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SORET EFFECT ON DOUBLE DIFFUSIVE CONVECTION IN A SPARSELY PACKED ROTATING ANISOTROPIC POROUS LAYER

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ABSTRACT

The Soret effect on the onset of double-diffusive convection in a sparsely packed rotating anisotropic porous layer is investigated analytically using linear and nonlinear stability theories. Linear theory is based on the normal mode technique. The Brinkman model that includes the Coriolis term is employed to describe flow through porous media. The expressions of Rayleigh number for stationary and oscillatory modes along with a dispersion relation for frequency of oscillation are obtained analytically using linear theory. The effect of anisotropy parameters, Taylor number, Soret parameter, Darcy number, solute Rayleigh number, Lewis number, Darcy Prandtl number and normalized porosity on the stationary and oscillatory convection is shown graphically. The nonlinear theory is based on the truncated representation of Fourier series method. The domain of nonlinear double diffusive convection ensures the quantification of heat and mass transfer. The effect of various parameters on heat and mass transfer is presented graphically. Some existing results are reproduced as the particular cases of present study.

Keywords*: Double diffusive convection. Rotation. Brinkman model. Anisotropy. Heat mass transfer. Soret effect.*

Nomenclature

a	Wavenumber
c	Specific heat of solid
c_p	Specific heat of fluid at constant pressure
d	Height of the porous layer
D_1	Soret coefficient
Da	Darcy number (modified), $\mu_e K_Z / \mu_f \, d^2$
$\mathbf{g}$	Gravitational acceleration, $(0, 0, -g)$
$\mathbf{K}$	Inverse anisotropic permeability tensor, $K_x^{-1}\mathbf{ii} + K_y^{-1}\mathbf{jj} + K_z^{-1}\mathbf{kk}$
l, m	Horizontal wavenumbers
Le	Lewis number, κ_{Tz}/κ_S
Nu	Nusselt number
p	Pressure
Pr_D	Darcy-Prandtl number, $\gamma\varepsilon\nu d^2 / K_z \kappa_{Tz}$
$\mathbf{V}$	Velocity vector, (u, v, w)
Ra_T	Thermal Rayleigh number, $\beta_T g \Delta T d K_z / \nu\kappa_{Tz}$
Ra_S	Solute Rayleigh number, $\beta_S g \Delta S d K_z / \nu\kappa_{Tz}$
S	Solute concentration
Sh	Sherwood number
S_T	Soret parameter, $D_1\beta_S / \kappa_{Tz} \, \beta_T$
t	Time
T	Temperature
Ta	Taylor number, $(2\Omega K_Z / \varepsilon\nu)^2$
ΔS	Salinity difference between the walls
ΔT	Temperature difference between the walls
x, y, z	Space coordinates

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Greek symbols

β_T	Thermal expansion coefficient
β_S	Solute expansion coefficient
γ	Ratio of specific heat, $\dfrac{(\rho c)_m}{(\rho c_p)_f}$
ε	Porosity
ρ_0	Reference density
η	Thermal anisotropy parameter, κ_{Tx}/κ_{Tz}
$\boldsymbol{\kappa}_T$	Anisotropic thermal diffusion tensor, $\kappa_{Tx}\mathbf{ii}+\kappa_{Ty}\mathbf{jj}+\kappa_{Tz}\mathbf{kk}$
κ_S	Solute Diffusivity
μ_f	Fluid viscosity
μ_e	Effective viscosity
ν	Kinematic viscosity
Θ	Dimensionless amplitude of temperature perturbation
σ	Growth rate
$\mathbf{\Omega}$	Angular velocity of rotation, $(0,0,\Omega)$
ξ	Mechanical anisotropy parameter, K_x/K_z
λ	Normalized porosity, ε/γ
Φ	Dimensionless amplitude of concentration perturbation
ψ	Stream function

Other symbols

∇_h^2	$\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$
∇^2	$\nabla_h^2+\dfrac{\partial^2}{\partial z^2}$

Subscripts

b	Basic state
c	Critical
f	Fluid
h	Horizontal
s	Solid
m	Porous medium
0	Reference value

Superscripts

*	Dimensionless quantity
'	Perturbed quantity
F	Finite amplitude
Osc	Oscillatory state
St	Stationary state

1. INTRODUCTION

Interest in the double diffusive convection in porous media subject to the Soret effect has recently increased in view of its potential occurrence in nature and wide range of applications such as high-quality crystal production, liquid gas storage, migration of moisture in fibrous insulation, transport of contaminants in saturated soil, solidification of molten alloys, and geothermally heated lakes and magmas, underground disposal of nuclear wastes, liquid re-injection, electro-chemical and drying processes. Double-diffusive convection occurs when the faster-diffusing component has an unstable distribution. In the ocean, this happens when cold, fresh water sits above warmer, saltier and denser water. The importance of double diffusion lies in its ability to affect water mass structure with its differential transport rates for heat and salt.

The problem of double diffusive convection in porous medium has been extensively investigated and the growing volume of work devoted to this area is well documented by Ingham and Pop [1], Nield and Bejan [2], Vafai [3-4] and

Vadasz [5]. Early studies on the phenomenon of thermohaline convection in porous media primarily focused on the problem of convective instability in a horizontal layer heated and salted from below. The study of the double diffusive generalization of the Horton–Rogers–Lapwood problem was first undertaken by Nield [6] on the basis of linear stability theory for various thermal and solutal boundary conditions. Soret instability in an anistropic porous medium with temperature-dependent viscosity has been studied by Patil and Subramanian [7].

A major part of the investigations on convection in porous media dealt with isotropic materials. However, in many practical situations, the porous materials are anisotropic in their mechanical and thermal properties. Due to the structure of the solid material in which the fluid flows, there can be a pronounced anisotropy in properties such as permeability or thermal diffusivity. The novelties introduced by anisotropy have only recently been studied. In geothermal system with a ground structure composed of many strata of different permeabilities, the overall horizontal permeability may be up to ten times as large as the vertical component. Process such as sedimentation, compaction, frost action, and reorientation of the solid matrix are responsible for the creation of anisotropic natural porous medium. Anisotropy can also be a characteristic of artificial porous material like pelleting used in chemical engineering process, fiber materials used in insulating purposes. Tyvand [8] studied the thermohaline instability in anisotropic porous media. The linear stability analysis was used by Malashetty [9] to investigate the effect of anisotropic thermo convective currents, in the presence of Soret and Dufour effects, on the critical Rayleigh number for both marginal and overstable motions. The review of research on convective flow through anisotropic porous media has been well documented by Storesletten [10]. More recently, Malashetty and Swamy [11] have studied the onset of convection in a binary fluid saturated anisotropic porous layer.

Nonlinear rotating convection in a porous medium uniformly heated from below is of considerable interest in geophysical fluid dynamics, as this phenomenon may occur within the Earth's outer core. Earth's outer core consists of molten Iron and lighter alloying element, sulphur in its molten form. This lighter alloying element present in the liquid phase is released as the new iron freezes due to supercooling onto the solid Inner core. Hence we get mushy layer near the inner core boundary where the problem becomes convective instability in a porous medium Roberts et al. [12]. An excellent review of research on thermal convection in a rotating porous media has been given by Vadasz [13]. Recently many authors have studied the effect of anisotropy and/or rotation on the onset of convection in a porous layer (see e.g., Govender [14]; Malashetty and Swamy [15]; Malashetty and Heera [16]; Malashetty et al. [17]).

Further, when two transport processes take place simultaneously; they interfere with each other and produce cross-diffusion effect. The flux of mass caused by temperature gradient is known as Soret or thermal-diffusion effect and the flux of heat caused by concentration gradient is knows as Dufour or diffusion-thermo effect respectively. Thermal-diffusion is labeled "positive" when particles move from a hot to cold region and "negative" when the reverse is true. The Dufour coefficient is of order of magnitude smaller than the Soret coefficient in liquids, and the corresponding contribution to the heat flux may be neglected (see e.g. Straughan and Hutter [18]). In the past only a few studies have been carried out concerning the influence of the Soret effect on convection of binary fluids. A Non-Darcian effects on double diffusive convection in a two-component fluid in a sparsely packed porous layer has been investigated by Shivakumara and Smuthra [19]. Bahloul et al. [20] carried out an analytical and numerical study of the double diffusive convection in a shallow horizontal porous layer under the influence of Soret effect. Mansour et al. [21] have investigated the multiplicity of solutions induced by thermosolutal convection in a square porous cavity heated from below and subjected to horizontal concentration gradient in the presence of Soret effect. The conditions under which Soret-induced buoyancy forces may be important were discussed by these authors. Recently Gaikwad et al. [22] studied an analytical study of linear and nonlinear double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect.

For the low porosity media, the viscous effects near the boundary are negligible. In such situations Darcy's law is a good approximation for the momentum equation. Further, the classical Darcy model is valid for flow through regular structures over the whole spectrum of the porosity. This model is silent about the flow structure near the bounding surfaces where close packing of the porous material is not possible. Brinkman model is valid for a sparsely packed porous medium wherein there is more window fluid to flow so that the distortions of velocity give rise to the usual shear force. An analytical and numerical study of double diffusive convection with parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux was performed using a Brinkman model by Amahmid et al. [23]. It is now well known that many applications in engineering disciplines as well as in circumstances linked to modern porous media involve high permeability porous media and in such situations the Darcy equation fails to give satisfactory results. Therefore use of non-Darcian models, which takes care of boundary and/or inertia effects, is of fundamental and practical interest to obtain accurate results for high permeability porous media. It may be noted that most of the previous investigators have assumed that the fluid viscosity is same as the effective viscosity in their study. However, Givler and Altobelli [24] have determined experimentally that $\mu_e = (5 \sim 12)\mu_f$ where μ_e is the effective viscosity and μ_f is the fluid viscosity, for water flowing through high porosity porous media. Therefore, consideration of the ratio of effective viscosity to the fluid viscosity different from unity is warranted to know its influence on the critical stability.

Recently, Tagare and Benerji Babu [25] have investigated the problem of nonlinear convection in a sparsely packed porous medium due to thermal and compositional buoyancy. The effect of rotation on the onset of double diffusive convection in a sparsely packed anisotropic porous layer is studied by Malashetty and Begum [26]. Although literature on the use of non-Darcian models to study flow and heat transfer in porous media is available extensively in the recent past, the works on double diffusive convection in a rotating porous layer in the presence of Soret effect based upon the non-Darcian models are very sparse. Therefore, the objective of the present study is to investigate the combined effect of rotation and anisotropy in the presence of Soret effect on the double diffusive convection in a horizontal sparsely packed porous layer using linear and nonlinear stability analyses.

2. MATHEMATICAL FORMULATION

Consider a sparsely packed, anisotropic porous layer, saturated with Boussinesq fluid of infinite horizontal extent confined between the planes z = 0 and $z = d$, with the vertically downward gravity force $\mathbf{g}$ acting on it. A constant temperatures $\Delta T + T_0$ and T_0 with solute concentrations $\Delta S + S_0$ and S_0 respectively are maintained between the lower and upper boundaries. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. The porous layer rotates uniformly about the z-axis with a constant angular velocity $\mathbf{\Omega} = (0,0,\Omega)$. We also note that we are restricting our study to liquids and hence Dofour effect is negligible. We however assume that Soret effect is weak and hence assume moderate values for the Soret coefficient.

The basic state of the fluid is assumed to be quiescent, and we superpose infinitesimal perturbations on this basic state. The governing equations for the perturbations are

$$\nabla.\mathbf{V} = 0, \tag{2.1}$$

$$\rho_0\left(\frac{1}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \frac{2}{\varepsilon}\mathbf{\Omega}\times\mathbf{V}\right) + \mu_f \mathbf{K}\cdot\mathbf{V} = -\nabla p - \rho_0\left(\beta_T T - \beta_S S\right)\mathbf{g} + \mu_e \nabla^2 \mathbf{V}, \tag{2.2}$$

$$\gamma\frac{\partial T}{\partial t} + \left(\mathbf{V}.\nabla\right)T + w\frac{\partial T_b}{\partial z} = \nabla\cdot(\boldsymbol{\kappa}_T\cdot\nabla T), \tag{2.3}$$

$$\varepsilon\frac{\partial S}{\partial t} + \left(\mathbf{V}.\nabla\right)S + w\frac{\partial S_b}{\partial z} = \kappa_S \nabla^2 T + D_1 \nabla^2 T, \tag{2.4}$$

where $\mathbf{V} = (u, v, w)$ is the velocity, p the pressure, T the temperature, S the concentration, ε the porosity, $\mathbf{\Omega} = (0,0,\Omega)$ constant angular velocity, $\mathbf{K} = K_x^{-1}\mathbf{ii} + K_y^{-1}\mathbf{jj} + K_z^{-1}\mathbf{kk}$ is the inverse of the anisotropic permeability tensor, $\boldsymbol{\kappa}_\mathbf{T} = \kappa_{Tx}\mathbf{ii} + \kappa_{Ty}\mathbf{jj} + \kappa_{Tz}\mathbf{kk}$ the anisotropic thermal diffusion tensor and D_1 is the Soret coefficient. We restrict consideration to horizontal isotropy in mechanical and thermal properties of the porous medium, i.e. $K_x = K_y$ and $\kappa_{Tx} = \kappa_{Ty}$. The permeability and thermal diffusivity tensors of the porous medium are assumed to have principal axes aligned with the coordinate system. The quantities ρ_0, μ_f, μ_e, κ_S, β_T and β_S denote the density, fluid viscosity, effective viscosity, solute diffusivity, thermal and solute expansion coefficients respectively. Further $\gamma = \frac{(\rho c)_m}{(\rho c_p)_f}$, $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon(\rho c_p)_f$, c_p is the specific heat of the fluid at constant pressure, c is the specific heat of the solid, the subscripts f, s and m denote fluid, solid and porous medium values respectively.

By operating curl twice on Eq. (2.2), we eliminate p' from it and then render the resulting equation and the Eqs. (2.3)-(2.4) dimensionless by setting

$$\left(x', y', z'\right) = \left(x^*, y^*, z^*\right)d,\ t' = t^*\left(\gamma d^2/\kappa_{Tz}\right),\ \left(u', v', w'\right) = \left(\kappa_{Tz}/d\right)\left(u^*, v^*, w^*\right),$$
$$T' = \left(\Delta T\right)T^*, S' = \left(\Delta S\right)S^*, \tag{2.5}$$

to obtain non-dimensional equations as

$$\left[\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 + \nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2} - Da\nabla^4\right]\left[\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + \frac{1}{\xi} - Da\nabla^2\right) + Ta\frac{\partial^2}{\partial z^2}\right]w = \left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + \frac{1}{\xi} - Da\nabla^2\right)\left(Ra_T\nabla_h^2 T - Ra_S\nabla_h^2 S\right) \tag{2.6}$$

$$\left[\frac{\partial}{\partial t}-\left(\eta\nabla_h^2+\frac{\partial^2}{\partial z^2}\right)+\mathbf{V}\cdot\nabla\right]T-w=0, \tag{2.7}$$

$$\left[\lambda\frac{\partial}{\partial t}-\frac{1}{Le}\nabla^2+\mathbf{V}\cdot\nabla\right]S-w=S_T\frac{Ra_T}{Ra_S}\nabla^2 T. \tag{2.8}$$

The dimensionless parameters, that appear are $Pr_D=\gamma\varepsilon\nu d^2/K_z\kappa_{Tz}$, the Darcy-Prandtl number, $Da=\mu_e K_Z/\mu_f d^2$, the modified Darcy number, $Ta=\left(2\Omega K_Z/\varepsilon\nu\right)^2$, the Taylor number, $S_T=D_1\beta_S/\kappa_{Tz}\beta_T$, the Soret parameter $Ra_T=\beta_T g\Delta Td K_z/\nu\kappa_{Tz}$, the thermal Rayleigh number, $Ra_S=\beta_S g\Delta SdK_z/\nu\kappa_{Tz}$, the solute Rayleigh number, $Le=\kappa_{Tz}/\kappa_S$, the Lewis number, $\xi=K_x/K_z$, the mechanical anisotropy parameter, $\eta=\kappa_{Tx}/\kappa_{Tz}$, the thermal anisotropy parameter, $\lambda=\varepsilon/\gamma$, normalized porosity. Eqs. (2.6)-(2.8) are to be solved for the stress free, isothermal and isosolutal boundary conditions

$$w=\frac{\partial^2 w}{\partial z^2}=T=S=0, \text{ at } z=0,1. \tag{2.9}$$

3. LINEAR STABILITY ANALYSIS

We predict the thresholds of both stationary and oscillatory convections using linear theory. The Eigen value problem defined by Eqs. (2.6)-(2.8) subject to the boundary conditions (2.9) is solved using the time-dependent periodic disturbances in a horizontal plane. Assuming that the amplitudes of the perturbations are very small, we write

$$\begin{pmatrix} w \\ T \\ S \end{pmatrix}=\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix} exp\left[i(lx+my)+\sigma t\right], \tag{3.1}$$

where l, m are horizontal wavenumbers and σ is the growth rate. Infinitesimal perturbations of the rest state may either damp or grow depending on the value of the parameter σ. Substituting Eq. (3.1) into the linearized version of Eqs. (2.6)-(2.8) we obtain

$$\begin{bmatrix} \left(\frac{\sigma}{Pr_D}\left(D^2-a^2\right)-a^2+\frac{1}{\xi}D^2-Da\left(D^2-a^2\right)^2\right) \\ \left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}-Da\left(D^2-a^2\right)\right)+Ta\,D^2 \end{bmatrix} W=\begin{pmatrix} \left(\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}-Da\left(D^2-a^2\right)\right)\right) \\ \left(-Ra_T\,a^2\,\Theta+Ra_S\,a^2\,\Phi\right) \end{pmatrix} \tag{3.2}$$

$$\left[\sigma-\left(D^2-\eta a^2\right)\right]\Theta-W=0, \tag{3.3}$$

$$\left[\lambda\sigma-\frac{1}{Le}\left(D^2-a^2\right)\right]\Phi-W=S_T\frac{Ra_T}{Ra_S}(D^2-a^2)\Theta, \tag{3.4}$$

where $D=d/dz$ and $a^2=l^2+m^2$.

We assume the solutions of Eqs. (3.2)- (3.4) satisfying the boundary conditions (2.9) in the form

$$\begin{pmatrix} W(z) \\ \Theta(z) \\ \Phi(z) \end{pmatrix}=\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} \mathrm{Sin}\, n\pi z, \ (n=1,2,3,......). \tag{3.5}$$

The most unstable mode corresponds to $n=1$ (fundamental mode). Therefore substituting Eq. (3.5) with $n=1$ into Eqs. (3.2)-(3.4), we obtain a matrix equation

$$\begin{pmatrix} M_{11} & -a^2 Ra_T & a^2 Ra_S \\ -1 & \sigma+\delta_2^2 & 0 \\ -1 & S_T \dfrac{Ra_T}{Ra_S}\delta^2 & \lambda\sigma+\delta^2 Le^{-1} \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.6}$$

where $M_{11}, \delta^2, \delta_1^2$ and δ_2^2 are given in the appendix.

The condition of nontrivial solution of above system of homogeneous linear equations (3.6) yields the expression for thermal Rayleigh number in the form

$$Ra_T = \frac{\left(\sigma+\delta_2^2\right)}{a^2}\left\{\left[\delta^2\left(\frac{\sigma}{Pr_D}+Da\,\delta^2\right)+\delta_1^2+\frac{\pi^2\,Ta}{\left(\sigma\,Pr_D^{-1}+\dfrac{1}{\xi}+Da\,\delta^2\right)}\right]\right\}. \tag{3.7}$$
$$\left. \frac{\left(\lambda\sigma+\delta^2\,Le^{-1}\right)}{\left(\lambda\sigma+\delta^2\left(Le^{-1}+S_T\right)\right)}+\frac{a^2 Ra_S}{\left(\lambda\sigma+\delta^2\left(Le^{-1}+S_T\right)\right)} \right.$$

3.1. STATIONARY STATE

For the validity of principle of exchange of stabilities (i.e. steady case), we have $\sigma=0$ (i.e. $\omega_r=\omega_i=0$) at the margin of stability. Then the Rayleigh number at which marginally stable steady mode exists becomes

$$Ra_T^{St} = \frac{\left(\eta a^2+\pi^2\right)}{a^2\left(1+Le\,S_T\right)}\left[\left(a^2+\pi^2\xi^{-1}\right)+Da\left(a^2+\pi^2\right)^2+\frac{\pi^2\,Ta}{\left(\xi^{-1}+Da\left(a^2+\pi^2\right)\right)}\right]+\frac{\left(\eta a^2+\pi^2\right)}{\left(a^2+\pi^2\right)\left(1+Le\,S_T\right)}Le\,Ra_S. \tag{3.8}$$

The minimum value of the Rayleigh number Ra_T^{St} occurs at the critical wave number $a=a_c^{St}$ where $a_c^{St}=\sqrt{h}$ satisfies a polynomial equation of degree seven in h (given in appendix).

In the absence of Soret effect i.e., $S_T=0$ Eq. (3.8) implies

$$Ra_T^{St} = \frac{\left(\eta a^2+\pi^2\right)}{a^2}\left[\left(a^2+\frac{\pi^2}{\xi}\right)+Da\left(a^2+\pi^2\right)^2+\frac{\pi^2\,Ta}{\dfrac{1}{\xi}+Da\left(a^2+\pi^2\right)}\right]+\frac{\left(\eta a^2+\pi^2\right)}{\left(a^2+\pi^2\right)}Le\,Ra_S. \tag{3.9}$$

This exactly coincides with the result of Malashetty and Begum (2010).

In the limit as $Da\to 0$ i.e., for a densely packed porous medium Eq. (3.9) reduces to

$$Ra_T^{St} = \frac{\left(\eta a^2+\pi^2\right)\left[\left(a^2+\dfrac{\pi^2}{\xi}\right)+\pi^2\,\xi\,Ta\right]}{a^2}+\left(\frac{\eta a^2+\pi^2}{a^2+\pi^2}\right)Le\,Ra_S. \tag{3.10}$$

This is exactly the one given by Malashatty and Heera (2008). When $Da\to 0$ and $Ta=0$, i.e. for a densely packed porous medium in the absence of rotation, Eq. (3.10) reduces to

$$Ra_T^{St} = \frac{1}{a^2}\left(a^2 + \frac{\pi^2}{\xi}\right)\left(\eta a^2 + \pi^2\right) + \left(\frac{\eta a^2 + \pi^2}{a^2 + \pi^2}\right) Le\, Ra_S\,, \tag{3.11}$$

given by Malashatty and Swamy (2009). Further, for an isotropic porous media, that is when $\xi = \eta = 1$, Eq. (3.9) gives

$$Ra_T^{St} = \frac{\pi^4\left(1+\alpha^2\right)^3 \beta}{\alpha^2} + \frac{Ta}{\alpha^2 \beta} + \frac{Ra_S}{\tau}\,, \tag{3.12}$$

where $\beta = \frac{1}{\pi^2}\left(\pi^2 Da + \left(1 + \frac{a^2}{\pi^2}\right)^{-1}\right)$, $\alpha^2 = \frac{a^2}{\pi^2}$, $\tau = \frac{1}{Le}$, which is the one obtained by Rudraiah et al. (1986).

3.2. OSCILLATIRY STATE

We now set $\sigma = i\,\omega_i$ in Eq. (3.7) and clear the complex quantities from the denominator, to obtain

$$Ra_T = \Delta_1 + i\,\omega_i\,\Delta_2\,, \tag{3.13}$$

Since Ra_T is a physical quantity, it must be real. Hence, from Eq. (3.13) it follows that either $\omega_i = 0$ (steady onset) or $\Delta_2 = 0$ ($\omega_i \neq 0$, oscillatory onset).

For oscillatory onset $\Delta_2 = 0$ ($\omega_i \neq 0$) and this gives a dispersion relation of the form (on dropping the subscript *i*)

$$a_0\left(\omega^2\right)^2 + a_1\left(\omega^2\right) + a_2 = 0\,, \tag{3.14}$$

where the coefficients a_0, a_1 and a_2 are given in appendix. Now Eq. (3.13) with $\Delta_2 = 0$, gives

$$\begin{aligned} Ra_T^{Osc} = {} & \frac{Ra_S Le\left(\delta_2^2\,\delta^2 l_1 + Le\,\omega^2\lambda\right)}{\left(\delta^4 l_1^2 + Le^2\omega^2\lambda^2\right)} + \frac{\left(\pi^2 Ta\,\xi\left(\begin{aligned}&\omega^2\left(Pr_D\delta^4\xi l_1 + Le^2\lambda\left(\omega^2\lambda\xi - Pr_D^2 S_T\delta^2 l_2\right)\right) + \\ &\left(-Le\omega^2\delta^2\lambda\xi + LePr_D l_1\omega^2\delta^2\lambda\xi + Pr_D^2\left(\delta^4 l_1 + Le^2\omega^2\lambda^2\right)l_2\right)\delta_2^2\end{aligned}\right)\right)}{\left(a^2\left(\delta^4 l_1^2 + Le^2\omega^2\lambda^2\right)\left(Pr_D^2 l_2^2 + \omega^2\xi^2\right)\right)} \\ & + \frac{\left(\begin{aligned}&-\omega\delta^2\left(Le^2\omega^2\lambda^2 + \delta^4\left(l_1\left(1 + Da\,LePr_D\lambda\right)\right) + Le^2 Pr_D S_T\lambda\delta_1^2\right) + \\ &\left(\delta^4\left(-Le^2 S_T\omega^2\lambda + Da\,Pr_D\left(\delta^4 l_1 + Le^2\omega^2\lambda^2\right)\right) + Pr_D\left(\delta^4 l_1 + Le^2\omega^2\lambda^2\right)\delta_1^2\right)\delta_2^2\end{aligned}\right)}{\left(a^2 Pr_D\left(\delta^4 l_1^2 + Le^2\omega^2\lambda^2\right)\right)}.\end{aligned} \tag{3.15}$$

4. WEAK NONLINEAR THEORY

In this section we consider the nonlinear analysis using a truncated representation of Fourier series considering only two terms. Although the linear stability analysis is sufficient for obtaining the stability condition of the motionless solution and the corresponding Eigen functions describing qualitatively the convective flow, it can neither provide information about the values of the convection amplitudes, nor regarding the rate of heat transfer. To obtain this additional information, we perform the nonlinear analysis, which is useful to understand the physical mechanism with minimum amount of mathematics and is a step forward towards understanding the full nonlinear problem.

For simplicity of analysis, we confine ourselves to the two-dimensional rolls, so that all the physical quantities are independent of *y*. We introduce stream function ψ such that $u = \frac{\partial\psi}{\partial z}, w = -\frac{\partial\psi}{\partial x}$ into the perturbed form of Eq. (2.2), eliminate pressure and non-dimensionalize the resulting equation and Eqs. (2.3)- (2.4) using the transformations (2.5) to obtain

$$\left\{\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 + \frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2} - Da\nabla^4\right\}\psi - Ta^{1/2}\frac{\partial V}{\partial z} + Ra_T\nabla_h^2\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0\,, \tag{4.1}$$

$$\left\{\frac{1}{Pr_D}\frac{\partial}{\partial t}+\frac{1}{\xi}-Da\nabla^2\right\}\frac{\partial V}{\partial z}+Ta^{1/2}\frac{\partial^2\psi}{\partial z^2}=0, \tag{4.2}$$

$$\frac{\partial T}{\partial t}-\left(\eta\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)T-\frac{\partial(\psi,T)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=0, \tag{4.3}$$

$$\lambda\frac{\partial S}{\partial t}-\frac{1}{Le}\nabla^2 S-\frac{\partial(\psi,S)}{\partial(x,z)}+\frac{\partial\psi}{\partial x}=S_T\frac{Ra_T}{Ra_S}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)T. \tag{4.4}$$

The first effect of non-linearity is to distort the temperature and concentration fields through the interaction of ψ, T and also ψ, S. The distortion of these fields will corresponds to a change in the horizontal mean, i.e. a component of the form $\sin(2\pi z)$ will be generated. Thus a minimal Fourier series which describes the finite amplitude free convection is given by,

$$\psi = A(t)\sin(ax)\sin(\pi z), \tag{4.5}$$

$$T = B(t)\cos(ax)\sin(\pi z)+C(t)\sin(2\pi z), \tag{4.6}$$

$$S = D(t)\cos(ax)\sin(\pi z)+E(t)\sin(2\pi z), \tag{4.7}$$

$$V = F(t)\sin(ax)\cos(\pi z)+G(t)\sin(2\pi x), \tag{4.8}$$

where the amplitudes $A(t),\ B(t),\ C(t),\ E(t),\ F(t)$ and $\mathrm{G(t)}$ are to be determined from the dynamics of the system.

Substituting equations (4.5)-(4.8) into equations (4.1)-(4.4) and equating the coefficients of like terms we obtain the following non-linear autonomous system of differential equations

$$\frac{dA}{dt}=-\frac{Pr_D}{\delta^2}\left(\delta_1^2 A+A\,Da\,\delta^4-\pi\,Ta^{1/2}F+a\,Ra_T B-a\,Ra_S D\right), \tag{4.9}$$

$$\frac{dB}{dt}=-a\,A-\delta_2^2 B-\pi\,a\,AC, \tag{4.10}$$

$$\frac{dC}{dt}=-4\pi^2 C+\frac{\pi\,a}{2}AB, \tag{4.11}$$

$$\frac{dD}{dt}=\frac{-1}{\lambda}\left(a\,A+\frac{\delta^2 D}{Le}+\pi\,a\,AE+S_T\frac{Ra_T}{Ra_S}\delta^2 B\right), \tag{4.12}$$

$$\frac{dE}{dt}=\frac{1}{\lambda}\left(\frac{\pi\,a\,AD}{2}-\frac{4\pi^2 E}{Le}-4\pi^2 C\,S_T\frac{Ra_T}{Ra_S}\right), \tag{4.13}$$

$$\frac{dF}{dt}=\frac{Pr_D}{\pi}\left(\pi\,Da\,\delta^2 F-\frac{\pi}{\xi}F-\pi^2 Ta^{1/2}A\right), \tag{4.14}$$

The non-linear system of autonomous differential equations is not suitable to analytical treatment for the general time-dependent variable and we have to solve it using a numerical method. However, one can make qualitative predictions as discussed below. The system of equations (4.9)-(4.14) is uniformly bounded in time and possesses many properties of the full problem. Thus volume in the phase space must contract. In order to prove volume contraction, we must show that velocity field has a constant negative divergence. Indeed,

$$\frac{\partial}{\partial A}\left(\frac{dA}{dt}\right)+\frac{\partial}{\partial B}\left(\frac{dB}{dt}\right)+\frac{\partial}{\partial C}\left(\frac{dC}{dt}\right)+\frac{\partial}{\partial D}\left(\frac{dD}{dt}\right)+\frac{\partial}{\partial E}\left(\frac{dE}{dt}\right)+\frac{\partial}{\partial F}\left(\frac{dF}{dt}\right)+\frac{\partial}{\partial G}\left(\frac{dG}{dt}\right) = -\left[\frac{Pr_D\,\delta_1^2}{\delta^2}+\left(\delta_2^2+4\pi^2\right)+\frac{1}{Le\,\lambda}(\delta^2+4\pi^2)+\frac{Pr_D\,\pi}{\xi}\right], \tag{4.15}$$

which is always negative and therefore the system is bounded and dissipative. As a result, the trajectories are attracted to a set of measure zero in the phase space; in particular they may be attracted to a fixed point, a limit cycle or, perhaps, a strange attractor. From Eq. (4.15) we conclude that if a set of initial points in phase space occupies a region $V(0)$ at time $t = 0$, then after some time t, the end points of the corresponding trajectories will fill a volume

$$V(t)=V(0)\exp\left[-\left(\frac{Pr_D\,\delta_1^2}{\delta^2}+\left(\delta_2^2+4\pi^2\right)+\frac{1}{Le\,\lambda}(\delta^2+4\pi^2)+\frac{Pr_D\,\pi}{\xi}\right)t\right]. \tag{4.16}$$

This expression indicates that the volume decreases exponentially with time. We can also infer that, the large Darcy Prandtl number and very small Lewis number ($Le<1$) tend to enhance dissipation. Finally we note that the system of Eqs. (4.9)-(4.14) are invariant under the symmetry transformation

$\left(A,B,C,D,E,F,G\right)\rightarrow\left(-A,-B,C,-D,-E,-F,-G\right)$.

4.1. STEADY FINITE AMPLITUDE MOTION

From qualitative predictions we look into the possibility of an analytical solution. In the case of steady motions, Eqs. (4.1)-(4.4) can be solved in closed form. Setting the left hand sides of Eqs. (4.9)-(4.14) equal to zero, we get

$$\delta_1^2 A + A\,Da\,\delta^4 - \pi\,Ta^{1/2}\,F + a\,Ra_T\,B - a\,Ra_S\,D = 0\,, \tag{4.17}$$

$$a\,A - \delta_2^2\,B - \pi\,a\,AC = 0\,, \tag{4.18}$$

$$8\pi^2\,C - \pi\,a\,AB = 0\,, \tag{4.19}$$

$$a\,A + \frac{\delta^2}{Le}D + \pi\,a\,AE + S_T\,\frac{Ra_T}{Ra_S}\delta^2\,B = 0\,, \tag{4.20}$$

$$\frac{8\pi^2}{Le}E - \pi\,a\,AD + 8\pi^2\,S_T\,\frac{Ra_T}{Ra_S}C = 0, \tag{4.21}$$

$$\pi\,Da\,\delta^2\,F - \frac{\pi}{\xi}F - \pi^2\,Ta^{1/2}\,A = 0. \tag{4.22}$$

Writing B, C, D, E and F in terms of A, using Eqs. (4.18)-(4.22) and substituting these in Eq. (4.17), with $A^2 \div 8 = x$ we get

$$A_1\,x^2 + A_2\,x + A_3 = 0\,, \tag{4.23}$$

The required root of Eq. (4.23) is,

$$x = \frac{1}{2A_1}\left(-A_2 + \left(A_2^2 - 4A_1\,A_3\right)^{1/2}\right). \tag{4.24}$$

When we let the radical in the above equation to vanish, we obtain the expression for finite amplitude Rayleigh number Ra^F , which characterizes the onset of finite amplitude steady motions. The finite amplitude Rayleigh number can be obtained in the form

$$Ra^F = \frac{1}{2B_1}\left(-B_2 + \left(B_2{}^2 - 4B_1\,B_3\right)^{1/2}\right), \tag{4.25}$$

where A_1, A_2, A_3, B_1, B_2 and B_3 are given in appendix.

4.2. HEAT AND MASS TRANSPORT

In the study of convection in fluids, the quantification of heat and mass transport is important. This is because the onset of convection, as Rayleigh number is increased, is more readily detected by its effect on the heat and mass transport. In the basic state, heat and mass transport is by conduction alone.

If H and J are the rate of heat and mass transport per unit area respectively, then

$$H = -\kappa_{Tz} \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}, \tag{4.26}$$

$$J = -\kappa_{Sz} \left\langle \frac{\partial S_{total}}{\partial z} \right\rangle_{z=0} - D_1 \left\langle \frac{\partial T_{total}}{\partial z} \right\rangle_{z=0}, \tag{4.27}$$

where the angular bracket corresponds to a horizontal average and

$$T_{total} = T_0 - \Delta T \frac{z}{d} + T(x,z,t), \tag{4.28}$$

$$S_{total} = S_0 - \Delta S \frac{z}{d} + S(x,z,t). \tag{4.29}$$

Substituting Eqs. (4.6) and (4.7) in Eqs. (4.28) and (4.29) respectively and using the resultant equations in Eqs. (4.26) and (4.27), we get

$$H = \frac{D_1 \Delta T}{d}\left(1 - 2\pi C\right), \tag{4.30}$$

$$J = \frac{\kappa_{Sz}\,\Delta S}{d}\left(1 - 2\pi E\right). \tag{4.31}$$

The Nusselt number and Sherwood number are defined by

$$Nu = \frac{H}{D_1\,\Delta T/d} = \left(1 - 2\pi C\right), \tag{4.32}$$

$$Sh = \frac{J}{\kappa_{Sz}\,\Delta S/d} = \left[\left(1 - 2\pi E\right) + S_T\, Le \frac{Ra_T}{Ra_S}\left(1 - 2\pi C\right)\right]. \tag{4.33}$$

Writing C and E in terms of A, using Eqs. (4.19)-(4.21), and substituting in Eqs. (4.32) and (4.33) respectively, we obtain

$$Nu = 1 + \frac{2x}{\left(\frac{\delta_2^2}{a^2} + x\right)}, \tag{4.34}$$

$$Sh = 1 + \frac{2x}{\left(\frac{\delta^2}{a^2\, Le^2 + x}\right)} + Le\, S_T \frac{Ra_T}{Ra_S}\left[1 + \frac{2x\left(1 - \delta^2\, Le - \delta^2\right)}{\left(\frac{\delta_2^2}{a^2} + x\right)}\right]. \tag{4.35}$$

The second term on the right hand side of Eqs. (4.34) and (4.35) represent the convective contribution to heat and mass transport respectively.

5. RESULT AND DISCUSSION

The effect of rotation on the onset of double-diffusive convection in a sparsely packed anisotropic porous layer, in the presence of Soret effect is investigated analytically using the linear and nonlinear stability theories. In the linear stability theory the threshold of both stationary and oscillatory Rayleigh number is obtained analytically along with the dispersion relation for frequency of oscillation. The nonlinear theory provides the quantification of heat and mass transports and also explains the possibility of the finite amplitude motions.

The neutral stability curves in the $Ra_T - a$ plane for various parameter values are as shown in Figs.1-9. We fixed the values for the parameters except the varying parameter. From these figures it is clear that the neutral curves are connected in a topological sense. This connectedness allows the linear stability criteria to be expressed in terms of the critical Rayleigh number $Ra_{T\,c}$, below which the system is stable and unstable above.

Fig. 1 shows the neutral stability curves for different values of mechanical anisotropy parameter ξ for fixed values of other parameters. We observe from this figure that the convection sets in as oscillatory mode prior to the stationary mode. It can be observed that the critical value of the Rayleigh number increases with the increasing ξ for both oscillatory and stationary convection. Thus ξ has stabilizing effect on stationary and oscillatory convection. Fig. 2 indicates the effect of thermal anisotropy parameter η on the neutral stability curves for the fixed values of other parameters. It is observed that critical value of the Rayleigh number for stationary and oscillatory mode increases with increasing η, indicating that the effect of η is to inhibit the onset of stationary and oscillatory convection. Fig. 3 depicts the effect of Taylor number Ta on the neutral stability curves. We find that the effect of increasing Ta is to increase the critical value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus the Taylor number Ta has a stabilizing effect on the double diffusive convection in sparsely packed anisotropic porous medium. Fig. 4 presents the effect of Darcy number Da on the neutral stability curves. We find that critical value of the stationary Rayleigh number increases with Da, indicating that the effect of Darcy number Da is to inhibit the onset of stationary convection. Whereas for the oscillatory convection, the critical value of oscillatory Rayleigh number decreases with increasing Da, upto certain value of $Da = Da^*$ and with further increase in the value of Da, the critical Rayleigh number increases. Thus Da has dual effect on oscillatory convection.

Fig. 5 depicts the effect of solute Rayleigh number Ra_S on the neutral stability curves for stationary and oscillatory modes. We find that the effect of increasing Ra_S is to increase the critical value of the Rayleigh number for stationary and oscillatory modes and the corresponding wavenumber. Thus the solute Rayleigh number Ra_S has a stabilizing effect on the double diffusive convection in sparsely packed anisotropic porous medium. In Fig. 6 the marginal stability curves for different values of Lewis number Le are drawn. It is observed that with the increase of Le the critical values of Rayleigh number and the corresponding wavenumber for oscillatory mode decrease while those for stationary mode increase. Therefore, the effect of Le is to advance the onset of oscillatory convection while its effect is to inhibit the stationary convection.

The effect of normalized porosity parameter λ is depicted in the Fig. 7. We find that an increase in λ decreases the minimum of the Rayleigh number for oscillatory mode, indicating that the effect of increasing λ is to advance the onset of oscillatory convection. The neutral stability curves for different values of Darcy-Prandtl number Pr_D are presented in Fig.8, from this figure it is evident that for small and moderate values of Pr_D the critical value of oscillatory Rayleigh number decreases with the increase of Pr_D, however this trend is reversed for large values of Pr_D. Fig.9 indicates the effect of Soret parameter S_T on the neutral stability curves for stationary and oscillatory modes. It is observed that as S_T increases positively, the critical values of Rayleigh number and the corresponding wavenumber for oscillatory mode increase while as S_T increases negatively, those decrease. Whereas the effect is reversed for the stationary mode. Thus Soret parameter has stabilizing effect on oscillatory convection and destabilizing effect on stationary convection.

The detailed behavior of oscillatory critical Rayleigh number with respect to the Taylor number is analyzed in the $Ra_{Tc} - Ta$ plane through Figs. 10-17. We observe from these figures that the critical Rayleigh number increases with the increase of Ta, indicating that the effect of rotation is to inhibit the onset of thermal convection and it is in agreement with the corresponding problem of isotropic and pure fluid layer (Chandrasekhar, [27]).

In Fig. 10, we display the variation of critical Rayleigh number Ra_{Tc} with Taylor number Ta for different values of mechanical anisotropy parameter ξ for the fixed values of other parameters. It is important to note that Ra_{Tc} decreases with the increase of ξ for small values of Ta, and for large value of Ta, the critical Rayleigh number increases. Thus the mechanical anisotropy parameter ξ has dual effect on oscillatory and stationary convection. Fig. 11 indicates the variation of Ra_{Tc} with Ta for different values of thermal anisotropy parameter η. It is observed that the critical Rayleigh number Ra_{Tc} increases with the increase of η indicating that the effect of thermal anisotropy parameter is to inhibit the onset of stationary and oscillatory convection. Fig. 12 presents the variation of Ra_{Tc} with Ta for different values of Darcy number Da. We find that the critical Rayleigh number Ra_{Tc} increases with the increase of Da for small values of Ta, and for large value of Ta, the critical Rayleigh number decreases. Thus Da has dual effect on oscillatory and stationary convection.

The variation of Ra_{Tc} with Ta for different values of solute Rayleigh number Ra_S and Darcy-Prandtl number Pr_D on the onset criteria is shown in Figs. 13 and 14 respectively. We observe from these figures that Ra_{Tc} increases with the increasing Ra_S and Pr_D. Thus the effect of Ra_S is to inhibit the onset of convection for both stationary and oscillatory modes. And the effect of the Darcy-Prandtl number is also to inhibit the onset of oscillatory convection. In Figs. 15 and 16 the variation of Ra_{Tc} with Ta for different values of Lewis number Le and normalized porosity parameter λ is shown for the fixed values of other parameters. It is observed that Ra_{Tc}, increases with the increase of Le, for the stationary convection while decreases for the oscillatory convection and it decreases with the increase of λ, indicating that Le has stabilizing effect on stationary convection while destabilizing effect on oscillatory convection and λ has destabilizing effect on oscillatory convection. The variation of Ra_{Tc} with Ta for different values of Soret parameter is presented in the Fig. 17. We find that as S_T increases positively, the critical values of Rayleigh number and the corresponding wavenumber decrease for stationary mode while as S_T increases negatively, those increase. And the trend is reversed for oscillatory mode. Thus positive S_T has destabilizing effect for stationary mode and stabilizing effect for oscillatory mode.

The quantity of heat and mass transfer across the layer is computed by the thermal Nussle number and Sherwood number. This is depicted in the Rayleigh-Nusselt number plane through the Figs. 18-22. We observe that as Ra_T increases with its critical value, the heat and mass transports increase and as Ra_T is increased further, they remain almost constant. In Figs. 18 and 19 the effect of mechanical anisotropy parameter and the Taylor number is displayed. It is found that with the increase of these parameters both the thermal Nusselt number and Sherwood number decrease, indicating that their effect is to enhance the heat and mass transport. From Fig. 20 it is observed that with the increasing thermal anisotropy parameter, the thermal Nusselt number and Sherwood number increase. Thus the heat and mass transport is reinforced by it. Fig. 21 shows the effect of Darcy number on heat and mass transport. We find that thermal Nusselt number decreases with the increasing Darcy number while Sherwood number increases. Thus the effect of Darcy number is to suppress the heat transport while mass transport is reinforced. The effect of Soret parameter on heat and mass transport is displayed in Fig.22. It is found that heat transport is suppressed (almost insignificant) while mass transport is reinforced by Soret parameter.

6. CONCLUSIONS

The effect of rotation on the onset of double-diffusive convection in a sparsely packed anisotropic porous layer, in the presence of Soret effect is investigated analytically using the linear and nonlinear stability theories. The usual normal mode technique is used to solve the linear problem. The truncated Fourier series method is used to make the finite amplitude analysis. The following conclusions are drawn:

1. The mechanical anisotropy parameter ξ has stabilizing effect on stationary and oscillatory modes. However the convection sets in as oscillatory mode prior to the stationary mode.
2. The effect of thermal anisotropy parameter η is to inhibit the onset of stationary and oscillatory convection.
3. The Taylor number Ta has a stabilizing effect on the double diffusive convection in sparsely packed anisotropic porous medium.
4. The effect of Darcy number Da is to inhibit the on the onset of stationary convection while it has dual effect on oscillatory convection.
5. The effect of solute Rayleigh number is to delay both stationary and oscillatory convection. And the effect of Lewis number is to delay the onset of stationary convection while it advances the oscillatory convection.
6. The effect of normalized porosity is to advance the onset of oscillatory convection. And the Darcy Prandtl Pr_D has a dual effect on the oscillatory mode.
7. The Soret parameter has stabilizing effect on oscillatory convection and destabilizing effect on stationary convection.

8. The effect of mechanical anisotropy parameter ξ and Taylor number Ta is to enhance the heat and mass transport.
9. The heat and mass transport is reinforced by the thermal anisotropy parameter η.
10. The Darcy number Da and Soret parameter S_T suppresses the heat transport while they reinforce the mass transport is reinforced by it

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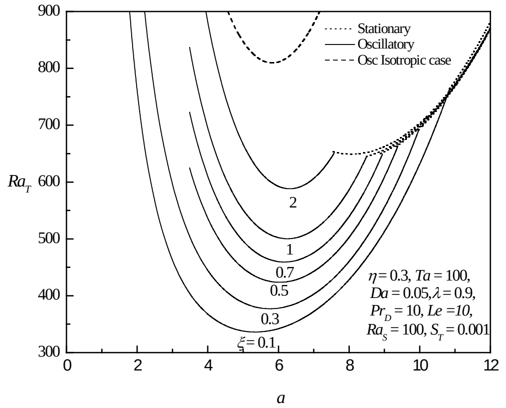

Fig. 1. Neutral stability curves for differents values of ξ

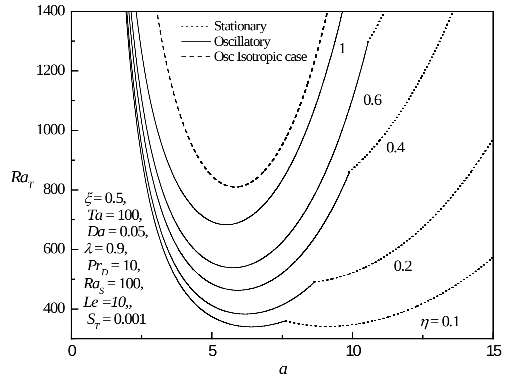

Fig. 2. Neutral stability curves for differents values of η

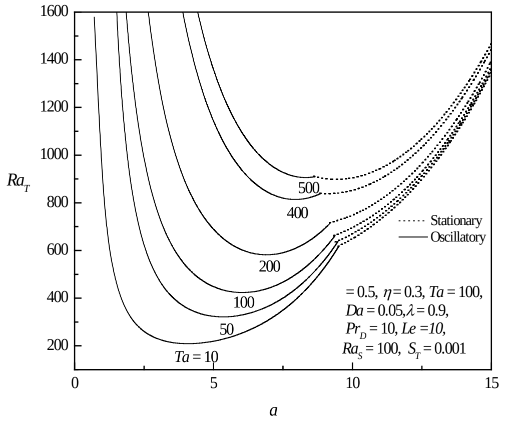

Fig. 3. Neutral stability curves for differents values of Ta.

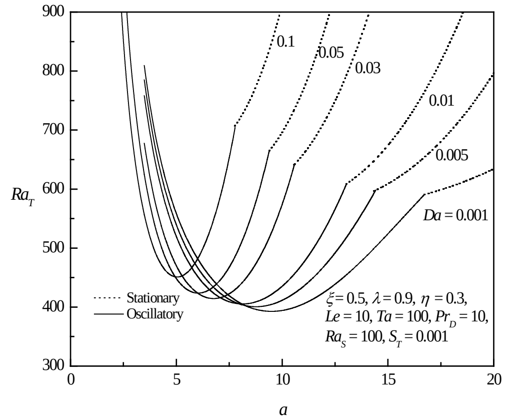

Fig. 4. Neutral stability curves for differents values of Da.

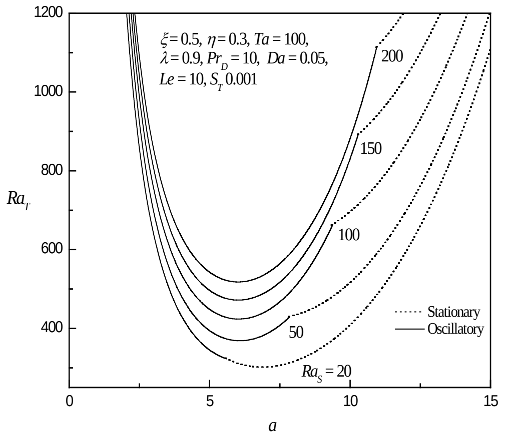

Fig. 5. Neutral stability curves for differents values of Ra_S.

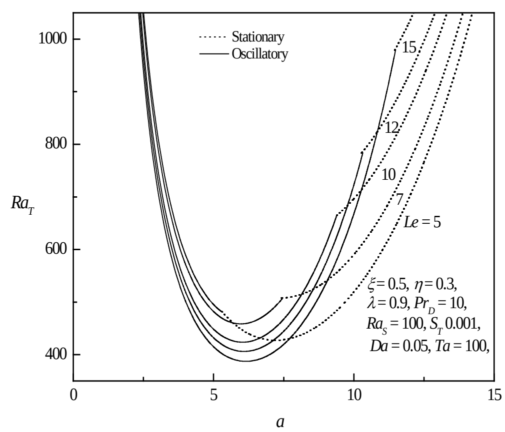

Fig. 6. Neutral stability curves for differents values of Le.

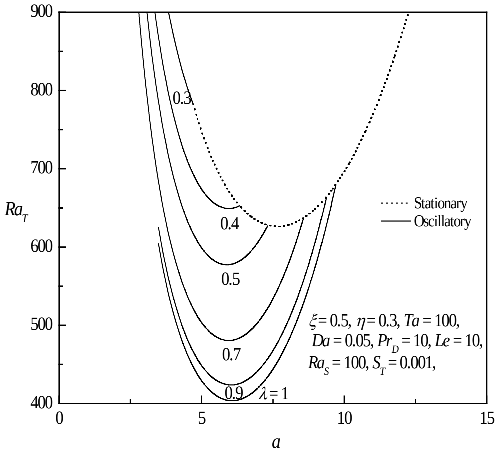

Fig. 7. Neutral stability curves for differents values of λ.

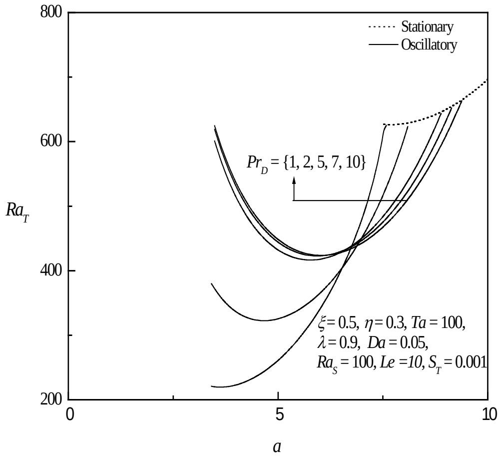

Fig. 8. Neutral stability curves for differents values of Pr_D.

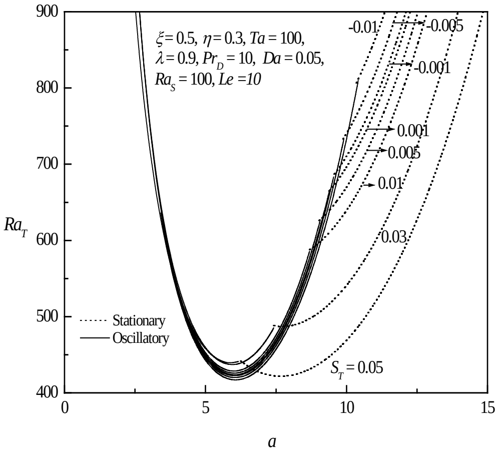

Fig. 9. Neutral stability curves for differents values of S_T.

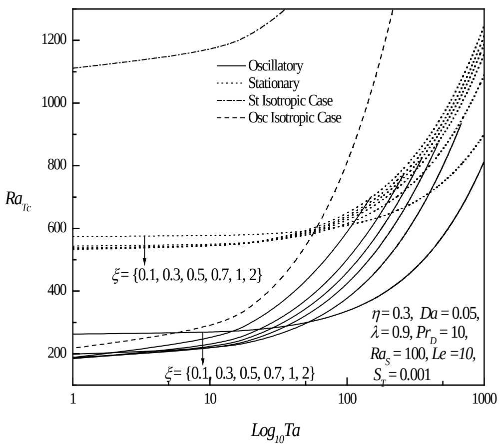

Fig. 10. Variation of critical Rayleigh number for differents values of ξ.

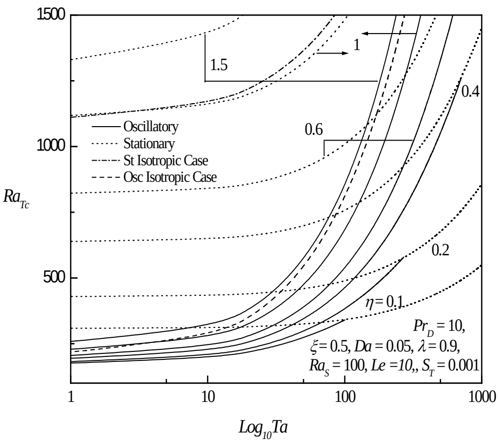

Fig. 11. Variation of critical Rayleigh number for differents values of η.

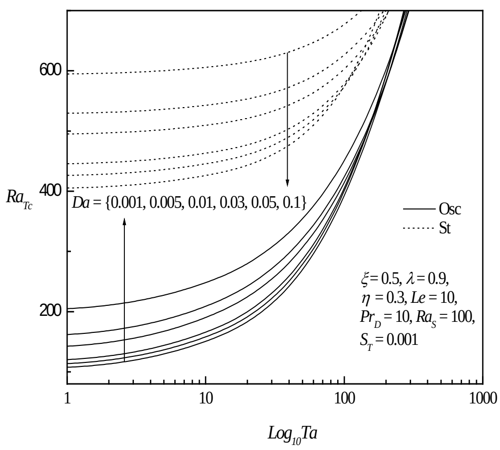

Fig. 12. Variation of critical Rayleigh number for differents values of Da.

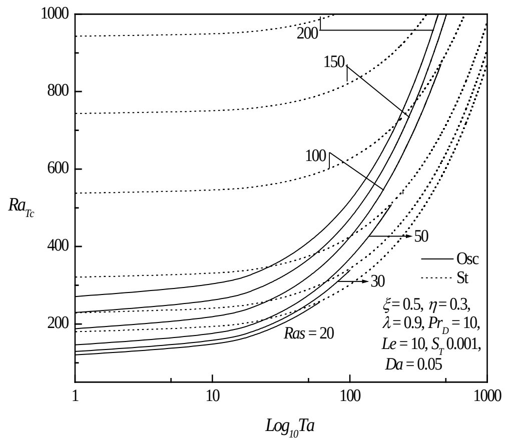

Fig. 13. Variation of critical Rayleigh number for differents values of Ra_S.

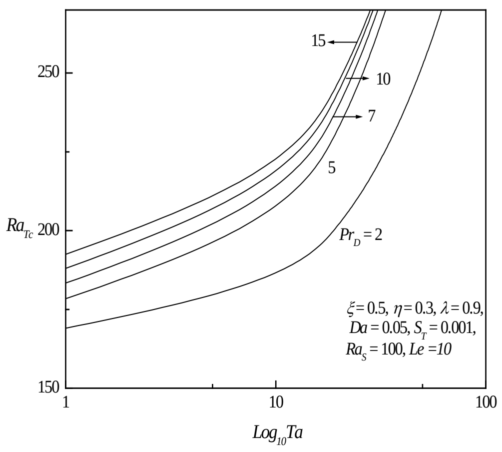

Fig. 14. Variation of critical Rayleigh number for differents values of Pr_D.

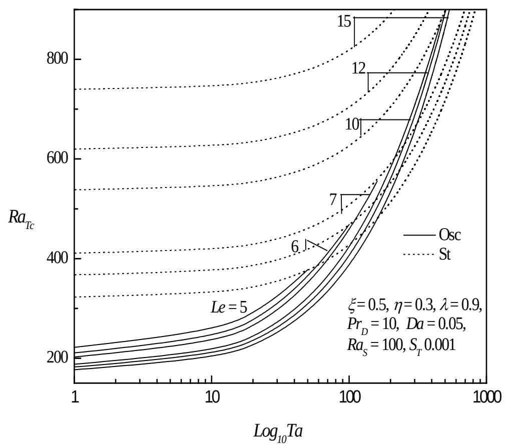

Fig. 15. Variation of critical Rayleigh number for differents values of *Le*.

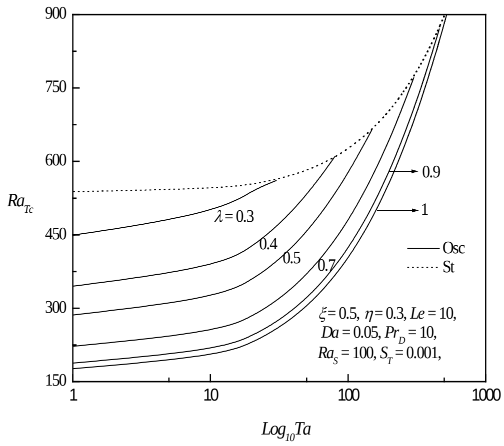

Fig. 16. Variation of critical Rayleigh number for differents values of λ.

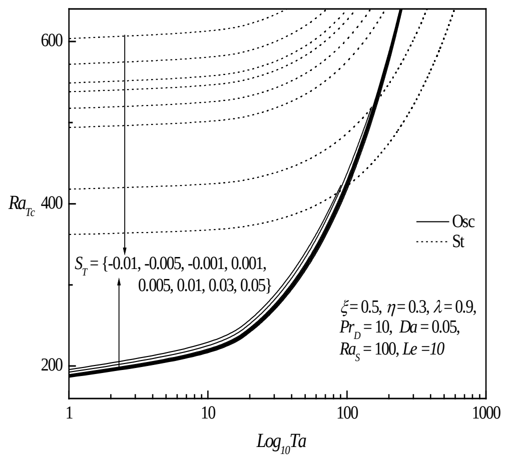

Fig. 17. Variation of critical Rayleigh number for differents values of S_T.

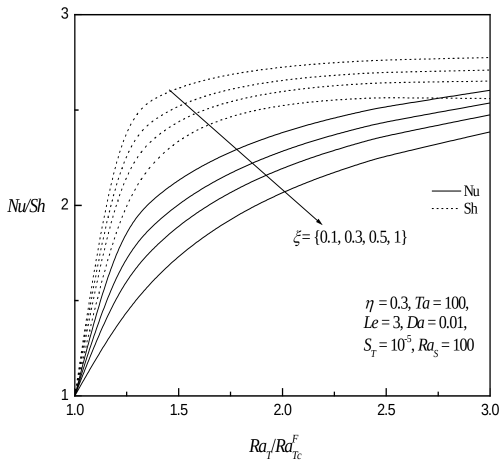

Fig.18. Variation of Nusselt number and Sherwood number with critical Rayleigh numberfor different values of ξ

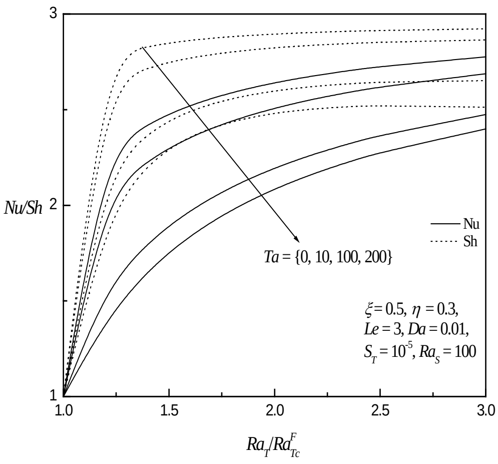

Fig.19. Variation of Nusselt number and Sherwood number with critical Rayleigh numberfor different values of *Ta*.

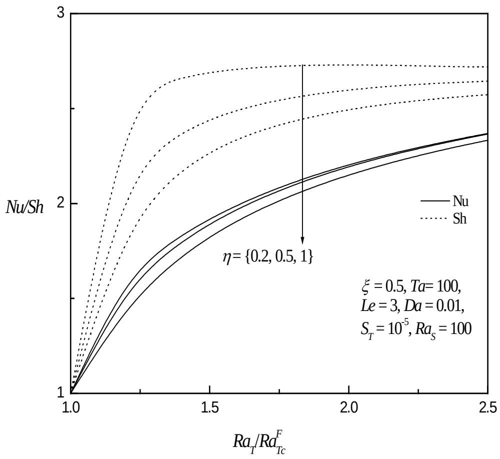

Fig.20. Variation of Nusselt number and Sherwood number with critical Rayleigh numberfor different values of η.

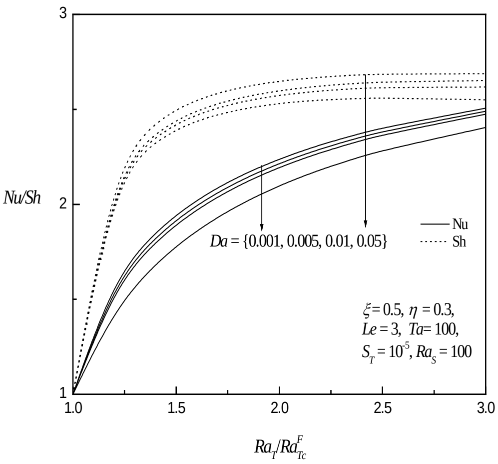

Fig.21. Variation of Nusselt number and Sherwood number with critical Rayleigh numberfor different values of *Da*.

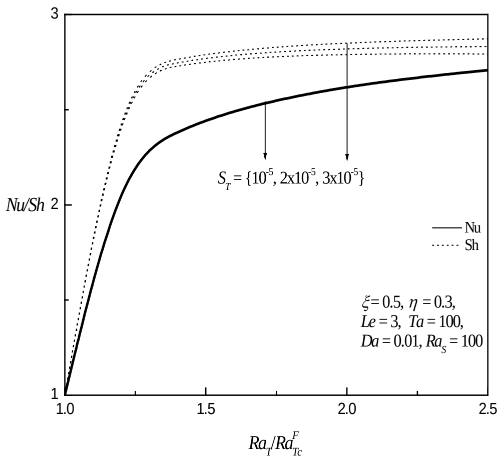

Fig.22. Variation of Nusselt number and Sherwood number with critical Rayleigh number for different values of S_T.

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APPENDIX

$$M_{11} = \delta^2 \sigma Pr_D^{-1} + \delta_1^2 + Da\,\delta^4 + \pi^2 Ta\left(\sigma Pr_D^{-1} + \xi^{-1} + Da\,\delta^2\right)^{-1}$$

$\delta^2 = \pi^2 + a^2$, $\delta_1^2 = \pi^2 \xi^{-1} + a^2$ and $\delta_2^2 = \pi^2 + \eta a^2$.

$$S_0 y^7 + S_1 y^6 + S_2 y^5 + S_3 y^4 + S_4 y^3 + S_5 y^2 + S_6 y + S_7 = 0$$

where

$$S_0 = 2\eta D^3 a,$$

$$S_1 = \frac{D^2 a \quad (4\eta + (\eta + Da\pi^2(1+10\eta))\xi}{\xi},$$

$$S_2 = \frac{2Da(\eta + (\eta + Da\pi^2(1+8\eta))\xi + 2Da\pi^2(\eta + Da\pi^2(1+5\eta))\xi^2)}{\xi^2},$$

$$S_3 = \frac{\eta + Da\pi^2(1+6\eta + (6\eta + Da\pi^2(5+24\eta))\xi + (-Ta + Da(6\pi^2\eta + 5Da\pi^4 (1+4\eta) + Le(Ra_s - Ra_s\eta)))\xi^2)}{\xi^2},$$

$$S_4 = \frac{2\pi^2(Da\xi(LeRa_s + \pi^2(Da\,Le\,Ra_s - Ta)\xi) + \eta(1+Da\pi^2\xi)(1+Da(5Da\pi^4\xi - LeRa_s\xi^2 + \pi^2(3+2\xi))))}{\xi^2},$$

$$S_5 = \frac{1}{\xi^3}\begin{pmatrix} -Le\pi^2 Ra_s(-1+\eta)\xi + D^3 a\pi^{10}(-5+2\eta)\xi^3 \\ +Da\pi^6\xi(-6+2\eta+2\eta\xi - (Ta + DaLeRa_s - 1 + \eta\xi^2 + D^2 a\pi^8\xi^2 \\ -10+\eta\,4+\xi+\pi^4 -1+\xi\eta - 2DaLeRa_s - 1 + \eta\,\xi \end{pmatrix},$$

$$S_6 = \frac{-2\pi^6\left(1+Da\pi^2\xi\right)^2\left(1+2Da\pi^2\xi\right)}{\xi^3} \quad \&$$

$$S_7 = \frac{-\pi^8\left(1+Da\pi^2\xi\right)^3}{\xi^3}.$$

$$l_1 = \left(1+LeS_T\right), l_2 = \left(1+Da\delta^2\xi\right), l_3 = \left(\delta_1^2 + Da\delta^4\right), l_3 = \left(\xi Da\delta^2 - 1\right), l_5 = \left(l_3\, l_4 - \pi^2 Ta\xi\right).$$

$$a_0 = \frac{\lambda\left(\delta^4\left(-S_T + Da\,Pr_D\,\lambda\right) + Pr_D\,\lambda\,\delta_1^2 + \delta^2\,\lambda\,\delta_2^2\right)}{Pr_D^{\;3}},$$

$$a_1 = \frac{Ra_S a^2\left(l_1\delta^2 - Le\lambda\delta_2^2\right)}{LePr_D^2} + \frac{1}{LePr_D^3\xi}\left(Ta\,\pi^2\lambda\left(LePr_D^2\lambda + \delta^2\left(-1+Pr_D + LePr_D S_T + Da\,LePr_D^2\right)\xi - Le\lambda\xi\delta_2^2\right)\right)$$
$$+\frac{1}{Le^2 Pr_D^3\xi^2}\begin{pmatrix} Pr_D\delta^4\left(Da\,\delta^4\,\xi^2 l_1 + Pr_D\lambda\left(-S_T + Da\,Pr_D\lambda\right)Le^2 l_2^2\right) + \\ Pr_D\left(\delta^4\xi^2 l_1 + Le^2 Pr_D^2\lambda^2\right)l_2^2 + \delta^2\left(\delta^4\xi^2 l_1 + Le^2 Pr_D\lambda\right) \\ \left(Da\,S_T\delta^4\xi^2 + Pr_D\lambda l_2^2 + Le^2 Pr_D S_T\lambda\,\xi^2\delta_1^2\right)\delta_2^2 \end{pmatrix},$$

$$a_2 = \frac{\left(Ra_S a^2 l_2^2\left(l_1\delta^2 - Le\lambda\delta_2^2\right)\right)}{Le\xi^2} + \frac{1}{Le^2 Pr_D\xi}\left(Ta\,\pi^2\delta^2\left(Pr_D l_1\delta^2 l_2 + \left(Le^2 Pr_D S_T\lambda + \delta^2\left(-1+LeS_T\left(-1+Da\,LePr_D\lambda\right)\right)\xi\right)\delta_2^2\right)\right)$$
$$+\frac{1}{Le^2 Pr_D\xi^2}\left(\delta^2 l_2^2\left(Pr_D\,l_1\delta^2 l_3 + \left(\delta^4\left(l_1\left(1+Da\,LePr_D\lambda\right)\right) + Le^2 Pr_D S_T\lambda\,\delta_1^2\right)\delta_2^2\right)\right).$$

$$A_1 = a^4 Le^2 l_5,$$

$$A_2 = l_5 a^2\left(\delta_2^2 Le^2 + \delta^2\right) - a^4 Lel_4\left(LeRa_T + Ra_S l_1\right),$$

$$A_3 = l_5\delta_2^2\delta^2 + a^2 l_4\left(Ra_S Le\left(\delta_2^2 - S_T\delta^2\right) - Ra_T\delta^2\right).$$

$$B_1 = a^8 Le^4 l_4^2,$$

$$B_2 = 2a^6 Le^2 l_4 \begin{pmatrix} a^2 LeRa_S l_1 \left(1 - Da\,\delta^2 \xi\right) + \left(\delta - Le\delta_2\right)\left(\delta + Le\delta_2\right) \\ \left(-\delta_1^2 - \pi^2 Ta\xi + Da\,\delta^2\left(-\delta^2 + Da\,\delta^4 \xi + \delta_1^2 \xi\right)\right) \end{pmatrix},$$

$$B_3 = a^8 Le^2\, Ra_S^2 l_1^2 l_4^2 + 2a^6 LeRa_S \left(\left(1 + Le\left(1 + 2Le\right) S_T\right)\delta^2 + Le^2\left(-1 + LeS_T\right)\delta_2^2\right) l_4$$
$$\left(-\delta_1^2 - \pi^2 Ta\,\xi + Da\,\delta^2\left(-\delta^2 + Da\,\delta^4 \xi + \delta_1^2 \xi\right)\right) + a^4\left(\delta - Le\delta_2\right)^2\left(\delta + Le\delta_2\right)^2$$
$$\left(\delta_1^2 + \pi^2 Ta\,\xi + Da\,\delta^2\left(\delta^2 - l_3 \xi\right)\right)^2.$$

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