International Journal of Mathematical Archive-3(8), 2012, 3055-3066
Available online through www.ijma.info ISSN 2229 – 5046

$g**I-continuous$ functions

Sr. Pauline Mary Helen*
Associate Professor, Nirmala College, Coimbatore, India

Mrs. Ponnuthai Selvarani
Associate Professor, Nirmala College, Coimbatore, India

Mrs. Veronica Vijayan,
Aassociate Professor, Nirmala College, Coimbatore, India

Mrs. Punitha Tharani,
Associate Professor,St. Mary's College, Tuticorin, India

(Received on: 06-07-12; Accepted on: 16-08-12)

ABSTRACT

In this paper, $I^{*S}-continuous$ functions, $g^{**S}I-continuous$ functions, strongly $g^{**S}I-continuous$ functions, weakly $g^{**S}I-continuous$ functions are introduced and their properties are investigated. $g^{**}I-compact$, $g^{**S}I-compact$, $g^{**}I-connected$, $g^{**S}I-connected$, $g^{**}I-normal$ and $g^{**S}I-normal$ spaces are defined and studied

Keywords: $I^{*S}-continuous$ functions, $g^{**S}I-continuous$ functions, strongly $g^{**S}I-continuous$ functions, weakly $g^{**S}I-continuous$ functions $g^{**}I-compact$, $g^{**S}I-compact$, $g^{**}I-connected$, $g^{**S}I-connected$, $g^{**}I-normal$ and $g^{**S}I-normal$ spaces

1. INTRODUCTION

Ideals in topological spaces have been considered since 1930. In 1990, Jankovic and Hamlett **[2]** once again investigated applications of topological ideals. M.E.Abd EI Monsef,E.F.Lashien and A.A. Nasef **[1]** in 1992 and quite recently Khan and Noiri have studied semi-local functions in ideal topological spaces. In this paper $I^{*S}-continuous$ functions, $g^{**S}I-continuous$ functions, strongly $g^{**S}I-continuous$ functions, weakly $g^{**S}I-continuous$ functions , $g^{**}I-compact$, $g^{**S}I-compact$, $g^{**}I-connected$, $g^{**S}I-connected$, $g^{**}I-normal$ and $g^{**S}I-normal$ spaces are introduced and their properties are investigated.

2. PRELIMINARIES

Definition 2.1: An ideal**[3]** I on a non empty set X is a collection of subsets of X which satisfies the following properties.(i) $A \in I$, $B \in I \Rightarrow A \cup B \in I$ (ii) $A \in I$, $B \subset A \Rightarrow B \in I$ .A topological space (X,τ) with an ideal I on X is called an ideal topological space and is denoted by (X,τ,I).Let Y be a subset of X. $I_Y = \{I \cap Y / I \in I\}$ is an ideal on Y and by $(Y, \tau/Y, I_Y)$ we denote the ideal topological subspace.

Definition 2.2: Let $P(X)$ be the power set of X, then a set operator ( )*: $P(X) \to P(X)$ called the local function**[7]** of A with respect to τ and I is defined as follows: For $A \subset X$, $A^*(I,\tau) = \{x \in X / U \cap A \notin I$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(I,\tau)$ in case there is no confusion.

Corresponding author: *Sr. Pauline Mary Helen**
Associate Professor, Nirmala College, Coimbatore, India

A Kuratowski closure operator $cl^*(\ )$ for a topology $\tau^*(I,\tau)$, called the τ^* - topology is defined by $Cl^*(A)=A\cup A^*$ For A, B in (X,τ,I) we have

(i) If $A\subset B$ then $A^*\subset B^*$

(ii) $\left(A^*\right)^*\subseteq A^*$

(iii) $A^*\cup B^*=(A\cup B)^*$

(iv) $(A\cap B)^*\subseteq A^*\cap B^*$

(v) If $I=\{\phi\}$, $A^*=cl(A)$ and $cl^*(A)=cl(A)$

(vi) If $I=P(X)$ then $A^*=\phi$ and $cl^*(A)=A$(vii) $A^*=cl(A^*)\subset cl(A)$ and A^* is a closed subset of cl(A).

Definition 2.3: A subset A of a space (X,τ) is said to be semi-open **[4]** if $A\subset cl(\text{int}(A))$

Definition 2.4: A set operator **[1]** $(\ )^{*S}:P(X)\to P(X)$ called a semi local function and $cl^{*s}(\ )$ of A with respect to τ and I are defined as follows: For $A\subset X$, $A^{*S}(I,\tau)=\{x\in X/U\cap A\notin I$ for every semi open set U containing $x\}$. and $Cl^{*S}(A)=A\cup A^{*S}$. For a subset A of X, $cl(A)$ (resp. $scl(A)$) denotes the closure (resp. semi closure) of A in (X,τ). Similarly $cl^*(A)$ and $\text{int}^*(A)$ denote the closure of A and interior of A in (X,τ^*).

Definition 2.5: A subset A of X is called * closed **[6]** (resp. $*s$ closed**[1]**) if $A^*\subseteq A$ (resp. $A^{*S}\subseteq A$). Their complements are called * open (resp. $*s$ **open)**

Lemma 2.6: [1] For A, B in (X,τ,I) we have

(i) If $A\subset B$ then $A^{*S}\subset B^{*S}$

(ii) $\left(A^{*S}\right)^{*S}\subseteq A^{*S}$

(iii) $A^{*S}\cup B^{*S}\supseteq(A\cup B)^{*S}$

(iv) $(A\cap B)^{*S}\subseteq A^{*S}\cap B^{*S}$ (v) If $I=\{\phi\}$, $A^{*S}=scl(A)$ and $cl^{*S}(A)=scl(A)$ (vi) If $I=P(X)$ then $A^{*S}=\phi$ and $cl^{*S}(A)=A$(vii) $A^{*S}=scl(A^{*S})\subset scl(A)$ and A^{*S} is semi closed.

In general $A^{*S}\cup B^{*S}\neq(A\cup B)^{*S}$

Definition 2.7: An ideal space (X,τ,I) is said to be

(iii) $*s-finitely$ additive if $\left[\bigcup_{i=1}^{n}A_i\right]^{*S}=\bigcup_{i=1}^{n}(A_i)^{*S}$ for every positive integer n.

(iv) $*S-$countably additive if $\left[\bigcup_{i=1}^{\infty}A_i\right]^{*S}=\bigcup_{i=1}^{\infty}(A_i)^{*S}$

(v) $*S-$ additive if $\left[\bigcup_{\alpha\in\Omega}A_\alpha\right]^{*S}=\bigcup_{\alpha\in\Omega}(A_\alpha)^{*S}$ for all indexing sets Ω.

In $*s-finitely$ additive space $cl^{*S}(A\cup B)=cl^*(A)\cup cl^*(B)$

Definition 2.8: A subset A of an ideal space (X,τ,I) is said to be g- closed **[5]**, if $cl(A)\subseteq U$ whenever $A\subseteq U$ and U is $open$ in X. The complement of $g-closed$ set is said to be $g-open$.

Definition 2.9:A subset A of an ideal space (X,τ,I) is said to be g^*- closed **[8]**, if $cl\ (A)\subseteq U$ whenever $A\subseteq U$ and U is $g-open$ in X. The complement of $g^*-closed$ set is said to be g^*-open.

Definition 2.10: A subset A of an ideal space (X,τ,I) is said to be $g^{**}-$ closed **[6]**, if $cl\ (A)\subseteq U$ whenever $A\subseteq U$ and U is $g^{*}-open$ in X. The complement of $g^{**}-closed$ set is said to be $g^{**}-open$

Definition 2.11: A subset A of an ideal space (X,τ,I) is said to be $g^{**}I-$ closed **[5]**, if $cl^{*}(A)\subseteq U$ whenever $A\subseteq U$ and U is $g^{*}-open$ in X. The complement of $g^{**}I-closed$ set is said to be $g^{**}I-open$

Definition 2.12: A subset A of an ideal space (X,τ,I) is said to be $g^{**s}I-$ closed **[5]**, if $cl^{*s}(A)\subseteq U$ whenever $A\subseteq U$ and U is $g^{*}-open$ in X. The complement of $g^{**s}I-closed$ set is said to be $g^{**s}I-open$

Remark 2.13:

1. In an ideal topological space (X,τ,I), union of two $g^{**}I-closed$ sets is $g^{**}I-closed$
2. In a finitely additive ideal topological space (X,τ,I), union of two $g^{**s}I-closed$ sets is $g^{**s}I-closed$
3. $g^{**}I-continuous$ **functions,** $g^{**S}I-continuous$ **functions**

We introduce the following definitions

Definition 3.1: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be (i) $*-continuous$ if $f^{-1}(V)$ is * open in X wherever V is open in Y. (ii) $*s\ -continuous$ if $f^{-1}(V)$ is *s- open in X whenever V is open in Y.

Definition 3.2: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be weakly $I*-continuous$ if for each $x\in X$ and for every open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(U)\subseteq cl^{*}(V)$.

Definition 3.3: A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be weakly $I^{*S}-continuous$ if for each $x\in X$ and for every open set V in Y containing $f(x)$, there exists an open set U containing x such that $f(U)\subseteq cl^{*S}(V)$.

Definition 3.4: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be $g^{**}I-continuous$ if for every V in σ, $f^{-1}(V)$ is $g^{**}I-open$ in X. Equivalently for every closed set V in Y, $f^{-1}(V)$ is $g^{**}I-closed$ in X.

Definition 3.5: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be $g^{**S}I-continuous$ if for every V in σ, $f^{-1}(V)$ is $g^{**S}I-open$ in X. Equivalently for every closed set V in Y, $f^{-1}(V)$ is $g^{**S}I-closed$ in X.

Definition 3.6: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be strongly $g^{**}J-continuous$ if for every $g**J-open\ \ set$ V in Y, $f^{-1}(V)$ is open in X. Equivalently for every $g^{**}J-closed$ set V in Y, $f^{-1}(V)$ is $closed$ in X.

Definition 3.7: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be strongly $g^{**S}J-continuous$ if for every $g^{**S}J-open$ set V in Y, $f^{-1}(V)$ is open in X. Equivalently for every $g^{**S}J-closed$ set V in Y, $f^{-1}(V)$ is closed in X.

Definition 3.8: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be weakly $g^{**}I-continuous$ if for every $x\in X$ and for every V in σ containing $f(x)$, there exists an $g^{**}I-open$ set U in X such that $x\in U$ and $f(U)\subseteq cl^{*}(V)$.

Definition 3.9: A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be weakly $g^{**S}I-continuous$ if for every $x\in X$ and for every V in σ containing $f(x)$, there exists an $g^{**S}I-open$ sets U in X such that $x\in U$ and $f(U)\subseteq cl^{*s}(V)$.

Definition 3.10: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be $g^{**}I-irresolute$ if for every $g^{**}J-open$ set V in Y, $f^{-1}(V)$ is $g^{**}I-open$ in X. Equivalently for every $g^{**}J-closed$ set V in Y, $f^{-1}(V)$ is $g^{**}I-closed$ in X.

Definition 3.11: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be $g^{**s}I-irresolute$ if for every $g^{**s}J-open$ set V in Y, $f^{-1}(V)$ is $g^{**s}I-open$ in X. Equivalently for every $g^{**s}J-closed$ set V in Y, $f^{-1}(V)$ is $g^{**s}I-closed$ in X.

Remark 3.12:
(i) Every $*-continuous$ function is $g**I-continuous$.
(ii) Every $*s-continuous$ function is $g^{**S}I-continuous$.

The converse is not true as seen in the following example.

Example 3.13: Let (X,τ) be an indiscrete and $I=\{\phi,x_0\}$. $Y=X$, $\sigma=P(X)$ the discrete topology and $J=I$. In X, all subsets are $g^{**}I-open$ and $g^{**S}I-open$. $*-open$ sets are $*S-open$ sets are $\{\phi,X,X-\{x_0\}\}$.

Let $f:X\to Y$ be identity function. Then f is $g^{**}I-continuous$, $g^{**S}I-continuous$ but not $*-continuous$ and not $*s-continuous$.

Remark 3.14: Every continuous function is $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$.

The converse is not true as seen in the following example.

Example 3.15: Let $X=\{a,b,c\},\tau=\{\phi,\{a\},X\}$, $\sigma=\{\phi,\{b\},Y\}$ and $I=J=\{\phi\}$.

Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be the identity map. Then f is $g^{**}I-continuous$ but not continuous.

Remark 3.16: Every strongly $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$ function is continuous and hence it is $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$

The converse is not true as seen in the following example.

Example 3.17: Let (X,τ) be an indiscrete topological space Y = X, $\tau=\sigma$ and $I=\{\phi,x_0\}=J$. Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be the identity map. In (X,τ,I) all the subsets are $g^{**}I-closed$ and $g^{**S}I-closed$. $*S-open$ sets are $\{\phi,X,X-x_0\}$ and $*-open$ sets are $\{\phi,X,X-x_0\}$. The map f is continuous, $g^{**}I-continuous$, $g^{**S}I-continuous$, $g^{**}I-irresolute$ and $g^{**S}I-irresolute$, $*continuous$, $*S-continuous$ but not strongly $g^{**}I-continuous$ and not strongly $g^{**S}I-continuous$.

Here any proper subset A of Y is $g^{**}I-open$ and $g^{**S}I-open$. But $f^{-1}(A)$ is not open in X.

Remark 3.18: Every $g^{**}I-continuous$ function is weakly $g^{**}I-continuous$ and every $g^{**s}I-continuous$ function is weakly $g^{**s}I-continuous$.
The converse is not true as seen in the following example.

Example 3.19: Let $X=Y=\{a,b,c,d\}$, $\tau=\sigma=\{\phi,\{a,b\},X\}$, $I=\{\phi\}=J$. Define $f:X\to Y$ is such that $f(a)=c$. $f(b)=d$ $f(c)=a$ $f(d)=c$ Then f is weakly $g^{**}I-continuous$ and. weakly $g^{**s}I-continuous$ but $A=\{c,d\}$ is closed in Y and $f^{-1}(A)=\{a,b\}$ is not $g^{**}I-closed$.

Remark 3.20: Every weakly $I^{*}-continuous$ function is weakly $g^{**}I-continuous$ and every weakly $I^{*s}-continuous$ function is weakly $g^{**s}I-continuous$

The converse is not true as seen in the following example.

Example 3.21: Let (X,τ) be an indiscrete topological space Y = X, $\sigma = P(X)$ and $I=\{\phi, x_0\}=J$ Let $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ be the identity map. In (X,τ,I) all the subsets are $g^{**}I-closed$ and $g^{**S}I-closed$. Then f is weakly g***I*-continuous and weakly $g^{**S}I-continuous$. Now $f(x_0) = x_o \in V=\{x_0\}$ which is open in Y. But there is no open set U containing x such that $f(U)\subseteq cl^{*}(V)$ and $f(U)\subseteq cl^{*S}(V)$ Therefore f is not weakly $I^{*}-continuous$ and not weakly $I^{*s}-continuous$

Remark 3.22: Every strongly $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$ function is $g^{**}I-irresolute$ $(resp.g^{**s}I-irresolute)$.

The converse is not true as seen in the following example.

Example 3.23: Let $X=Y=\{a,b,c,d\}$, $\tau=\{\phi,\{a,b\},X\}=\sigma$, $I=J=\{\phi\}$. Let $f:X\rightarrow Y$ be the identity function. Then f is $g^{**}I-irresolute$ and $g^{**S}I-irresolute$. $A=\{b,c\}$ is $g^{**}I-closed$ and $g^{**S}I-closed$ in Y. But $f^{-1}(A)=\{b,c\}$ is not closed in X. Therefore f is not strongly $g^{**}I-continuous$ and not strongly $g^{**S}I-continuous$.

Remark 3.24: Every $g^{**}I-irresolute$ function is $g^{**}I-continuous$ and every $g^{**s}I-irresolute$ function is $g^{**s}I-continuous$

The converse is not true as seen in the following example.

Example 3.25: Let $X=\{a,b,c,d\}$, $\tau=\{\phi,\{a,b\},X\}$, $I=\{\phi\}$, $Y=X$, $\tau=\sigma$, $I=J$. Define $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ is such that $f(a)=d$, $f(b)=a$, $f(c)=c$, $f(d)=b$. Then f is $g^{**}I-continuous$ and $g^{**s}I-continuous$ $A=\{d\}$ is $g^{**}I-closed$ and $g^{**S}I-closed$ in Y. But $f^{-1}(A)=\{a\}$ is not $g^{**}I-closed$ and $g^{**S}I-closed$ in X. Therefore f is not $g^{**}I-irresolute$ and not $g^{**S}I-irresolute$.

Remark 3.26: Every strongly $g^{**}I-continuous$ $(resp.g^{**S}I-continuous)$ function is weakly $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$.

The result follows from remark (3.16) and (3.18).

The converse is not true as seen in example (3.17).

Theorem 3.27: Let $f:(X,\tau,I)\rightarrow(Y,\sigma,J)$ and $g:(Y,\sigma,J)\rightarrow(Z,\eta,K)$ then $g_o f$ is

(i) continuous if f is strongly $g^{**}I-continuous$ and g is $g^{**}I-continuous$.

(ii) $g^{**}I-continuous$ if f is strongly $g^{**}I-irresolute$ and g is $g^{**}I-continuous$.

(iii) $g^{**}I-irresolute$ if f is $g^{**}I-continuous$ and g is strongly $g^{**}I-irresolute$.

(iv) $g^{**}I-continuous$ if f is $g^{**}I-continuous$ and g is continuous.

(v) strongly $g^{**}I-continuous$ if f is strongly $g^{**}I-continuous$ and g is $g^{**}I-irresolute$.

(vi) $g^{**}I-irresolute$ if both f and g are $g^{**}I-irresolute$.

(vii) strongly $g^{**}I-continuous$ if both f and g are strongly $g^{**}I-continuous$.

(viii) $g^{**}I-irresolute$ if f is $g^{**}I-irresolute$ and g is strongly $g^{**}I-continuous$.

Proof follows from the definitions.

Definition 3.28: An ideal topological space (X,τ,I) is said to be $g^{**}I$ (resp. $g^{**S}I$) - multiplicative if arbitrary intersection of $g^{**}I$ (resp. $g^{**S}I$) - $closed$ set is $g^{**}I$ (resp. $g^{**S}I$) - $closed$.

In such spaces arbitrary union of $g^{**}I$ (resp. $g^{**S}I$) - $open$ set is $g^{**}I$ (resp. $g^{**S}I$) - $open$.

Definition 3.29: Let N be a subset of (X,τ,I) and $x\in X$. A subset N of X is called a $g^{**}I-open$ neighbourhood ($g^{**S}I-open$ neighbourhood) of x if there exists $g^{**}I-open$ ($g^{**S}I-open$ sets) U containing x such that $U\subseteq N$.

Theorem 3.30: Let (X,τ,I) be a $g^{**}I$ (resp. $g^{**S}I$) - $multiplicative$ ideal topological space. For a function $f:(X,\tau,I)\to(Y,\sigma)$ the following conditions are equivalent.

(i) f is $g^{**}I$ (resp. $g^{**S}I$) - $continuous$.

(ii) For each $x\in X$ and each open set V in Y with $f(x)\in V$, there exists a $g^{**}I$ ($g^{**S}I$) –open set U containing x such that $f(U)\subseteq(V)$.

(iii) For each $x\in X$ and each open set V in Y with $f(x)\in V$, $f^{-1}(V)$ is an $g^{**}I$ (resp. $g^{**S}I$) - open neighbourhood of x.

Proof:

(i) $\Rightarrow$ (ii) Let f be $g^{**}I-continuous$, $x\in X$ and V be open set contained in Y such that $f(x)\in V$. Then $U=f^{-1}(V)$ is $g^{**}I-open$, $x\in X$ and $f(U)\subseteq V$.

(ii) $\Rightarrow$ (iii) Let $x\in X$ and V be an open set in Y containing $f(x)$. By (ii) there exists a $g^{**}I-open$ set U such that $x\in U$ and $f(U)\subseteq V$. Therefore $f^{-1}(V)$ is a neighbourhood of x.

(iii) $\Rightarrow$ (i) Let V be an open set in Y. Let $x\in f^{-1}(V)$. Then by (iii), there exists a $g^{**}I-open$ set U_x such that $x\in U_x\subseteq f^{-1}(V)$. Therefore $f^{-1}(V)=\bigcup_{x\in f^{-1}(V)}\bigcup_x$. Since (X,τ,I) is $g^{**}I$ - $multiplicative$ $f^{-1}(V)$ is $g^{**}I-open$.

Proof is similar in the case of $g^{**s}I-continuous$ *function.*

Theorem 3.31: Let (X,τ,I) be a $g^{**}I-multiplicative$ ideal topological space in which every open set is $*-closed$. Then a function $f:(X,\tau,I)\to(Y,\sigma)$ is $g^{**}I-continuous$ if and only if it is $g^{**}I-weakly$ $continuous$

Proof: Obviously $g^{**}I-continuity$ $\Rightarrow$ $g^{**}I-\text{weak}\,continuity$. Conversely, let f be Let $x\in U$ and $f(x)\in V$ which is open in Y. Then there exists a $g^{**}I-open$ set U in X such that $x\in U$ and $f(U)\subset cl^{*}(V)=V$, since V is $*-closed$. Therefore by theorem (3.30), f is $g^{**}I-continuous$.

Theorem 3.32: Let (X,τ,I) be a $g^{**S}I-multiplicative$ ideal topological space in which every open set is $*S-closed$. Then a function $f:(X,\tau,I)\to(Y,\sigma)$ is weakly $g^{**S}I-continuous$ if and only if it is $g^{**S}I-continuous$.

Proof is similar to the proof of theorem (4.31).

Theorem 3.33: Let (X,τ,I) be an ideal topological space. Let $f:(X,\tau,I)\to(Y,\Omega)$ be $g^{**}I-continuous$ and U be $g^{**}I-open$ in X. Then $f/U:(U,\tau_U,I_U)\to(Y,\Omega)$ is $g^{**}I-continuous$ **Proof:** Let V be open in Y. Then $f^{-1}(V)$ is $g^{**}I-open$ in X. Therefore $(f/U)^{-1}(V)=U\cap f^{-1}(V)$ is also $g^{**}I-open$ since intersection of two $g^{**}I-open$ sets is $g^{**}I-open$. Therefore f/U is $g^{**}I-continuous$.

Theorem 3.34: Let (X,τ,I) be an $*s-finitely\ \ additive$ ideal topological space. Let $f:(X,\tau,I)\to(Y,\Omega)$ be $g^{**}I-continuous$ and U be $g^{**S}I-open$ in X. Then $f/U:(U,\tau_U,I_U)\to(Y,\Omega)$ is $g^{**S}I-continuous$.

Proof is similar to the proof of the above theorem since in a $*s-additive$ space intersection of $g^{**S}I-open$ sets in $g^{**S}I-open$.

Theorem 3.35: Let (X,τ,I) be a $g^{**}I-multiplicative$ ideal topological space. Then $f:(X,\tau,I)\to(Y,\Omega)$ is $g^{**}I-continuous$ if and only if graph functions $g:X\to X\times Y$ defined by $g(x)=(x,f(x))$ for each $x\in X$ is $g^{**}I-continuous$.

Proof: Necessity: Let $x\in X$ and W an open set in $X\times Y$ containing $g(x)=(x,f(x))$. Then there exists a basic open set $U\times V$ such that $g(x)\in U\times V\subseteq W$. Then $f(x)\in V$. By theorem (3.34), there exists a $g^{**}I-open$ set U_1 in X such that $x\in U_1$ and $f(U_1)\subseteq V$. $U\cap U_1$ is $g^{**}I-open$ in X. Then $x\in U_1\cap U$ and $g(U_1\cap U)\subset U\times V\subset W$. Therefore g is $g^{**}I-continuous$.

Sufficiency: Let $g:X\to X\times Y$ be $g^{**}I-continuous$. Let $x\in X$ and V be an open set in Y such that $f(x)\in V$. Then $X\times V$ is an open set in $X\times Y$. Since g is $g^{**}I-continuous$, there exists $g^{**}I-open$ set U in X such that $x\in U$ and $g(U)\subseteq X\times V$. Therefore $x\in U$ and $f(U)\subseteq V$ which proves f is $g^{**}I-continuous$.

Theorem 3.36: If (X,τ,I) is a $*s-finitely$ additive, $g^{**S}I-multiplicative$ ideal topological space then $f:(X,\tau,I)\to(Y,\sigma)$ is $g^{**s}I-continuous$ if and only if graph functions $g:X\to X\times Y$ defined by $g(x)=(x,f(x))$ for each $x\in X$ is $g^{**s}I-continuous$.

Proof is similar as in the case of $g^{**}I-continuous$ function because in $*s-finitely$ additive space, intersection of two $g^{**S}I-open\,sets\ is\ g^{**S}I-open$

Theorem 3.37: Let $\{X_\alpha/\alpha\in\nabla\}$ be any family of topological spaces. If $f:(X,\tau,I)\to\Pi X_\alpha$ is $g^{**}I-continuous$ (resp. $g^{**}I-continuous$) then $P_\alpha of:X\to X_\alpha$ is $g^{**}I-continuous$ (resp. $g^{**}I-continuous$) for each $\alpha\in\nabla$ where P_α is the projection of ΠX_α onto X_α.

Proof: Consider a fixed $\alpha_0\in\nabla$. Let G_α be open in X_α. Since P_α is continuous, $P_\alpha^{-1}(G_\alpha)$ is open in X_α. Therefore $(P_{\alpha 0}of)^{-1}(G_{\alpha_0})=f^{-1}\left[P_{\alpha_0}^{-1}(G_{\alpha_0})\right]$ is $g^{**}I-open$ (resp. $g^{**S}I-open$). Therefore $P_{\alpha_0}of$ is $g^{**}I-continuous$ (resp. $g^{**S}I-continuous$).

Definition 3.38: A collection $\{A_\alpha/\alpha\in\Omega\}$ of $g^{**}I-open$ (resp. $g^{**S}I-open$) sets is called $g^{**}I-open$ cover (resp. $g^{**S}I-open$ cover) of a subset B of X if $B\subseteq\bigcup_{\alpha\in\Omega}A_\alpha$.

Definition 3.39: An ideal topological space (X,τ,I) is called $g^{**}I-compact$ (resp. $g^{**s}I-compact$) if for every $g^{**}I-open$ cover (resp. $g^{**S}I-open$ cover) $\{A_\alpha / \alpha\in\Omega\}$ in (X,τ,I) there exists a finite subset Ω_0 of Ω such that $X=\bigcup_{\alpha\in\Omega} A_\alpha$.

Definition 3.40: An ideal topological space (X,τ,I) is called $g^{**}I-compact$ (resp. $g^{**}I-compact$) modulo *I* if for every $g^{**}I-open$ cover (resp. $g^{**S}I-open$ cover) $\{A_\alpha / \alpha\in\Omega\}$ in (X,τ,I) there exists a finite subset Ω_0 and Ω such that $X-\bigcup_{\alpha\in\Omega} A_\alpha \in I$.

The following examples show that spaces which are $g^{**}I-compact$, $g^{**}I-compact$ modulo *I* and spaces which are not $g^{**}I-compact$, and not $g^{**}I-compact$ modulo *I* do exist.

Example 3.41: Let X be an infinite set and τ a cofinite topology (i.e). $\tau=\{\phi, X, A/A^c \text{ is finite }\}$, $I=\{\phi\}$, Then $G^{**}IO(X)=\{\phi, X, A/A^c \text{ is finite }\}$. Let $\{A_\alpha/\alpha\in\Omega\}$ be a $g^{**}I-open$ cover for X. Fix $\alpha_0\in\Omega$. Then $A_{\alpha_0}\in G^{**}IO(X)$ and so $X-A_{\alpha_0}$ is finite. Let $X-A_{\alpha_0}=\{x_1,...x_n\}$. Then there exists $\alpha_i, i=1,2,...n$ such that $x_i\in A_{\alpha_i}$. Then $A_{\alpha_0}\cup A_{\alpha_1}\cup.....\cup A_{\alpha_n}=X$. Therefore $X-\bigcup_{i=0}^{n} A_{\alpha_i}=\phi\in I$.

Therefore the space is $g^{**}I-compact$ and $g^{**}I-compact$ modulo *I*. This space is also $g^{**s}I-compact$ and $g^{**s}I-compact$ modulo *I*.

Example 3.42: Let (X,τ) be infinite indiscrete space and $I=\{\phi,\{x_0\}\}$. All subsets are $g^{**}I-open$ and $g^{**S}I-open$. $\{\{x\}/x\in X\}$ is a $g^{**}I-open$ cover for X. But it has no finite sub cover modulo *I*. Therefore this is not $g^{**}I-compact$, not $g^{**}I-compact$ modulo *I,not* $g^{**s}I-compact$ and not $g^{**s}I-compact$ modulo *I*.

Remark 3.43: In an ideal topological space (X,τ,I)

1. $g^{**}I-compactness \Rightarrow g^{**}I-compactness$ modulo *I*. (When $I=\{\phi\}$ both the concepts coincide).
2. $g^{**S}I-compactness \Rightarrow g^{**S}I-compactness$ modulo *I*. (When $I=\{\phi\}$ both the concepts coincide).
3. $g^{**S}I-compactness \Rightarrow g^{**}I-compactness$ (since $g^{**}I$ open sets are $g^{**S}I-$ open)
4. $g^{**S}I-compactness$ modulo *I*. $\Rightarrow g^{**}I-compactness$ modulo *I*. (since $g^{**}I$ open sets are $g^{**S}I-$ open)
5. Every finite ideal space (X,τ,I) is $g^{**}I-compact$, $g^{**S}I-compact$, $g^{**}I-compact$ modulo *I*, $g^{**S}I-compact$ modulo *I*.

Theorem3.44: A $g^{**}I-closed$ (resp. $g^{**S}I-closed$) subset of $g^{**}I-compact$ (resp. $g^{**S}I-compact$) ideal space is $g^{**}I-compact$ (resp. $g^{**S}I-compact$).

Proof: Let (X,τ,I) be a $g^{**}I-compact$ (resp. $g^{**S}I-closed$) ideal topological space and let B be a $g^{**}I-closed$ $(resp. g^{**s}I-closed)$ subset of X. Then $X-B$ is $g^{**}I-open$ $(resp. g^{**s}I-open)$.

Let $\{A_\alpha\}_{\alpha\in\Omega}$ be a $g^{**}I-open$ $(resp. g^{**s}I-open)$ cover for B. Then $\{X\setminus B, A_\alpha/\alpha\in\Omega\}$ is a $g^{**}I-open$ $(resp. g^{**s}I-open)$ cover for X. Since X is $g^{**}I-compact$ $(resp. g^{**s}I-compact)$ there exists a finite subset Δ_0 of Δ such that $[\bigcup_{\alpha\in\Delta_0} A_\alpha \cup (X-B)] = X$. Then $B\subseteq \bigcup_{\alpha\in\Delta_0} A_\alpha$. Therefore B is $g^{**}I-compact$ $(resp. g^{**s}I-compact)$.

Theorem 3.45: A $g^{**}I-closed$ (resp. $g^{**S}I-closed$) subset of a $g^{**}I-compact$ (resp. $g^{**S}I-compact$) modulo I space is $g^{**}I-compact$ (resp. $g^{**S}I-compact$)modulo I.

Proof: Let (X,τ,I) be a $g^{**}I-compact$ (resp. $g^{**S}I-compact$) modulo I ideal topological space and let B be a $g^{**}I-closed$ (resp. $g^{**S}I-closed$) subset of X. Then $X-B$ is $g^{**}I-open$ $(resp.g^{**s}I-open)$. Let $\{A_\alpha\}_{\alpha\in\Omega}$ be a $g^{**}I-open$ $(resp.g^{**s}I-open)$ cover for B. Then $\{X\setminus B, A_\alpha / \alpha\in\Delta\}$ is a $g^{**}I-open$ $(resp.g^{**s}I-open)$ cover for X. Since X is $g^{**}I-compact$ $(resp.g^{**s}I-compact)$ modulo *I,* there exists a finite subset Δ_0 of Δ such that $X-[\bigcup_{\alpha\in\Delta_0} A_\alpha \cup (X-B)]\in I$ which implies $[X-\bigcup_{\alpha\in\Delta_0} A_\alpha]\cap B\in I.$ $\therefore [B-\bigcup_{\alpha\in\Delta_0} A_\alpha]\in I$ Hence B is $g^{**}I-compact$ modulo I.

Theorem 3.46: The image of $g^{**}I-compact$ (resp. $g^{**s}I-compact$) space under a $g^{**}I-continuous$ (resp. $g^{**s}I-continuous$) function f is compact .

Proof: Let (X,τ,I) be $g^{**}I-compact$ $(resp.g^{**s}I-compact)$ and $f:(X,\tau,I)\to(Y,\eta)$ be an onto $g^{**}I-continuous$ $(resp.g^{**s}I-continuous)$ function. Let $\{A_\alpha\}_{\alpha\in\Delta}$ be an open cover for Y. Then $\{f^{-1}\{A_\alpha\}\}_{\alpha\in\Delta}$ is a $g^{**}I-open$ $(resp.g^{**s}I-open)$ cover for X. Since X is $g^{**}I-compact$ $(resp.g^{**}I-compact)$ there exists a finite subset Δ_0 of Δ such that $X=\bigcup_{\alpha\in\Delta_0} f^{-1}(A_\alpha).$

Therefore Y = f(X) = $\bigcup_{\alpha\in\Delta_0}(A_\alpha)$.which implies Y is $compact$

Theorem 3.47: The image of $g^{**}I-compact$ (resp. $g^{**s}I-compact$) space modulo I under a $g^{**}I-continuous$ (resp. $g^{**s}I-continuous$) function f is compact modulo $f(I)$.

Proof: f (I) is an ideal in Y, the rest of the proof is similar to the proof of the above theorem.

Theorem 3.48: The image of $g^{**}I-compact$ (resp. $g^{**s}I-compact$) space under a $g^{**}I-irresolute$ (resp. $g^{**s}I-irresolute$) function f is $g^{**}I-compact$.

Proof: Similar to the proof of theorem (4.46).

Theorem 3.49: The image of $g^{**}I-compact$ (resp. $g^{**s}I-compact$) space modulo I under a $g^{**}I-irresolute$ (resp. $g^{**s}I-irresolute$) function f is $g^{**}I-compact$ modulo $f(I)$

Proof: Similar to the proof of theorem (4.46).

Theorem 3.50: The image of $compact$ space under strongly $g^{**}I-continuous$ (resp.strongly $g^{**s}I-continuous$) function f is $g^{**}I-compact$ $(resp.g^{**s}I-compact)$.

Proof: Similar to the proof of theorem (4.46).

Theorem 3.51: The image of compact modulo I space under strongly $g^{**}I-continuous$ (resp.strongly $g^{**s}I-continuous$) function f is $g^{**}I-compact$ $(resp.g^{**s}I-compact)$ modulo $f(I)$

Proof: Similar to the proof of theorem (4.46).

Definition 3.52: An ideal topological space (X,τ,I) is said to be $g^{**}I-connected$ (resp. $g^{**s}I-connected$) If X cannot be written as disjoint union of $g^{**}I-open$ $(g^{**s}I-open)$ sets. Otherwise X is said to be $g^{**}I-disconnected$.

The following example shows the existence of such spaces.

Example 3.53: Let X be an infinite set and τ a cofinite topology and $I=\{\phi\}$, $G^{**s}IO(X)=$ $G^{**}IO(X)=\{\phi, X, A/A^c$ is finite $\}$. Suppose $X=A\cup B$ where A and B are disjoint $g^{**}I-open$ $(resp.g^{**s}I-open)$ sets then $A\cap B=\phi$. $A^C\cup B^C=X$ which is not true since A^C and B^C are finite. Therefore this space is $g^{**}I-connected$ $(resp.g^{**s}I-connected)$.

Example 3.54: Let (X,τ) be infinite indiscrete space and $I=\{\phi,\{x_0\}\}$. All subsets are $g^{**}I-open$ *and* $g^{**s}I-open$. Let A be any proper subset of X. Then $X=A\cup A^C$ where A and A^c are $g^{**}I-open$ and $g^{**s}I-open$ Therefore the space is $g^{**}I-disconnected$ and not $g^{**s}I-disconnected$.

Remark 3.54: Let (X,τ,I) be an ideal topological space

1. When $I=P(X)$, since $cl^*(A)=A=cl^{*S}(A)$, every subset is $*-open$,$*s-open$, $*-closed$ *and* $*s-closed$. Therefore all subsets are $g^{**}I-open$, $g^{**S}I-open$, $g^{**}I-closed$ and $g^{**S}I-closed$. Therefore $(X,\tau,P(X))$ is $g^{**}I-disconnected$ and $g^{**S}I-disconnected$.
2. (X,τ,I) is $g^{**}I-connected$ $(resp.g^{**S}I-connected)$ $\Leftrightarrow$ there exists no subset which is both $g^{**}I-open$ $(resp.g^{**S}I-open)$ and $g^{**}I-closed$. $(resp.g^{**S}I-closed)$
3. A is $g^{**S}I-connected \Rightarrow g^{**}I-connected$. (Since $g^{**}I-open$ sets are $g^{**S}I-open$)

Theorem 3.56: Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be an onto function.

1. X is $g^{**}I-connected$ (resp. $g^{**S}I-connected$) and f is $g^{**}I-continuous$ (resp. $g^{**S}I-continuous$) $\Rightarrow$ Y is *connected*
2. X is $g^{**}I-connected$ (resp. $g^{**S}I-connected$) and f is *continuous* $\Rightarrow$ Y is *connected*
3. X is $g^{**}I-connected$ (resp. $g^{**S}I-connected$) and f is $g^{**}I-irresolute$ (resp. $g^{**S}I-irresolute$) $\Rightarrow$ Y is $g^{**}J-connected$ (resp. $g^{**S}J-connected$).
4. X is connected and f is strongly $g^{**}J-continuous$ (resp. $g^{**S}J-continuous$) $\Rightarrow$ Y is $g^{**}J-connected$ (resp. $g^{**S}J-connected$).
5. X is connected and f is strongly $g^{**}J-continuous$ (resp. $g^{**S}J-continuous$) $\Rightarrow$ Y is connected.

Proof:
(1) Suppose Y is disconnected, there exists disjoint open sets A, B such that $Y=A\cup B$. Then $f^{-1}(Y)=f^{-1}(A)\cup f^{-1}(B)$. $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint $g^{**}I-open$ sets in X

which is a contradiction since X is $g^{**}I-connected$.

Proof of (2), (3), (4), (5) and (6) are similar to the proof of (1)

Definition 3.57: An ideal topological space (X,τ,I) is said to be $g^{**}I-normal$ $(respg^{**s}I-normal)$ if for every two disjoint closed sets F_1 and F_2 in X, there exists disjoint $g^{**}I-open$ $(respg^{**}I-open)$ sets U_1 and U_2 such that $F_1\subseteq U_1$, $F_2\subseteq U_2$.

Definition 3.58: An ideal topological space (X,τ,I) is said to be $g^{**}I-normal$ $(respg^{**s}I-normal)$ modulo I if for every two disjoint closed sets F_1 and F_2 in X, there exists disjoint $g^{**}I-open$ $(respg^{**s}I-open)$ sets U_1 and U_2 such that $F_1\subseteq U_1$, $F_2\subseteq U_2$ and $U_1\cap U_2\in I$.

Example 3.59: In example (3.42), (X,τ,I) is $g^{**}I-normal$, $g^{**s}I-normal$, $g^{**}I-normal$ modulo I and $g^{**s}I-normal$ modulo I .In example (3.41), (X,τ,I) is not $g^{**}I-normal$ not $g^{**s}I-normal$ not, $g^{**}I-normal$ modulo I and not $g^{**s}I-normal$ modulo I

Remark 3.60: In an ideal space (X,τ,I) .

1. $g^{**}I-normal \Rightarrow g^{**}I-normal$ modulo *I*.When $I=\{\phi\}$ both concepts coincide
2. $g^{**S}I-normal \Rightarrow g^{**S}I-normal$ modulo I. When $I=\{\phi\}$ both concepts coincide
3. $Normal \Rightarrow g^{**}I-normal \Rightarrow g^{**S}I-normal$. (Since open sets are $g^{**}I-open$, and $g^{**}I-open$ sets are $g^{**S}I-open$)
4. $(X,\tau,P(X))$ is always $g^{**}I-normal$ and $g^{**S}I-normal$ since all subsets are $g^{**}I-open$ and $g^{**S}I-open$

Definition 3.61: A function $f:(X,\tau,I)\to(Y,\sigma,J)$ is said to be

(i) $g^{**}J-open$ $(resp.g^{**s}J-open)$ if $f(V)$ is $g^{**}J-open$ $(resp.g^{**s}J-open)$ in Y wherever V is open in X.

(ii) $g^{**}J-closed$ $(resp.g^{**s}J-closed)$ if $f(V)$ is $g^{**}J-closed$ $(resp.g^{**s}J-closed)$ in Y wherever V is closed in X.

(iii) $g^{**}I-strongly$ $(resp.g^{**s}I-strongly)$ open if $f(V)$ is open in Y wherever V is $g^{**}I-open$ $(resp.g^{**s}I-open)$ in X.

Theorem 3.61: Let $f:(X,\tau,I)\to(Y,\sigma,J)$ be a bijective function.Then the following are equivalent.

1. f^{-1} is $g^{**}J-continuous$ (resp. $g^{**S}J-continuous$).
2. f is $g^{**}J-open$ (resp. $g^{**S}J-open$).
3. f is $g^{**}J-closed$ (resp. $g^{**S}J-closed$).

Theorem3.62: Let $f:(X,\tau,I)\to(Y,\sigma,J)$ where $J=f(I)$ be an injection function.

1. X is normal and f is $g^{**}J-open$ $(respg^{**s}J-open)$ and continuous $\Rightarrow$ Y is $g^{**}J-normal$. $(respg^{**s}I-normal)$
2. X is $g^{**}I-normal$ $(respg^{**s}I-normal)$, f is $g^{**}I-strongly$ $(respg^{**s}I-strongly)$ open and continuous $\Rightarrow$ Y is $g^{**}I-normal$ $(respg^{**s}I-normal)$ and normal
3. X is $g^{**}I-normal$ $(respg^{**s}I-normal)$ modulo *I*, and f is $g^{**}I-strongly$ $(respg^{**}I-strongly)$ open and continuous $\Rightarrow$ Y is $g^{**}J-normal$ $(respg^{**s}J-normal)$ modulo J and normal modulo J
4. X is normal modulo I and f is $g^{**}J-open$ $(respg^{**s}J-open)$ and continuous $\Rightarrow$ Y is $g^{**}J-normal$. $(respg^{**s}J-normal)$ modulo J

Proof:

(1) Let F_1 and F_2 two disjoint closed sets in Y. Then $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint closed sets in X. Since X is normal there exists disjoint open sets U_1 and U_2 such that $f^{-1}(F_1)\subseteq U_1$ and $f^{-1}(F_2)\subseteq U_2$. Since f is $g^{**}J-open$ $(respg^{**s}J-open)$, $f(U_1)$ and $f(U_2)$ are $g^{**}I-open$ $(respg^{**s}I-open)$ in X such that $F_1\subseteq f(U_1)$ and $F_2\subseteq f(U_2)$ and $F_1\cap F_2=\phi$.

Therefore f is 1 – 1. Therefore Y is $g^{**}I-normal$ $(respg^{**}I-normal)$

(2) Proof is similar to the proof of(1)

(3) F_1 and F_2 are two disjoint closed sets in Y

$\Rightarrow$ $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint closed sets in X.

$\Rightarrow$ there exists $g^{**}I-open$ $(respg^{**s}I-open)$ sets U_1 and U_2 in X such that $f^{-1}(F_1)\subseteq U_1$ and $f^{-1}(F_2)\subseteq U_2$. and $U_1\cap U_2\in I$

$\Rightarrow$ $f(U_1)$ and $f(U_2)$ are disjoint $open$ sets and hence $g^{**}J-open$ $(respg^{**s}J-open)$ sets in Y containing F_1 and F_2 respectively and $f(U_1)\cap f(U_2)\in f(I)=J$.

$\Rightarrow$ Y is $g^{**}J-normal$ $(respg^{**s}J-normal)$ modulo J and normal modulo J

(4) Proof is similar to the proof of (3)

REFERENCES

[1] M.E. Abd EI Mosef, E.F. Lashien and A.A. Nasef, some topological operators via ideals, kyungpook Mathematical Journal, vol.32 (1992)

[2] D. Jankovic and T.R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly, 1990, 97 (4).

[3] K.Kuratowski, Topologie, I.Warszawa, 1933

[4] N. Levine, Semiopen sets and semi continuity in topological spaces, Amer.Math.Monthly 70 (1963), 36-41.

[5] N. Levine, Rend. Cire. Math. Palermo, 19 (1970), 89 – 96.

[6] Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces, IJM A 3(5), (2012), 1-15.

[7] R. Vaidyanathasamy, The localization theory in set-topology, Proc. Indian Acad. Sci., 20 (1945)51-61.

[8] M.K.R.S. Veera Kumar, Mem. Fac. Sci. Kochi Univ. (Math.), 21 (2000), 1 – 19.

Source of support: Nil, Conflict of interest: None Declared

