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COMMON FIXED POINTS OF THREE SELF MAPPINGS IN COMPLEX VALUED METRIC SPACES

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ABSTRACT

We prove a common fixed point theorem for three self mappings in complex valued metric spaces. Our result generalizes some recent results in the literature due to Azam et. al.[1] and Sintunavarat et. al.[14]. Also, an example is given to illustrate our obtained result.

Keywords and phrases: *Complex valued metric space, point of coincidence, weakly compatible mappings, common fixed point.*

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1. INTRODUCTION

Banach's fixed point theorem plays a major role in fixed point theory. It has applications in many branches of mathematics. Because of its usefulness, a lot of articles have been dedicated to the improvement and generalization of that result. Most of these generalizations have been made by considering different contractive type conditions in different spaces. In 2011, Azam et. al.[1] made one such generalization by introducing a complex valued metric space. In fact, they obtained a sufficient condition for the existence of common fixed points of a pair of mappings satisfying some contractive type conditions in this setting. Very recently, Sintunavarat et. al. [14] generalized this result by replacing the constants of contraction by some control functions. The purpose of this work is to obtain a common fixed point result for three self mappings in complex valued metric spaces which generalizes the results of [1] and [14].

2. PRELIMINARIES

Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. We can define a partial ordering $\preceq$ on $\mathbb{C}$ as follows: $z_1 \preceq z_2$ if and only if $\mathrm{Re}(z_1) \le \mathrm{Re}(z_2)$ *and* $\mathrm{Im}(z_1) \le \mathrm{Im}(z_2)$.

Thus, $z_1 \preceq z_2$ if one of the following conditions is satisfied:

(i) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ *and* $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$;

(ii) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ *and* $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$;

(iii) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ *and* $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$;

(iv) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$ *and* $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$.

In particular, we will write $z_1 \underset{\neq}{\prec} z_2$ if $z_1 \neq z_2$ and one of (ii), (iii), and (iv) is satisfied and we will write $z_1 \prec z_2$ if only (iv) is satisfied. It follows that

(i) $0 \preceq z_1 \preceq z_2 \Rightarrow |z_1| \le |z_2|$;

(ii) $0 \preceq z_1 \underset{\neq}{\prec} z_2 \Rightarrow |z_1| < |z_2|$;

(iii) $z_1 \preceq z_2$ and $z_2 \prec z_3 \Rightarrow z_1 \prec z_3$;

(iv) $a, b \in R,\ 0 \le a \le b$ *and* $z_1 \preceq z_2 \Rightarrow az_1 \preceq bz_2$.

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Definition 2.1. ([1]) Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \mathbb{C}$ satisfies the following conditions:

(i) $0 \preceq d(x, y),\ for\ \ all\ \ x, y \in X\ \ and\ d(x, y) = 0\ if\ and\ only\ if\ \ x = y$;

(ii) $d(x, y) = d(y, x),\ \ for\ \ all\ \ x, y \in X$;

(iii) $d(x, y) \preceq d(x, z) + d(z, y),\ \ for\ all\ x, y, z \in X$.

Then d is called a complex valued metric on X and (X, d) is called a complex valued metric space. Note that $d(x, y) \preceq 1 + d(x, y)$ and so, $\left|\dfrac{d(x, y)}{1 + d(x, y)}\right| \le 1$.

Example 2.2. ([14]) Let $X = \mathbb{C}$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(z_1, z_2) = e^{ik}\ |z_1 - z_2|$, where $k \in R$. Then (X, d) is a complex valued metric space.

Definition 2.3. ([1]) Let (X, d) be a complex valued metric space, (x_n) be a sequence in X and $x \in X$.

(i) If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x) \prec c$, then (x_n) is said to be convergent, (x_n) converges to x and x is the limit point of (x_n). We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x\ \ as\ n \to \infty$.

(ii) If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n, x_{n+m}) \prec c$, where $m \in N$, then (x_n) is said to be Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex valued metric space.

Lemma 2.4. ([1]) Let (X, d) be a complex valued metric space and let (x_n) be a sequence in X. Then (x_n) converges to x if and only if $|d(x_n, x)| \to 0\ \ as\ n \to \infty$.

Lemma 2.5. ([1]) Let (X, d) be a complex valued metric space and let (x_n) be a sequence in X. Then (x_n) is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0\ \ as\ n \to \infty$, where $m \in N$.

Definition 2.6. ([4]) Let T and S be self mappings of a set X. If $w = Tx = Sx$ for some x in X, then x is called a coincidence point of T and S and w is called a point of coincidence of T and S.

Definition 2.7. ([7]) Let T and S be self mappings of a nonempty set X. The mappings T and S are weakly compatible if $TSx = STx$ whenever $Tx = Sx$.

Definition 2.8. A mapping $T : X \to X$ in a complex valued metric space (X, d) is said to be expansive if there is a real constant $c > 1$ satisfying

$$cd(x, y) \preceq\ d(Tx, Ty)$$

for all $x, y \in X$.

3. MAIN RESULTS

In this section, we always suppose that $\mathbb{C}$ is the set of complex numbers and $\preceq$ is a partial ordering on $\mathbb{C}$. Throughout the paper we denote by N the set of all positive integers.

Lemma 3.1. ([2]) Let X be a nonempty set and the mappings $S, T,\ f : X \to X$ have a unique point of coincidence v in X. If $(S,\ f)$ and $(T,\ f)$ are weakly compatible, then S, T and f have a unique common fixed point.

Theorem 3.2. Let (X, d) be a complex valued metric space and $f, S, T : X \to X$. Suppose there exist mappings $\wedge_1, \wedge_2 : X \to [0,1)$ such that for all $x, y \in X$:

(i) $\wedge_i(Sx) \le \wedge_i(fx)$ *and* $\wedge_i(Tx) \le \wedge_i(fx)$ *for* $i = 1, 2$;

(ii) $\wedge_1(fx) + \wedge_2(fx) < 1$;

(iii) $d(Sx, Ty) \precsim \wedge_1(fx)\, d(fx, fy) + \dfrac{\wedge_2(fx)\, d(fx, Sx)\, d(fy, Ty)}{1 + d(fx, fy)}$.

If $S(X) \cup T(X) \subseteq f(X)$ and $f(X)$ is complete, then f, S and T have a unique point of coincidence. Moreover, if (S, f) and (T, f) are weakly compatible , then f, S and T have a unique common fixed point in X .

Proof. Let $x_0 \in X$ be arbitrary. Choose a point $x_1 \in X$ such that $fx_1 = Sx_0$ which is possible since $S(X) \subseteq f(X)$. Also, we may choose a point $x_2 \in X$ satisfying $fx_2 = Tx_1$ since $T(X) \subseteq f(X)$. Continuing in this way, we can construct a sequence (fx_n) in $f(X)$ such that

$$
\begin{aligned}
fx_n &= Sx_{n-1}, \text{ if } n \text{ is odd} \\
&= Tx_{n-1}, \text{ if } n \text{ is even}.
\end{aligned}
$$

If $n \in N$ is odd, then by using hypothesis we obtain

$$
\begin{aligned}
d(fx_n, fx_{n+1}) &= d(Sx_{n-1}, Tx_n) \\
&\precsim \wedge_1(fx_{n-1})\, d(fx_{n-1}, fx_n) + \frac{\wedge_2(fx_{n-1})\, d(fx_{n-1}, Sx_{n-1})\, d(fx_n, Tx_n)}{1 + d(fx_{n-1}, fx_n)} \\
&= \wedge_1(fx_{n-1})\, d(fx_{n-1}, fx_n) + \frac{\wedge_2(fx_{n-1})\, d(fx_{n-1}, fx_n)\, d(fx_n, fx_{n+1})}{1 + d(fx_{n-1}, fx_n)}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
|d(fx_n, fx_{n+1})| &\le \wedge_1(fx_{n-1})\, |d(fx_{n-1}, fx_n)| + \wedge_2(fx_{n-1})\, |d(fx_n, fx_{n+1})| \left| \frac{d(fx_{n-1}, fx_n)}{1 + d(fx_{n-1}, fx_n)} \right| \\
&\le \wedge_1(fx_{n-1})\, |d(fx_{n-1}, fx_n)| + \wedge_2(fx_{n-1})\, |d(fx_n, fx_{n+1})| \\
&= \wedge_1(Tx_{n-2})\, |d(fx_{n-1}, fx_n)| + \wedge_2(Tx_{n-2})\, |d(fx_n, fx_{n+1})| \\
&\le \wedge_1(fx_{n-2})\, |d(fx_{n-1}, fx_n)| + \wedge_2(fx_{n-2})\, |d(fx_n, fx_{n+1})| \\
&= \wedge_1(Sx_{n-3})\, |d(fx_{n-1}, fx_n)| + \wedge_2(Sx_{n-3})\, |d(fx_n, fx_{n+1})| \\
&\le \wedge_1(fx_{n-3})\, |d(fx_{n-1}, fx_n)| + \wedge_2(fx_{n-3})\, |d(fx_n, fx_{n+1})| \\
&\vdots \\
&\le \wedge_1(fx_0)\, |d(fx_{n-1}, fx_n)| + \wedge_2(fx_0)\, |d(fx_n, fx_{n+1})|
\end{aligned}
$$

which implies that

$$
|d(fx_n, fx_{n+1})| \le \frac{\wedge_1(fx_0)}{1 - \wedge_2(fx_0)}\, |d(fx_{n-1}, fx_n)|.
$$

If $n \in N$ is even, then

$$
\begin{aligned}
d(fx_n, fx_{n+1}) &= d(Tx_{n-1}, Sx_n) = d(Sx_n, Tx_{n-1}) \\
&\precsim \wedge_1(fx_n)\, d(fx_n, fx_{n-1}) + \frac{\wedge_2(fx_n)\, d(fx_n, Sx_n)\, d(fx_{n-1}, Tx_{n-1})}{1 + d(fx_n, fx_{n-1})}
\end{aligned}
$$

$$= \wedge_1(fx_n)d(fx_n, fx_{n-1}) + \frac{\wedge_2(fx_n)d(fx_n, fx_{n+1})d(fx_{n-1}, fx_n)}{1+d(fx_n, fx_{n-1})}.$$

Therefore,

$$\begin{aligned}
\left|d(fx_n, fx_{n+1})\right| &\le \wedge_1(fx_n)\left|d(fx_n, fx_{n-1})\right| + \wedge_2(fx_n)\left|d(fx_n, fx_{n+1})\right| \left|\frac{d(fx_{n-1}, fx_n)}{1+d(fx_n, fx_{n-1})}\right| \\
&\le \wedge_1(fx_n)\left|d(fx_n, fx_{n-1})\right| + \wedge_2(fx_n)\left|d(fx_n, fx_{n+1})\right| \\
&= \wedge_1(Tx_{n-1})\left|d(fx_n, fx_{n-1})\right| + \wedge_2(Tx_{n-1})\left|d(fx_n, fx_{n+1})\right| \\
&\le \wedge_1(fx_{n-1})\left|d(fx_n, fx_{n-1})\right| + \wedge_2(fx_{n-1})\left|d(fx_n, fx_{n+1})\right| \\
&= \wedge_1(Sx_{n-2})\left|d(fx_n, fx_{n-1})\right| + \wedge_2(Sx_{n-2})\left|d(fx_n, fx_{n+1})\right| \\
&\;\vdots \\
&\le \wedge_1(fx_0)\left|d(fx_n, fx_{n-1})\right| + \wedge_2(fx_0)\left|d(fx_n, fx_{n+1})\right|
\end{aligned}$$

which gives that

$$\left|d(fx_n, fx_{n+1})\right| \le \frac{\wedge_1(fx_0)}{1-\wedge_2(fx_0)}\left|d(fx_n, fx_{n-1})\right|.$$

Thus for any positive integer n, it must be the case that

$$\left|d(fx_n, fx_{n+1})\right| \le \frac{\wedge_1(fx_0)}{1-\wedge_2(fx_0)}\left|d(fx_{n-1}, fx_n)\right|. \tag{3.1}$$

If we let $\alpha := \dfrac{\wedge_1(fx_0)}{1-\wedge_2(fx_0)}$, then by repeated application of (3.1)

$$\begin{aligned}
\left|d(fx_n, fx_{n+1})\right| &\le \alpha\left|d(fx_{n-1}, fx_n)\right| \\
&\le \alpha^2\left|d(fx_{n-2}, fx_{n-1})\right| \\
&\;\vdots \\
&\le \alpha^n\left|d(fx_0, fx_1)\right|.
\end{aligned}$$

Now, for all $m, n \in N, m > n$, we have

$$d(fx_n, fx_m) \precsim d(fx_n, fx_{n+1}) + d(fx_{n+1}, fx_{n+2}) + \cdots + d(fx_{m-1}, fx_m).$$

Therefore,

$$\begin{aligned}
\left|d(fx_n, fx_m)\right| &\le \left|d(fx_n, fx_{n+1})\right| + \left|d(fx_{n+1}, fx_{n+2})\right| + \cdots + \left|d(fx_{m-1}, fx_m)\right| \\
&\le \left(\alpha^n + \alpha^{n+1} + \cdots + \alpha^{m-1}\right)\left|d(fx_0, fx_1)\right| \\
&\le \frac{\alpha^n}{1-\alpha}\left|d(fx_0, fx_1)\right|.
\end{aligned}$$

Since $\alpha \in [0,1)$, taking limit as $m, n \to \infty$, we have $\left|d(fx_n, fx_m)\right| \to 0$ which implies that (fx_n) is a Cauchy sequence in $f(X)$. By completeness of $f(X)$, there exist $u, v \in X$ such that $fx_n \to v = fu$.

Now,

$$\begin{aligned}
d(fu, Tu) &\precsim d(fu, fx_{2n+1}) + d(fx_{2n+1}, Tu) \\
&= d(fu, fx_{2n+1}) + d(Sx_{2n}, Tu) \\
&\precsim d(fu, fx_{2n+1}) + \wedge_1(fx_{2n})\, d(fx_{2n}, fu) + \frac{\wedge_2(fx_{2n})\, d(fx_{2n}, Sx_{2n})\, d(fu, Tu)}{1+d(fx_{2n}, fu)}
\end{aligned}$$

which implies that

$$\begin{aligned}
|d(fu,Tu)| &\le |d(fu, fx_{2n+1})| + \wedge_1(fx_{2n})\,|d(fx_{2n}, fu)| + \frac{\wedge_2(fx_{2n})\,|d(fx_{2n}, Sx_{2n})|\,|d(fu,Tu)|}{|1+d(fx_{2n}, fu)|} \\
&\le |d(fu, fx_{2n+1})| + \wedge_1(fx_{2n})\,|d(fx_{2n}, fu)| + \wedge_2(fx_{2n})\,|d(fx_{2n}, Sx_{2n})|\,|d(fu,Tu)|, \\
&\qquad\qquad \text{since } 1 \preceq 1 + d(fx_{2n}, fu). \\
&\le |d(fu, fx_{2n+1})| + \wedge_1(fx_0)\,|d(fx_{2n}, fu)| + \wedge_2(fx_0)\,|d(fx_{2n}, fx_{2n+1})|\,|d(fu,Tu)|.
\end{aligned}$$

Taking $n \to \infty$, it follows that $|d(fu,Tu)| = 0$ and hence $d(fu,Tu) = 0$. Therefore, $fu = Tu = v$. Similarly, we can show that $fu = Su = v$.

Thus, $fu = Su = Tu = v$ and so v becomes a common point of coincidence of f, S and T.

For uniqueness, let there exists another point $w(\ne v) \in X$ such that $fx = Sx = Tx = w$ for some $x \in X$. Thus,

$$\begin{aligned}
d(v,w) &= d(Su,Tx) \\
&\preceq \wedge_1(fu)\, d(fu, fx) + \frac{\wedge_2(fu)\, d(fu,Su)\, d(fx,Tx)}{1+d(fu, fx)} \\
&= \wedge_1(v)\, d(v,w) + \frac{\wedge_2(v)\, d(v,v)\, d(w,w)}{1+d(v,w)} \\
&= \wedge_1(v)\, d(v,w)
\end{aligned}$$

which implies that

$|d(v,w)| \le \wedge_1(v)\,|d(v,w)|$.

Since $0 \le \wedge_1(v) < 1$, it follows that $|d(v,w)| = 0$ and so $v = w$. If (S, f) and (T, f) are weakly compatible, then by Lemma 3.1, f, S and T have a unique common fixed point in X.

As an application of Theorem 3.2, we have the following results.

Corollary 3.3. [[14], Theorem 3.1] Let (X,d) be a complete complex valued metric space and $S,T : X \to X$. Suppose there exist mappings $\wedge_1, \wedge_2 : X \to [0,1)$ such that for all $x, y \in X$:

(i) $\wedge_i(Sx) \le \wedge_i(x)$ *and* $\wedge_i(Tx) \le \wedge_i(x)$ *for* $i = 1, 2$;

(ii) $\wedge_1(x) + \wedge_2(x) < 1$;

(iii) $d(Sx,Ty) \preceq \wedge_1(x)\, d(x,y) + \dfrac{\wedge_2(x)\, d(x,Sx)\, d(y,Ty)}{1+d(x,y)}$.

Then S and T have a unique common fixed point in X.

Proof. The result follows from Theorem 3.2 by taking $f = I$, the identity mapping.

Corollary 3.4. [[1], Theorem 4] Let (X,d) be a complete complex valued metric space and $S,T : X \to X$. If S and T satisfy

$$d(Sx,Ty) \preceq \lambda d(x,y) + \frac{\mu\, d(x,Sx)\, d(y,Ty)}{1+d(x,y)}$$

for all $x, y \in X$, where λ, μ are nonnegative reals with $\lambda + \mu < 1$, then S and T have a unique common fixed point.

Proof. The desired result can be obtained from Theorem 3.2 by setting $\wedge_1(x) = \lambda, \wedge_2(x) = \mu \text{ and } f = I$.

Corollary 3.5. [[14], Theorem 3.7] Let (X, d) be a complex valued metric space, $f, T : X \to X$ be such that $T(X) \subseteq f(X)$ and $f(X)$ is complete. Suppose there exist mappings $\wedge_1, \wedge_2 : X \to [0,1)$ such that for all $x, y \in X$:

(i) $\wedge_i(Tx) \le \wedge_i(fx)$ $\quad for\ i = 1, 2$;

(ii) $\wedge_1(fx) + \wedge_2(fx) < 1$;

(iii) $d(Tx,Ty) \precsim \wedge_1(fx)\, d(fx, fy) + \dfrac{\wedge_2(fx)\, d(fx,Tx)\, d(fy,Ty)}{1 + d(fx, fy)}$.

Then f and T have a unique point of coincidence. Moreover, if f and T are weakly compatible, then f and T have a unique common fixed point in X .

Proof. The conclusion of the Corollary follows from Theorem 3.2 by considering $S = T$.

Corollary 3.6. Let (X, d) be a complex valued metric space and let $f, T : X \to X$ satisfy

$$d(Tx,Ty) \precsim \lambda\, d(fx, fy) + \frac{\mu\, d(fx,Tx)\, d(fy,Ty)}{1 + d(fx, fy)}$$

for all $x, y \in X$, where λ, μ are nonnegative reals with $\lambda + \mu < 1$. If $T(X) \subseteq f(X)$ and $f(X)$ is complete, then f and T have a unique point of coincidence. Moreover, if f and T are weakly compatible, then f and T have a unique common fixed point in X .

Proof. Putting $S = T, \wedge_1(x) = \lambda, \wedge_2(x) = \mu$ in Theorem 3.2, we can prove this result.

Corollary 3.7. Let (X, d) be a complete complex valued metric space and $T : X \to X$. Suppose there exist mappings $\wedge_1, \wedge_2 : X \to [0,1)$ such that for all $x, y \in X$:

(i) $\wedge_i(Tx) \le \wedge_i(x)$ $\quad for\ i = 1, 2$;

(ii) $\wedge_1(x) + \wedge_2(x) < 1$;

(iii) $d(Tx,Ty) \precsim \wedge_1(x)\, d(x, y) + \dfrac{\wedge_2(x)\, d(x,Tx)\, d(y,Ty)}{1 + d(x, y)}$.

Then T has a unique fixed point in X .

Proof. The conclusion of the Corollary follows from Theorem 3.2 by considering $S = T$ and $f = I$.

Theorem 3.8. Let (X, d) be a complete complex valued metric space and let $f : X \to X$ be an onto expansive mapping i.e., $f(X) = X$ and there exists a real constant $c > 1$ such that

$$c\, d(x, y) \precsim d(fx, fy)$$

for all $x, y \in X$. Then f has a unique fixed point in X .

Proof. We can prove this result by applying Corollary 3.6 with $T = I, \text{ and } \mu = 0$.
We conclude with an example.

Example 3.9. Let $X = [1, \infty)$. Define $T, f : X \to X$ by $Tx = 2x - 1$ and $fx = 5x - 4$. If d_u is the usual metric on X , then T and f are not contraction mappings as for all $x, y \in X$

$$d_u(Tx,Ty) = 2\,|x - y|$$

and

$$d_u(fx, fy) = 5\,|x-y|.$$

So, we can not apply Banach contraction theorem to find the unique fixed point 1 of T and f.

We consider a complex valued metric $d : X \times X \to \mathbb{C}$ by

$$d(x,y) = |x-y| + i\,|x-y|, \; for\ all\ x, y \in X.$$

Then (X,d) is a complete complex valued metric space.

Now,

$$\begin{aligned} d(Tx,Ty) &= 2\left[\,|x-y| + i\,|x-y|\,\right] \\ &= \frac{2}{5}\, d(fx, fy) \\ &\preceq \frac{1}{2}\, d(fx, fy). \end{aligned}$$

Since $T(X) = f(X) = X$, we have all the conditions of Corollary 3.6 with $\lambda = \dfrac{1}{2}, \mu = 0$. So, applying Corollary 3.6 we can obtain a unique common fixed point 1 of T and f in X.

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