

$(\tau_1, \tau_2)^* - Q^*$ CONTINUOUS MAPS IN BITOPOLOGICAL SPACES

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ABSTRACT

We introduce a new type of functions called $(\tau_1, \tau_2)^* - Q^*$ continuous maps, $(\tau_1, \tau_2)^* - Q^*$ irresolute and $(\tau_1, \tau_2)^* - Q^*$ Contra continuous map. We obtain several characterization of this functions and study its bitopological properties.

Keywords: $(\tau_1, \tau_2)^* - Q^*$ continuous map, $(\tau_1, \tau_2)^* - Q^*$ irresolute map, $(\tau_1, \tau_2)^* - Q^*$ Contra continuous map.

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1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non-empty set and τ_1, τ_2 are topologies on X is called a bitopological space and Kelly [11] initiated the study of such spaces. The notion of Q^* -closed sets in a topological space was introduced by Murugalingam and Lalitha[9] in 2010.

Levine [12] introduced the concept of generalized closed sets in topological spaces. Also he introduced the notion of semi open sets in topological spaces. Bhattacharyya and Lahiri[1] introduced a class of sets called semi generalized closed sets by means of semi open sets of Levine and obtained various topological properties .

Maheshwari and Prasad [15] introduced semi open sets in bitopological spaces in 1977 and further properties of this notion were studied by Bose in 1981.

In 1985, Fukutake[9] introduced the concepts of g - closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces .Also he defined one kind of semi open sets in bitopological spaces and studied their properties in 1989.

Sundaram et al. introduced and studied the concept of a class of maps, namely g - continuous maps. Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by F.H. Khedr and H.S. Alsadi.

Recently P. Padma and S. Udayakumar[10] introduced the concept of $(\tau_1, \tau_2)^* - Q^*$ closed sets in bitopological spaces.

In the present paper, we introduced $(\tau_1, \tau_2)^* - Q^*$ continuous maps, $(\tau_1, \tau_2)^* - Q^*$ irresolute map, $(\tau_1, \tau_2)^* - Q^*$ Contra continuous map. We obtain several characterization of this functions and study its bitopological properties.

2. PRELIMINARIES

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) . For a subset A of X , $\tau_i - cl(A)$, $\tau_i - Q^*cl(A)$ (resp. $\tau_i - int(A)$, $\tau_i - Q^*int(A)$) represents closure of A and Q^* closure of A (resp. interior of A , Q^* -interior of A) with respect to the topology τ_i . Now we shall require the following known definitions are prerequisites.

Definition 2.1 - A subset A of a bitopological spaces (X, τ_1, τ_2) is called

- i) $(\tau_1, \tau_2)^* - Q^*$ closed if $\tau_1\tau_2 - int(A) = \phi$ and A is $\tau_1\tau_2 - closed$.
- ii) $(\tau_1, \tau_2)^* - Q^*$ open if $X - A$ is $(\tau_1, \tau_2)^* - Q^*$ closed in X .

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Example 2.1 - Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$, $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then $\tau_1 \tau_2$ - open sets on X are $\emptyset, X, \{b, c\}, \{c\}$ and $\tau_1 \tau_2$ - closed sets on X are $\emptyset, X, \{a\}, \{a, b\}$. Clearly $\emptyset, \{a, b\}$ and $\{a\}$ are $(\tau_1, \tau_2)^* - Q^*$ closed and $X, \{c\}$ and $\{b, c\}$ are $(\tau_1, \tau_2)^* - Q^*$ open.

Definition 2.2 - Let (X, τ_1, τ_2) be a bitopological spaces. Let $A \subset X$. The intersection of all $(\tau_1, \tau_2)^* - Q^*$ closed sets of X containing a subset A of X is called **$(\tau_1, \tau_2)^* - Q^*$ closure of A** and is denoted by $(\tau_1, \tau_2)^* - Q^* \text{cl}(A)$.

Definition 2.3 - Let (X, τ_1, τ_2) be a bitopological spaces. Let $A \subset X$. The union of all $(\tau_1, \tau_2)^* - Q^*$ open sets contained in a subset A of X is called **$(\tau_1, \tau_2)^* - Q^*$ interior of A** and is denoted by $(\tau_1, \tau_2)^* - Q^* \text{int}(A)$.

3. PROPERTIES OF $(\tau_1, \tau_2)^* - Q^*$ CLOSED SETS

The family of all $(\tau_1, \tau_2)^* - Q^*$ closed subsets of a bitopological space (X, τ_1, τ_2) is denoted by $(\tau_1, \tau_2)^* - Q^*$.

Theorem 3.1. Let (X, τ_1, τ_2) be a bitopological spaces. The set of all $(\tau_1, \tau_2)^* - Q^*$ closed sets with X is a topology.

Proof: It follows from example 2.1

Lemma 3.1. For any subset S of X , $(\tau_1, \tau_2)^* - Q^* \text{int}[(\tau_1, \tau_2)^* - Q^* \text{cl}(S) - S] = \emptyset$.

Proof. The proof is obvious.

Proposition 3.1. Every $(\tau_1, \tau_2)^* - Q^*$ closed set is $\tau_1 \tau_2$ - closed.

Proof: Let A be a $(\tau_1, \tau_2)^* - Q^*$ closed set in X .

Then $X - A$ is $(\tau_1, \tau_2)^* - Q^*$ open.

We have to show that A is $(\tau_1, \tau_2)^* - Q^*$ closed.

Since every $(\tau_1, \tau_2)^* - Q^*$ - open set is $\tau_1 \tau_2$ - open, we have $X - A$ is $\tau_1 \tau_2$ - open.

Thus,

A is $\tau_1 \tau_2$ - closed.

Remark 3.1. The converse is not true in general.

i.e) $\tau_1 \tau_2$ - closed need not be a $(\tau_1, \tau_2)^* - Q^*$ closed.

Definition 3.1. Let (X, τ_1, τ_2) be a bitopological spaces. **$(\tau_1, \tau_2)^* - \text{contra } Q^* \text{cl}(A)$** is defined by the intersection of all $(\tau_1, \tau_2)^* - Q^*$ open sets containing A .

Theorem 3.2. Let (X, τ_1, τ_2) be a bitopological spaces. Then $(\tau_1, \tau_2)^* - \text{contra cl}(A) \neq (\tau_1, \tau_2)^* - \text{contra } Q^* \text{cl}(A)$.

Proof: The following example supports our claim.

Consider the example

Let $X = \{a, b, c\}$,

Let $\tau_1 = \{\emptyset, X, \{a, b\}, \{c\}\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$.

Let $A = \{a, b\}$

Then

$(\tau_1, \tau_2)^* - \text{contra cl}(A) = \{a, b\}$

$(\tau_1, \tau_2)^* - \text{contra } Q^* \text{cl}(A) = X$

Therefore

$$(\tau_1, \tau_2)^* \text{- contra cl } (A) \neq (\tau_1, \tau_2)^* \text{- contra } Q^* \text{ cl } (A).$$

Theorem 3.3. If every $(\tau_1, \tau_2)^* \text{- } Q^*$ open set is $(\tau_1, \tau_2)^* \text{- } Q^*$ closed then $(\tau_1, \tau_2)^* \text{- } Q^* \text{ cl } (A) = (\tau_1, \tau_2)^* \text{- contra } Q^* \text{ cl } (A)$.

Proof: Proofs follows from the definition

Example 3.1. Let $X = \{a, b, c\}$ and $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{a\}\}$.

Let $A = \{b, c\}$.

Then

$$(\tau_1, \tau_2)^* \text{- } Q^* \text{ cl } (A) = \{b, c\}$$

$$(\tau_1, \tau_2)^* \text{- contra } Q^* \text{ cl } (A) = \{b, c\}.$$

Therefore,

$$(\tau_1, \tau_2)^* \text{- } Q^* \text{ cl } (A) = (\tau_1, \tau_2)^* \text{- contra } Q^* \text{ cl } (A).$$

Remark 3.2. Intersection of two $(\tau_1, \tau_2)^* \text{- } Q^*$ closed sets are $(\tau_1, \tau_2)^* \text{- } Q^*$ closed . The following example supports our claim.

Example 3.2. In example 2.1, $\{a, b\}$, $\{a\}$ are $(\tau_1, \tau_2)^* \text{- } Q^*$ closed set. $\{a, b\} \cap \{a\} = \{a\}$ are $(\tau_1, \tau_2)^* \text{- } Q^*$ closed set.

4. $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous Map

In this section we study the continuous maps by using $(\tau_1, \tau_2)^* \text{- } Q^*$ closed sets.

Definition 4.1. A map $f: X \rightarrow Y$ is called $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous if the inverse image of each $(\sigma_1, \sigma_2)^* \text{- } Q^*$ closed in Y is $\tau_1 \tau_2$ - closed in X .

Example 4.1. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{b\}, \{b, c\}\}$ and $\sigma_1 = \{\phi, Y, \{b\}\}$, $\sigma_2 = \{\phi, Y, \{b\}, \{b, c\}\}$. Then $\phi, \{a\}, \{a, c\}$ are $(\sigma_1, \sigma_2)^* \text{- } Q^*$ closed in Y . Let $f: X \rightarrow Y$ be the identity map. Then $f(\phi) = \phi$, $f(\{a, c\}) = \{a, c\}$, $f(\{a\}) = \{a\}$. Since $\phi, \{a, c\}, \{a\}$ are $\tau_1 \tau_2$ - closed in X . Therefore, f is $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous .

Theorem 4.1. Every $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous map is $(\tau_1, \tau_2)^* \text{- continuous}$.

Proof: Let $f: X \rightarrow Y$ be the $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous.

We shall show that f is $(\tau_1, \tau_2)^* \text{- continuous}$.

Let U be any $(\tau_1, \tau_2)^* \text{- closed}$ set in Y .

Since f is $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous, we have

$$f^{-1}(U) \text{ is } (\tau_1, \tau_2)^* \text{- closed set in } X.$$

Since every $(\tau_1, \tau_2)^* \text{- } Q^*$ closed set is $(\tau_1, \tau_2)^* \text{- closed}$.

Then

$$f^{-1}(U) \text{ is } (\tau_1, \tau_2)^* \text{- closed set in } X.$$

Therefore,

$$f \text{ is } (\tau_1, \tau_2)^* \text{- continuous}.$$

Remark 4.1- The converse of the above theorem is not true in general. The following example supports our claim.

Example 4.2- Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$ and $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y\}$. Then $\phi, \{b, c\}$ are $(\sigma_1, \sigma_2)^* \text{- } Q^*$ closed in Y . Let $f: X \rightarrow Y$ be the identity map. Clearly, f is $(\tau_1, \tau_2)^* \text{- continuous}$ map but not $(\tau_1, \tau_2)^* \text{- } Q^*$ continuous since $f^{-1}(\{a, b\}) = \{a, b\}$ is not $(\tau_1, \tau_2)^* \text{- } Q^*$ open.

Remark 4.2. Since every $(\tau_1, \tau_2)^* - Q^*$ closed is $\tau_1 \tau_2$ - closed and $\tau_1 \tau_2$ - closed set is $(\tau_1, \tau_2)^* - g$ closed, $(\tau_1, \tau_2)^* - sg$ closed, $(\tau_1, \tau_2)^* - semi$ closed, we have every $(\tau_1, \tau_2)^* - Q^*$ continuous map is $(\tau_1, \tau_2)^* - g$ continuous, $(\tau_1, \tau_2)^* - sg$ continuous and $(\tau_1, \tau_2)^* - semi$ continuous. But none of the above is reversible. The following example supports our claim.

Example 4.3.

i) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$, $\tau_2 = \{\emptyset, X\}$ and $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be the identity map. Therefore, f is $(\tau_1, \tau_2)^* - sg$ continuous but not $(\tau_1, \tau_2)^* - Q^*$ continuous.

ii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, c\}, \{a\}, \{c\}\}$, $\tau_2 = \{\emptyset, X, \{a\}\}$ and $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y\}$. Let $f: X \rightarrow Y$ be the identity map. Clearly, f is $(\tau_1, \tau_2)^* - g$ continuous but not $(\tau_1, \tau_2)^* - Q^*$ continuous.

iii) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$, $\tau_2 = \{\emptyset, X\}$ and $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{a, b\}\}$. Let $f: X \rightarrow Y$ be the identity map. Therefore, f is $(\tau_1, \tau_2)^* - semi$ continuous but not $(\tau_1, \tau_2)^* - Q^*$ continuous.

Definition 4.2. A map $f: X \rightarrow Y$ is called $(\tau_1, \tau_2)^* - Q^*$ irresolute if the inverse image of each $(\sigma_1, \sigma_2)^* - Q^*$ closed set in Y is $(\tau_1, \tau_2)^* - Q^*$ closed set in X .

Example 4.4. In Example 4.1 $\phi, \{a\} \& \{a, c\}$ are $(\sigma_1, \sigma_2)^* - Q^*$ closed set in Y and $\phi, \{a\}, \{a, c\}$ are $(\tau_1, \tau_2)^* - Q^*$ closed set in X . Let $f: X \rightarrow Y$ be the identity map. Then $f(\phi) = \phi$, $f(\{a, c\}) = \{a, c\}$, $f(\{a\}) = \{a\}$. Therefore, f is $\tau_1 \tau_2 - Q^*$ irresolute Map.

Proposition 4.1. Every $(\tau_1, \tau_2)^* - Q^*$ irresolute Map is $(\tau_1, \tau_2)^* - Q^*$ continuous.

Proof: Let $f: X \rightarrow Y$ be the $(\tau_1, \tau_2)^* - Q^*$ irresolute.

We shall show that f is $(\tau_1, \tau_2)^* - Q^*$ continuous.

Let v be any $(\tau_1, \tau_2)^* - Q^*$ closed set in Y .

Since f is $(\tau_1, \tau_2)^* - Q^*$ irresolute, we have

$f^{-1}(v)$ is $(\tau_1, \tau_2)^* - Q^*$ closed set in X .

Since every $(\tau_1, \tau_2)^* - Q^*$ closed set is $\tau_1 \tau_2$ - closed.

Then

$f^{-1}(v)$ is $\tau_1 \tau_2$ - closed set in X .

Therefore,

f is $(\tau_1, \tau_2)^* - Q^*$ continuous.

Remark 4.4 - The converse is true in general

ie) Every $(\tau_1, \tau_2)^* - Q^*$ continuous Map is not $(\tau_1, \tau_2)^* - Q^*$ irresolute.

Example 4.5 - Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}\}$. Then $\tau_1 \tau_2$ - open sets on X are $\phi, X, \{a\}, \{c\}, \{a, c\}$ and $\tau_1 \tau_2$ - closed sets on X are $\phi, X, \{b, c\}, \{a, b\}, \{b\}$. Clearly $\{b\}$ is $(\tau_1, \tau_2)^* - Q^*$ closed in X .

Let $\sigma_1 = \{\emptyset, Y, \{a\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. Then $\sigma_1 \sigma_2$ - open sets on X are $\phi, Y, \{a\}$ and $\sigma_1 \sigma_2$ - closed sets on X are $\phi, X, \{b, c\}$. Clearly $\{b, c\}$ is $(\sigma_1, \sigma_2)^* - Q^*$ closed in Y .

Let $f: X \rightarrow Y$ be the identity map.

Let $g: Y \rightarrow Z$ be the identity map.

Clearly f is $(\tau_1, \tau_2)^* - Q^*$ continuous but not $(\tau_1, \tau_2)^* - Q^*$ irresolute.

Since the inverse image of $(\sigma_1, \sigma_2)^* - Q^*$ closed set $\{b, c\}$ in Y is not $(\tau_1, \tau_2)^* - Q^*$ closed in X .

Remark 4.5. A map $f: X \rightarrow Y$ is $(\tau_1, \tau_2)^* - Q^*$ irresolute if and only if the inverse image of every $(\sigma_1, \sigma_2)^* - Q^*$ open in Y is $(\tau_1, \tau_2)^* - Q^*$ open in X .

Remark 4.6. The composition of two $(\tau_1, \tau_2)^* - Q^*$ continuous map is not, in general, $(\tau_1, \tau_2)^* - Q^*$ continuous map as is illustrated in the following example.

Example 4.6. Let $X = Y = Z = \{a, b, c\}$ and let $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, X, \{a\}\}$.

Then $\tau_1\tau_2$ - open sets on X are $\phi, X, \{a\}$ and $\tau_1\tau_2$ - closed sets on X are $\phi, X, \{b, c\}$.

Let $\sigma_1 = \{\phi, Y, \{a\}\}$ and $\sigma_2 = \{\phi, Y\}$ Then $\sigma_1\sigma_2$ - open sets on Y are $\phi, Y, \{a\}$ and $\sigma_1\sigma_2$ - closed sets on Y are $\phi, Y, \{b, c\}$.

Let $U_1 = \{\phi, Z, \{a, c\}\}$ and $U_2 = \{\phi, Z\}$. Then U_1U_2 - open sets on Z are $\phi, Z, \{a, c\}$ and U_1U_2 - closed sets on Z are $\phi, Z, \{b\}$.

Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be the identity map.

Clearly f is $(\tau_1, \tau_2)^* - Q^*$ continuous map and g is $(\sigma_1, \sigma_2)^* - Q^*$ continuous map. But $g \circ f$ is not $(\tau_1, \tau_2)^* - Q^*$ continuous map. Since $f^{-1}(g^{-1}(\{b\})) = f^{-1}(\{b\}) = \{b\}$ is not $(\tau_1, \tau_2)^* - Q^*$ closed $[\tau_1\tau_2 - \text{cl}(\{b\}) = X]$.

Proposition 4.2. For any $(\tau_1, \tau_2)^* - Q^*$ irresolute map $f: X \rightarrow Y$ and any $(\tau_1, \tau_2)^* - Q^*$ continuous map $g: Y \rightarrow Z$ the composition $g \circ f: X \rightarrow Z$ is $(\tau_1, \tau_2)^* - Q^*$ continuous map.

Proof: Let V be any $(\sigma_1, \sigma_2)^* - Q^*$ closed set in Z .

Since g is $(\tau_1, \tau_2)^* - Q^*$ continuous, we have

$g^{-1}(V)$ is $\tau_1\tau_2$ - closed set in Y .

Since f is $\tau_1\tau_2 - Q^*$ - irresolute, we have

$f^{-1}[g^{-1}(V)]$ is $\tau_1\tau_2$ - closed set in X

Thus,

$g \circ f: X \rightarrow Z$ is $(\tau_1, \tau_2)^* - Q^*$ continuous map.

Definition 4.3. A bijection $f: X \rightarrow Y$ is called $(\tau_1, \tau_2)^* - Q^*$ homeomorphism, if f is $(\tau_1, \tau_2)^* - Q^*$ continuous and its inverse also $(\tau_1, \tau_2)^* - Q^*$ continuous.

Definition 4.4. A space X is called $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space, if every $(\tau_1, \tau_2)^* - Q^*$ closed set is $\tau_1\tau_2$ - closed.

Proposition 4.3. A space X is called $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space, iff every $(\tau_1, \tau_2)^* - Q^*$ closed set is $\tau_1\tau_2$ - closed.

Proposition 4.4. If (X, τ_1, τ_2) is $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space, then it is an $\tau_1\tau_2 - Q^* T_{\frac{1}{2}}$ space.

Proof: Let (X, τ_1, τ_2) be a $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space.

Claim: $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space is an $\tau_1\tau_2 - Q^* T_{\frac{1}{2}}$ space.

i.e) to prove every $\tau_1\tau_2 - Q^*$ closed set is τ_2 - closed.

Let A be $\tau_1\tau_2 - Q^*$ closed set in X .

$\Rightarrow A$ is $\tau_1\tau_2$ - closed. [Since, X is $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space]

$\Rightarrow A$ is τ_2 - closed.

Thus, every $\tau_1\tau_2 - Q^*$ closed set is τ_2 - closed.

Therefore, (X, τ_1, τ_2) is an $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space.

Remark 4.7. The converse of the above proposition is not true as shown in the following example .

Example 4.7. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a\}, \{b, c\}\}$. Then \emptyset and $\{b, c\}$ are $\tau_1\tau_2 - Q^*$ closed set. Clearly $\{b, c\}$ is not $(\tau_1, \tau_2)^* - Q^*$ closed.

Therefore, (X, τ_1, τ_2) is an $\tau_1\tau_2 - Q^* T_{\frac{1}{2}}$ space, but it is not $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space.

Example 4.8. Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}, \{b, c\}\}$. Then $\tau_1\tau_2 -$ open sets are $\emptyset, X, \{b, c\}, \{c\}$ and $\tau_1\tau_2 -$ closed sets are $\emptyset, X, \{a, b\}, \{a\}$. Then $\emptyset, \{a, b\}, \{a\}$ are $(\tau_1, \tau_2)^* - Q^*$ closed set. Therefore, X is $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space.

Definition 4.5. A bitopological space (X, τ_1, τ_2) is said to be **Strongly $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ space** if it is both $(\tau_1, \tau_2)^* - Q^* T_{\frac{1}{2}}$ and $(\tau_2, \tau_1)^* - Q^* T_{\frac{1}{2}}$.

Definition 4.6. A map $f: X \rightarrow Y$ is called **$(\tau_1, \tau_2)^* - Q^*$ Contra continuous** if the inverse image of each $(\sigma_1, \sigma_2)^* - Q^*$ closed in Y is $\tau_1\tau_2 -$ open in X .

Example 4.9. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$, $\tau_2 = \{\emptyset, X, \{b\}, \{b, c\}\}$ and $\sigma_1 = \{\emptyset, Y, \{a\}\}$, $\sigma_2 = \{\emptyset, Y, \{a\}, \{a, c\}\}$. Then $\emptyset, X, \{a, c\}$ are $\tau_1\tau_2 -$ open in X and $\emptyset, \{b, c\}, \{b\}$ are $(\sigma_1, \sigma_2)^* - Q^*$ closed in Y . Let $f: X \rightarrow Y$ be the identity map.

Then $f(\emptyset) = \emptyset$, $f(\{b, c\}) = \{b, c\}$, $f(\{b\}) = \{b\}$.

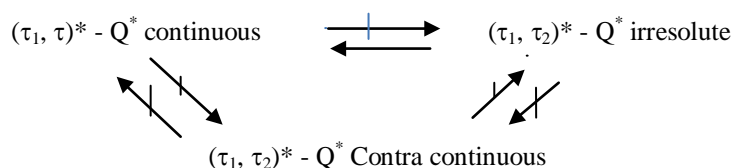
$\Rightarrow f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{b, c\}) = \{b, c\}$ and $f^{-1}(\{b\}) = \{b\}$.

Since $\emptyset, \{b\}$ & $\{b, c\}$ are $\tau_1\tau_2 -$ open in X . Therefore, f is $(\tau_1, \tau_2)^* - Q^*$ Contracontinuous.

Remark 4.8. A $(\tau_1, \tau_2)^* - Q^*$ Contra continuous map need not be $(\tau_1, \tau_2)^* - Q^*$ continuous. The following example supports our claim.

Example 4.10. Refer example 4.4, clearly $A = \{b, c\}$ is not $(\tau_1, \tau_2)^* -$ closed. Therefore, f is $(\tau_1, \tau_2)^* - Q^*$ Contra continuous map but not $(\tau_1, \tau_2)^* - Q^*$ continuous.

Remark 4.9. From the above maps, we have the following diagram of implications



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