonempty set X we mean a function $\mu: X \rightarrow [0, 1]$ and the complement of μ denoted by μ^c , is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

2.5 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy subalgebra of X if $\mu(x*y) \geq \min \{\mu(x), \mu(y)\}$, for all $x, y \in X$

2.6 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy ideal of X if

- i. $\mu(0) \geq \mu(x)$
- ii. $\mu(x) \geq \min \{\mu(x*y), \mu(y)\}$ for all $x, y \in X$

2.7 Definition: An intuitionistic fuzzy subset (IFS) A in a set X is defined as an object of the form $A = \{\langle X, \mu(x), \lambda(x) \rangle / x \in X\}$, where $\mu: X \rightarrow [0, 1]$ and $\lambda: X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively and for every $x \in X$, $0 \leq \mu(x) + \lambda(x) \leq 1$. For the sake of simplicity we use the symbol $A = (X, \mu, \lambda)$ for the intuitionistic fuzzy set $A = \{\langle X, \mu(x), \lambda(x) \rangle / x \in X\}$.

2.8 Definition: An intuitionistic fuzzy set $A = (X, \mu, \lambda)$ is called an intuitionistic fuzzy subalgebra of X if it satisfies

- i. $\mu(x*y) \geq \min \{\mu(x), \mu(y)\}$
- ii. $\lambda(x*y) \leq \max \{\lambda(x), \lambda(y)\}$, for all $x, y \in X$

2.1 Theorem: Every intuitionistic fuzzy subalgebra $A = (X, \mu, \lambda)$ of X satisfies the inequalities $\mu(0) \geq \mu(x)$ and $\lambda(0) \leq \lambda(x)$.

Proof: For all $x, y \in X$.

$$\mu(0) = \mu(x*x) \geq \min \{\mu(x), \mu(x)\} = \mu(x) \text{ and } \lambda(0) = \lambda(x*x) \leq \max \{\lambda(x), \lambda(x)\} = \lambda(x),$$

2.9 Definition: A non empty set I of a Q-algebra X is called an implicative Q-ideal of X if

- i. $0 \in I$
- ii. $(x*y)*z \in I$ and $y*z \in I$ then $x*z \in I$, for all $x, y, z \in X$.

2.10 Definition: A fuzzy set μ in a Q-algebra X is called a fuzzy implicative Q-ideal of X if

- i. $\mu(0) \geq \mu(x)$
- ii. $\mu(x*z) \geq \min \{\mu((x*y)*z), \mu(y*z)\}$ for all $x, y, z \in X$

2.11 Definition: An intuitionistic fuzzy set $A = (X, \mu, \lambda)$ in a Q-algebra X is called an intuitionistic fuzzy implicative Q-ideal of X if

- i. $\mu(0) \geq \mu(x)$ and $\lambda(0) \leq \lambda(x)$
- ii. $\mu(x*z) \geq \min \{\mu((x*y)*z), \mu(y*z)\}$
- iii. $\lambda(x*z) \leq \max \{\lambda((x*y)*z), \lambda(y*z)\}$ for all $x, y, z \in X$

2.12 Definition: An intuitionistic fuzzy set $A = (X, \mu, \lambda)$ in a Q-algebra X is called an intuitionistic fuzzy closed implicative Q-ideal of X if it satisfies

- i. $\mu(0*x) \geq \mu(x)$ and $\lambda(0*x) \leq \lambda(x)$
- ii. $\mu(x*z) \geq \min \{\mu((x*y)*z), \mu(y*z)\}$
- iii. $\lambda(x*z) \leq \max \{\lambda((x*y)*z), \lambda(y*z)\}$ for all $x, y, z \in X$

2.13 Definition: Let $A = (X, \mu, \lambda)$ be an intuitionistic fuzzy set in a Q-algebra X the set $U(\mu; s) = \{x \in X / \mu(x) \geq s\}$ is called an upper s -level of μ and the set $L(\lambda; t) = \{x \in X / \lambda(x) \leq t\}$ is called lower t -level of λ .

3. Intuitionistic Q- fuzzy implicative Q-ideal

In this section Q-fuzzy implicative Q-ideal, Q-fuzzy closed implicative Q-ideal, Intuitionistic Q-fuzzy implicative Q-ideal, Intuitionistic Q-fuzzy closed implicative Q-ideal, homomorphism of Q-algebra are defined and some related properties are discussed.

3.1 Definition: A Q-fuzzy subset $\mu: X \times Q \rightarrow [0,1]$ in a Q- algebra X is called a Q- fuzzy implicative Q- ideal of X if

- i. $\mu(0, q) \geq \mu(x, q)$
- ii. $\mu((x*y), q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \}$ for all $x, y, z \in X, q \in Q$.

3.2 Definition: An intuitionistic Q-fuzzy set A in an non empty set X is an object having the form $A = \{ (X, \mu(x, q), \lambda(x, q)) / x \in X, q \in Q \}$, where the function $\mu: X \times Q \rightarrow [0,1]$ and $\lambda: X \times Q \rightarrow [0,1]$ denote the degree of membership (namely $\mu(x, q)$) and degree of non-membership (namely $\lambda(x, q)$) and $0 \leq \mu(x, q) + \lambda(x, q) \leq 1$, for all $x \in X$ and $q \in Q$.

For the sake of simplicity, we use the symbol $A = (X, \mu, \lambda)$ for the intuitionistic Q- fuzzy set $A = \{ (X, \mu(x, q), \lambda(x, q)) / x \in X, q \in Q \}$.

3.3 Definition: An intuitionistic Q- fuzzy set A of a Q-algebra X is a subalgebra if

- i. $\mu(x*y, q) \geq \min \{ \mu(x, q), \mu(y, q) \}$
- ii. $\lambda(x*y, q) \leq \max \{ \lambda(x, q), \lambda(y, q) \}$ for all $x, y, z \in X$ and $q \in Q$.

3.4 Definition: An intuitionistic Q- fuzzy set $A = (X, \mu, \lambda)$ in a Q- algebra is called an intuitionistic Q- fuzzy implicative Q- ideal of X, if it satisfies the following axioms:

- i. $\mu(0, q) \geq \mu(x, q)$ and $\lambda(0, q) \leq \lambda(x, q)$
- ii. $\mu(x*z, q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \}$
- iii. $\lambda(x*z, q) \leq \max \{ \lambda((x*y)*z, q), \lambda(y*z, q) \}$ for all $x, y, z \in X$ and $q \in Q$.

3.1 Theorem: Let $A = (X, \mu, \lambda)$ be an intuitionistic Q- fuzzy implicative Q-ideal of a Q- algebra X. If $x \leq y$ in X then,
 $\mu(x, q) \geq \mu(y, q)$ and $\lambda(x, q) \leq \lambda(y, q)$.

Proof : Let $x \leq y$ implies $x*y=0$

$$\begin{aligned} \mu(x, q) &= \mu(x*0, q) \geq \min \{ \mu((x*y)*0, q), \mu(y*0, q) \} \\ &= \min \{ \mu(0*0, q), \mu(y*0, q) \} \\ &= \min \{ \mu(0, q), \mu(y, q) \} \\ &= \mu(y, q) \quad (\text{Since } \mu(0) \geq \mu(y)) \end{aligned}$$

Therefore, $\mu(x, q) \geq \mu(y, q)$

Similarly, $\lambda(x, q) \leq \lambda(y, q)$.

3.5 Definition: Let $A = (X, \mu, \lambda)$ be an intuitionistic Q- fuzzy set in X. then

- i. $\Box A = (X, \mu, \mu^c)$ and
- ii. $\Diamond A = (X, \lambda^c, \lambda)$

3.2 Theorem: If $A = (X, \mu, \lambda)$ is an intuitionistic Q- fuzzy implicative Q- ideal of a Q- algebra X, then $\Box A = (X, \mu, \mu^c)$ is also an intuitionistic Q- fuzzy implicative Q- ideal of X.

Proof: Let A is an intuitionistic Q- fuzzy implicative Q-ideal of X. We have $\mu(0, q) \geq \mu(x, q)$
 $1 - \mu^c(0, q) \geq 1 - \mu^c(x, q)$

$$\mu^c(0, q) \leq \mu^c(x, q) \text{ For any } x \in X \text{ and } q \in Q.$$

$$\text{and } \mu(x*z, q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \}$$

$$1 - \mu^c(x*z, q) \geq \min \{ 1 - \mu^c((x*y)*z, q), 1 - \mu^c(y*z, q) \}$$

$$\begin{aligned} \mu^c(x*z, q) &\leq 1 - \min \{ 1 - \mu^c((x*y)*z, q), 1 - \mu^c(y*z, q) \} \\ &\leq \max \{ \mu^c((x*y)*z, q), \mu^c(y*z, q) \} \end{aligned}$$

Hence $\Box A = (X, \mu, \mu^c)$ is an intuitionistic Q- fuzzy implicative Q- ideal of X.

3.3 Theorem: Let $A = (X, \mu, \lambda)$ be an intuitionistic Q- fuzzy implicative Q-ideal of a Q-algebra X then $\Diamond A = (X, \lambda^c, \lambda)$ is also an intuitionistic Q- fuzzy implicative Q-ideal of X.

Proof: Let A is an intuitionistic Q- fuzzy implicative Q-ideal of X. We have
 $\lambda(0, q) \leq \lambda(x, q)$

$$1-\lambda^c(0, q) \leq 1-\lambda^c(x, q)$$

$$\lambda^c(0, q) \geq \lambda^c(x, q) \text{ for any } x \in X \text{ and } q \in Q.$$

Now for any $x, y, z \in X$ and $q \in Q$.

$$\lambda(x*z, q) \leq \max \{ \lambda((x*y)*z, q), \lambda(y*z, q) \}$$

$$1-\lambda^c(x*z, q) \leq \max \{ 1-\lambda^c((x*y)*z, q), 1-\lambda^c(y*z, q) \}$$

$$\begin{aligned} \lambda^c(x*z, q) &\geq 1-\max \{ 1-\lambda^c((x*y)*z, q), 1-\lambda^c(y*z, q) \} \\ &= \min \{ \lambda^c((x*y)*z, q), \lambda^c(y*z, q) \} \end{aligned}$$

$$\lambda^c(x*z, q) \geq \min \{ \lambda^c((x*y)*z, q), \lambda^c(y*z, q) \}$$

Hence $\diamond A = (X, \lambda^c, \lambda)$ is an intuitionistic Q-fuzzy implicative Q-ideal of X .

3.4 Theorem: Let $A = (X, \mu, \lambda)$ be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra if and only if $\square A, \diamond A$ are intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X .

Proof: It is clear.

3.6 Definition: An intuitionistic Q-fuzzy set $A = (X, \mu, \lambda)$ in a Q-algebra X is called an intuitionistic Q-fuzzy closed implicative Q-ideal of X , if it satisfies the following condition

- i. $\mu(0*x, q) \geq \mu(x, q)$ and $\lambda(0*x, q) \leq \lambda(x, q)$
- ii. $\mu(x*z, q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \}$
- iii. $\lambda(x*z, q) \leq \max \{ \lambda((x*y)*z, q), \lambda(y*z, q) \}$ for all $x, y, z \in X$ and $q \in Q$.

3.5 Theorem: If $A = (X, \mu, \lambda)$ be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X , then $\square A = (X, \mu, \mu^c)$ is also an intuitionistic Q-fuzzy closed implicative Q-ideal of X .

Proof: For any $x \in X$ and $q \in Q$,

$$\mu(0*x, q) \geq \mu(x, q)$$

$$1-\mu^c(0*x, q) \geq 1-\mu^c(x, q)$$

$$\mu^c(0*x, q) \leq \mu^c(x, q)$$

Hence $\square A = (X, \mu, \mu^c)$ is an intuitionistic Q-fuzzy closed implicative Q-ideal of X .

3.6 Theorem: If $A = (X, \mu, \lambda)$ be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X , then $\diamond A = (X, \lambda^c, \lambda)$ is also an intuitionistic Q-fuzzy closed implicative Q-ideal.

Proof: For any $x \in X$ and $q \in Q$,

$$\lambda(0*x, q) \leq \lambda(x, q)$$

$$1-\lambda^c(0*x, q) \leq 1-\lambda^c(x, q)$$

$$\lambda^c(0*x, q) \geq \lambda^c(x, q).$$

Hence $\diamond A = (X, \lambda^c, \lambda)$ is an intuitionistic Q-fuzzy closed implicative Q-ideal of X .

3.7 Theorem: $A = (X, \mu, \lambda)$ be an intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X if and only if $\square A, \diamond A$ are intuitionistic Q-fuzzy closed implicative Q-ideal of a Q-algebra X .

Proof: It is clear.

3.8 Theorem: Let $A = (X, \mu, \lambda)$ be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X if and only if the non – empty upper s-level cut $U(\mu; s)$ and the non-empty lower t-level cut $L(\lambda; t)$ are implicative Q-ideals of X , for any $s, t \in [0, 1]$.

Proof: Suppose $A=(X, \mu, \lambda)$ be an intuitionistic Q-fuzzy implicative Q-ideal of a Q-algebra X. For any $s, t \in [0, 1]$ define the sets

$$U(\mu:s) = \{(x, q) \in X \times Q / \mu(x, q) \geq s\} \text{ and } L(\lambda;t) = \{(x, q) \in X \times Q / \lambda(x, q) \leq t\}$$

Since $U(\mu:s) \neq \emptyset$, for $(x, q) \in U(\mu:s)$

$$\mu(x, q) \geq s$$

$$\mu(0, q) \geq \mu(x, q) \geq s$$

$$\mu(0, q) \geq s \text{ implies } (0, q) \in U(\mu:s)$$

Let $(x*y)*z, q) \in U(\mu:s)$ and $(y*z, q) \in U(\mu:s)$

$$\mu((x*y)*z, q) \geq s \text{ and } \mu(y*z, q) \geq s$$

$$\text{Since } \mu((x*z), q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \} \\ \geq \min \{ s, s \} = s$$

$$\mu((x*z), q) \geq s \text{ implies } (x*z, q) \in U(\mu:s)$$

Hence $U(\mu:s)$ is an implicative Q-ideal of X.

Similarly we can prove that $L(\lambda;t)$ is an implicative Q-ideal of X

Conversely,

Suppose that for any $s, t \in [0, 1]$, $U(\mu:s)$ and $L(\lambda;t)$ are implicative Q-ideals of X.

If possible, assume $x_0, y_0 \in X$ and $q_0 \in Q$ such that $\mu(0, q_0) < \mu(x_0, q_0)$ and $\lambda(0, q_0) > \lambda(y_0, q_0)$

$$\text{take } s_0 = (1/2) [\mu(0, q_0) + \mu(x_0, q_0)]$$

$$\text{That is } s_0 < \mu(x_0, q_0) \text{ and } 0 \leq \mu(0, q_0) < s_0 < 1, (x_0, q_0) \in U(\mu:s_0) \text{ and } (0, q_0) \notin U(\mu:s_0)$$

Since $U(\mu:s_0)$ is an implicative Q-ideal of X we have $(0, q_0) \in U(\mu:s_0)$ and $\mu(0, q_0) \geq s_0$

Therefore Our assumption is wrong.

Hence $\mu(0, q) \geq \mu(x, q)$ for all $x \in X$ and $q \in Q$.

$$\text{Similarly by taking } t_0 = (1/2) [\lambda(0, q_0) + \lambda(y_0, q_0)]$$

We can show that $\lambda(0, q) \leq \lambda(y, q)$ for all $y \in X$ and $q \in Q$.

ii. If possible, assume that $x_0, y_0, z_0 \in X$ and $q_0 \in Q$ and such that

$$\mu(x_0 * z_0, q_0) < \min \{ \mu((x_0 * y_0) * z_0, q_0), \mu(y_0 * z_0, q_0) \}$$

$$\text{Take } s_0 = (1/2) [\mu(x_0 * z_0, q_0) + \min \{ \mu((x_0 * y_0) * z_0, q_0), \mu(y_0 * z_0, q_0) \}]$$

$$\text{and } s_0 < \min \{ \mu((x_0 * y_0) * z_0, q_0), \mu(y_0 * z_0, q_0) \} \text{ and } s_0 > \mu(x_0 * z_0, q_0)$$

$$\text{Therefore } s_0 < \mu((x_0 * y_0) * z_0, q_0), s_0 < \mu(y_0 * z_0, q_0) \text{ and } s_0 > \mu(x_0 * z_0, q_0) (x_0 * z_0, q_0) \notin U(\mu:s_0)$$

Since $U(\mu:s_0)$ is an implicative Q-ideal of X.

$$((x_0 * y_0) * z_0, q_0) \in U(\mu:s_0), (y_0 * z_0, q_0) \in U(\mu:s_0) \text{ imply that } (x_0 * z_0, q_0) \in U(\mu:s_0)$$

Therefore Our assumption is wrong.

Hence $\mu(x*z, q) \geq \min \{ \mu((x*y)*z, q), \mu(y*z, q) \}$ for any $x, y, z \in X$ and $q \in Q$.

Similarly we can prove that

$\lambda(x*z, q) \leq \max \{ \lambda((x*y)*z, q), \lambda(y*z, q) \}$ for any $x, y, z \in X$ and $q \in Q$.

Hence A is an intuitionistic Q -fuzzy implicative Q -ideal of a Q -algebra X .

3.9 Theorem: $A=(X, \mu, \lambda)$ be an intuitionistic Q -fuzzy closed implicative Q -ideal of a Q -algebra X if and only if the non empty upper s -level cut $U(\mu; s)$ and the non-empty lower t -level cut $L(\lambda; t)$ are closed implicative Q -ideals of X , for any $s, t \in [0, 1]$.

Proof: Suppose $A=(X, \mu, \lambda)$ be an intuitionistic Q -fuzzy closed implicative Q -ideal of a Q -algebra X . $\mu(0*x, q) \geq \mu(x, q)$ and $\lambda(0*x, q) \leq \lambda(x, q)$ for any $x \in X$ and $q \in Q$.

For any $(x, q) \in U(\mu; s) \Rightarrow \mu(x, q) \geq s \Rightarrow \mu(0*x, q) \geq s \Rightarrow (0*x, q) \in U(\mu; s)$ and

$$(x, q) \in L(\lambda; t) \Rightarrow \lambda(x, q) \leq t \Rightarrow \lambda(0*x, q) \leq t \Rightarrow (0*x, q) \in L(\lambda; t)$$

Hence $U(\mu; s)$ and $L(\lambda; t)$ are closed Q -ideals of X for $s, t \in [0, 1]$.

Conversely,

$U(\mu; s)$ and $L(\lambda; t)$ are closed Q -ideals of X for $s, t \in [0, 1]$.

To show $A=(X, \mu, \lambda)$ is an intuitionistic Q -fuzzy closed Q -ideal of X it is enough to show that $\mu(0*x, q) \geq \mu(x, q)$ and

$\lambda(0*x, q) \leq \lambda(x, q)$ for any $x \in X$ and $q \in Q$.

If possible assume that $x_0 \in X$ and $q_0 \in Q$ such that $\mu(0*x_0, q_0) < \mu(x_0, q_0)$

take $s_0 = (1/2) [\mu(0*x_0, q_0) + \mu(x_0, q_0)]$

$$\Rightarrow \mu(0*x_0, q_0) < s_0 < \mu(x_0, q_0)$$

$$\Rightarrow (x_0, q_0) \in U(\mu; s_0) \text{ but } (0*x_0, q_0) \notin U(\mu; s_0)$$

which is a contradiction to the definition of closed Q -ideal.

Hence $\mu(0*x, q) \geq \mu(x, q)$ for any $x \in X$ and $q \in Q$.

Similarly we can prove that $\lambda(0*x, q) \leq \lambda(x, q)$ for any $x \in X$ and $q \in Q$.

3.7 Definition: Let f be a mapping on a set $X \times Q$ and $A=(X, \mu, \lambda)$ an intuitionistic Q -fuzzy set in X . Then the fuzzy sets u and v on $f(X \times Q)$ defined by

$$u(y, q) = \sup_{(x, q) \in f^{-1}(y, q)} \mu(x, q) \text{ and}$$

$$v(y, q) = \inf_{(x, q) \in f^{-1}(y, q)} \lambda(x, q)$$

for all $(y, q) \in f(X \times Q)$ is called the image of A under f . If u, v are fuzzy sets in $f(X \times Q)$. Then the fuzzy sets $\mu = u \circ f$ and $\lambda = v \circ f$ are called the pre-images of u and v respectively under f .

3.8 Definition: A function $f: X \times Q \rightarrow Y \times Q$ is said to be a homomorphism of Q -algebras if $f[(x, q)*(y, q)] = f(x, q) * f(y, q) = f(x*y, q)$

3.10 Theorem: Let $f: X \times Q \rightarrow Y \times Q$ be an onto homomorphism of Q -algebras. If $B=(Y, u, v)$ is an intuitionistic Q -fuzzy implicative Q -ideal of Y then the pre-image of B under f is an intuitionistic Q -fuzzy implicative Q -ideal of X .

Proof: Let $A=(X,\mu,\lambda)$ where $\mu = u \circ f$ and $\lambda = v \circ f$ is the pre-image of $B=(Y, u, v)$ under f .

Since $B=(Y, u, v)$ is an intuitionistic Q- fuzzy implicative Q-ideal of Y we have

$$u(0^1, q) \geq u(f(x, q)) = u^\circ f(x, q) = \mu(x, q) \text{ and } v(0^1, q) \leq v(f(x, q)) = v^\circ f(x, q) = \lambda(x, q)$$

on the other hand,

$$u(0^1, q) = u(f(0, q)) = u^\circ f(0, q) = \mu(0, q) \text{ and } v(0^1, q) = v(f(0, q)) = v^\circ f(0, q) = \lambda(0, q)$$

$$\mu(0, q) = u(0^1, q) \geq \mu(x, q) \text{ implies } \mu(0, q) \geq \mu(x, q)$$

$$\lambda(0, q) = v(0^1, q) \leq \lambda(x, q) \text{ implies } \lambda(0, q) \leq \lambda(x, q). \text{ for all } x, y \in X \text{ and } q \in Q.$$

$$\text{ii. Now show that } \mu(x^*z, q) \geq \min \{ \mu((x^*y)^*z, q), \mu(y^*z, q) \} \text{ and}$$

$$\lambda(x^*z, q) \leq \max \{ \lambda((x^*y)^*z, q), \lambda(y^*z, q) \}, \text{ for all } x, y, z \in X, q \in Q.$$

$$\begin{aligned} \text{We have, } \mu(x^*z, q) &= u^\circ f(x^*z, q) = u(f(x^*z, q)) \\ &= u(f(x, q)^*f(z, q)) \\ &\geq \min \{ u(f(x^*y, q)^*f(z, q)), u(f(y^*z, q)) \} \\ &= \min \{ u(f((x^*y)^*z, q)), u(f(y^*z, q)) \} \\ &= \min \{ u^\circ f((x^*y)^*z, q), u^\circ f(y^*z, q) \} \\ &= \min \{ \mu((x^*y)^*z, q), \mu(y^*z, q) \} \end{aligned}$$

Hence $\mu(x^*z, q) \geq \min \{ \mu((x^*y)^*z, q), \mu(y^*z, q) \}$ is true for all $x, y \in X$ and $q \in Q$.

Similarly, we can prove that $\lambda(x^*z, q) \leq \max \{ \lambda((x^*y)^*z, q), \lambda(y^*z, q) \}$ for all $x, y \in X$ and $q \in Q$.

Hence the pre-image $A=(X,\mu,\lambda)$ of B is an intuitionistic Q- fuzzy implicative Q-ideal of X .

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