

## ON $R^*$ - CLOSED SETS IN TOPOLOGICAL SPACES

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### ABSTRACT

*In this paper, a new class of sets called  $R^*$ -closed sets in topological spaces is introduced and their properties are discussed.*

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### 1. INTRODUCTION

Way back in 1970, Levine [11] introduced the concept of generalized closed sets. This study has been furthered by general topologists. Stone [19], Cameron [5] Gnanamabal [9], N Palaniappan and Rao [15] introduced regular open sets, regular semi open sets, generalized pre regular closed sets, regular generalized closed sets respectively. N.

Nagaveni [14], Sundaram and SheikJohn[18], S.S. Benchalli and R.S. Wali [4], Sharmishta Battacharya [17], Sanjay Mishra[16] delved into the study of weak generalized closed sets, weakly closed sets, regular w-closed sets, regularized weak closed sets, regular generalized weakly- closed sets respectively. This paper is an attempt to study a new class of sets called  $R^*$ -closed sets.

Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicitly stated. The topology of a given space  $X$  is denoted by  $\tau$  and  $(X, \tau)$  is replaced by  $X$  if there is no confusion. For  $A \subset X$ , the closure and the interior of  $A$  in  $X$  are denoted by  $\text{cl}(A)$  and  $\text{int}(A)$  respectively.

### 2. PRELIMINARIES

**Definition 2.1.** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (1) a regular open [19] if  $A = \text{int}(\text{cl}(A))$  and regular closed [19] if  $A = \text{cl}(\text{int}(A))$
- (2) a pre open [13] if  $A \subseteq \text{int}(\text{cl}(A))$  and pre closed [13] if  $\text{cl}(\text{int}(A)) \subseteq A$ .
- (3) a semi open [10] if  $A \subseteq \text{cl}(\text{int}(A))$  and semi closed [10] if  $\text{int}(\text{cl}(A)) \subseteq A$ .
- (4) a semi-preopen [1] if  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$  and semi-pre closed [1] if  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$

The intersection of all regular closed(semi- closed, pre-closed, semi- pre-closed) subset of  $(X, \tau)$  containing  $A$  is called the regular closure (semi-closure, pre-closure, semi-pre-closure resp.) of  $A$  and is denoted by  $\text{rcl}(A)$  ( $\text{scl}(A)$ ,  $\text{pcl}(A)$ ,  $\text{spcl}(A)$  resp.)

**Definition 2.2.** [6] A subset  $A$  of a space  $(X, \tau)$  is called regular semi open set, if there is a regular open set  $U$  such that  $U \subset A \subset \text{cl}(U)$ . The family of all regular semi open sets of  $X$  is denoted by  $\text{RSO}(X)$ .

**Lemma 2.3.** [5] Every regular semi open set in  $(X, \tau)$  is semi open but not conversely.

**Lemma 2.4.** [8] If  $A$  is regular semi open in  $(X, \tau)$ , then  $X \setminus A$  is also an regular semi open.

**Lemma 2.5.** [8] In a space  $(X, \tau)$ , the regular closed sets, regular open sets and clopen sets are regular semi open.

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**Definition2.6.** [8] A subset  $A$  of a space  $(X, \tau)$  is said to be semi regular open if it is both semi open and semi closed.

**Definition2.7.** A subset of a topological space  $(X, \tau)$  is called

- (1) a generalized closed (briefly g- closed) [11] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (2) a semi generalized closed (briefly sg-closed) [3] if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi open in  $X$ .
- (3) a generalized semi closed (briefly gs-closed) [2] if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (4) a generalized semi preclosed (briefly gsp- closed) [6] if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (5) a regular generalized (briefly rg-closed) [15] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .
- (6) a generalized pre-closed (briefly gp- closed) [12] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (7) a generalized pre regular closed (briefly gpr-closed) [9] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .
- (8) a weakly generalized closed (briefly wg - closed) [14] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (9) a weakly closed (briefly w-closed) [18] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi open in  $X$ .
- (10) a semi weakly generalized closed (briefly swg-closed) [14] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi open.
- (11) a regular weakly generalized closed (briefly rwg-closed) [14] if  $\text{cl}(\text{int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $X$ .
- (12) a regular w-closed (briefly rw-closed)[4] if  $\text{cl}(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular semi open in  $X$ .
- (13) a regular generalized weak closed set (briefly rgw-closed) [16] if  $\text{cl}(\text{int}(A)) \subset U$  whenever  $A \subset U$  and  $U$  is regular semi open.

The complements of the above mention closed sets are their respective open sets.

### 3. $R^*$ -CLOSED SETS AND $R^*$ -OPEN SETS

**Definition 3.1.** A subset  $A$  of a space  $(X, \tau)$  is called  $R^*$ -closed if  $\text{rcl}(A) \subset U$  whenever  $A \subset U$  and  $U$  is regular semi open in  $(X, \tau)$ .

We denote the set of all  $R^*$ - closed sets in  $(X, \tau)$  by  $R^*C(X)$ .

**Example 3.2.** Let  $X = \{a, b, c\}$   $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$

$R^*$ -closed sets are  $\{X, \emptyset, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ .

**Remark 3.3.** Closed sets and  $R^*$ -closed sets are independent of each other.

**Example 3.4.** Let  $X = \{a, b, c, d\}$   $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$   $R^*$ -closed sets are  $\{X, \emptyset, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$  Closed sets are  $\{X, \emptyset, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$

**Theorem 3.5.**

- (1).Every  $R^*$ -closed sets is rg- closed.
- (2).Every  $R^*$ -closed set is gpr- closed.
- (3) Every  $R^*$ -closed set is rwg-closed.
- (4).Every  $R^*$ - closed sets is rw-closed.
- (5).Every  $R^*$ -closed set is pr-closed.
- (6).Every  $R^*$ -closed sets is rgw- closed.

**Proof:** Straight forward.

Converse of the theorem need not be true as seen in the following example.

**Example 3.6**

a) Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Let  $A = \{c\}$ .  $A$  is rg-closed, gpr closed, rwg closed, but not  $R^*$ -closed set.

b) Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Let  $A = \{d\}$  is rw-closed set but not  $R^*$ -closed set.

c) Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

$R^*C(X) = \{\{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X, \phi\}$

Let  $A = \{c\}$ .  $A$  is pr-closed set but not  $R^*$ -closed set.

d) Let  $X = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$

$R^*C(X) = \{X, \phi, \{c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$

$A = \{b\}$  is rgw closed set but not  $R^*$ -closed set.

**Remark 3.7.** g-closed, gs-closed, gp-closed, gsp- closed sets are independent with  $R^*$ -closed sets.

**Example 3.8.** In example 3.4.  $A = \{a, b\}$  is  $R^*$ -closed set but it is not g-closed, gs-closed, gp-closed and gsp-closed.

$B = \{d\}$  is g-closed, gs-closed, gp-closed and gsp-closed but not  $R^*$ -closed set.

**Remark 3.9.** The following example shows that  $R^*$ -closed sets are independent of wg-closed, w-closed, sg-closed, swg-closed.

**Example 3.10.** Let  $X = \{a, b, c, d\}$  and  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ .

(1) Closed sets are  $\{X, \phi, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}\}$

(2)  $R^*$ -closed set are  $\{X, \phi, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ .

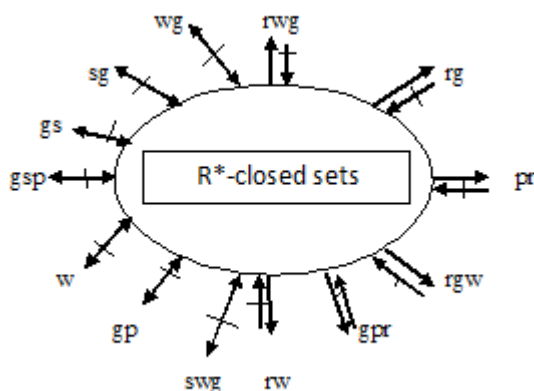
(3) wg-closed sets are  $\{\{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$ .

(4) w-closed sets are  $\{\{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$ .

(5) sg-closed sets are  $\{\{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$ .

(6) swg-closed sets are  $\{\{c\}, \{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \phi\}$ .

**Remark 3.11** From the above discussion we have the following diagram.



**Theorem 3.12.** The union of the two  $R^*$ -closed sets is an  $R^*$ -closed subset of  $X$ .

**Proof:** Assume that  $A$  and  $B$  are  $R^*$ -closed sets in  $X$ . Let  $U$  the regular semi open in  $X$  such that  $(A \cup B) \subset U$ . Then  $A \subset U$  and  $B \subset U$ . Since  $A$  and  $B$  are  $R^*$ -closed,  $\text{rcl}(A) \subset U$  and  $\text{rcl}(B) \subset U$  respectively. Hence  $\text{rcl}(A \cup B) \subset U$ . Therefore  $A \cup B$  is  $R^*$ -closed.

**Remark 3.13** The intersection of two  $R^*$ -closed sets in  $X$  need not be  $R^*$ -closed in  $X$ .

**Example 3.14.** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$ .  $R^*$ -closed sets are  $\{X, \phi, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$

Let  $A = \{a, d\}$  and  $B = \{a, b\}$

Therefore  $A \cap B = \{a\} \notin R^*$ - closed set.

**Theorem 3.15.** If a subset  $A$  of  $X$  is  $R^*$ -closed set in  $X$ , then  $\text{rcl}(A) \setminus A$  does not contain any non-empty regular semi open set in  $X$ .

**Proof:** Suppose that  $A$  is  $R^*$ -closed set in  $X$ . Let  $U$  be a regular semi open set such that  $\text{rcl}(A) \setminus A \supset U$  and  $U \neq \emptyset$ . Now  $U \subset X \setminus A$  implies  $A \subset X \setminus U$ . Since  $U$  is regular semi open, by Lemma 2.4  $X \setminus U$  is also regular semi open in  $X$ . Since  $A$  is  $R^*$ -closed in  $X$ , by definition  $\text{rcl}(A) \subset X \setminus U$ . So  $U \subset X \setminus \text{rcl}(A)$ , hence  $U \subset \text{rcl}(A) \cap X \setminus \text{rcl}(A) = \emptyset$ . This shows that  $U = \emptyset$ , which is a contradiction.

Hence  $\text{rcl}(A) \setminus A$  does not contain any non-empty regular semi open set in  $X$ .

**Remark 3.16.** If  $\text{rcl}(A) \setminus A$  contain no non-empty regular semi open subset of  $X$ , then  $A$  need not to be  $R^*$ -closed

**Example.3.17.** In example 3.1  $X = \{a, b, c, d\}$  and  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b, c\}\}$

Let  $A = \{a\}$ .  $\text{rcl}(A) \setminus A = \{c, d\}$  does not contain a non empty regular semi open set, but  $A = \{a\}$  is not  $R^*$ -closed.

**Corollary 3.18.** If a subset  $A$  of  $X$  is  $R^*$ -closed in  $X$ , then  $\text{rcl}(A) \setminus A$  does not contain any non empty regular open set in  $X$ .

**Proof:** Follows from theorem 3.13, since every regular open set is regular semi open.

**Corollary 3.19.** If a subset  $A$  of  $X$  is  $R^*$ -closed set in  $X$  then  $\text{rcl}(A) \setminus A$  does not contain any non empty regular closed set in  $X$ .

**Proof:** Follows the theorem 3.13 and the fact that every regular closed set is regular semi open.

**Theorem 3.20.** For any element  $x \in X$ . The set  $X \setminus \{x\}$  is  $R^*$ -closed or regular semi open.

**Proof:** Suppose  $X \setminus \{x\}$  is not regular semi open, then  $X$  is the only regular semi open set containing  $X \setminus \{x\}$ . This implies  $\text{rcl}\{X \setminus \{x\}\} \subset X$ . Hence  $X \setminus \{x\}$  is  $R^*$ -closed or regular semi open set in  $X$ .

**Theorem 3.21.** If  $A$  is regular open and  $R^*$ -closed. Then  $A$  is regular closed and hence  $r$ -clopen.

**Proof:** Suppose  $A$  is regular open and  $R^*$ -closed.  $A \subset A$  and by hypothesis  $\text{rcl}(A) \subset A$ . Also  $A \subset \text{rcl}(A)$ , so  $\text{rcl}(A) = A$ . Therefore  $A$  is regular closed and hence  $r$ -clopen.

**Theorem 3.22.** If  $A$  is an  $R^*$ -closed subset of  $X$  such that  $A \subset B \subset \text{rcl}(A)$ , then  $B$  is an  $R^*$ -closed set in  $X$ .

**Proof:** Let  $A$  be an  $R^*$ -closed set of  $X$  such that  $A \subset B \subset \text{rcl}(A)$ . Let  $U$  be a regular semi open set of  $X$  such that  $B \subset U$ , then  $A \subset U$ . Since  $A$  is  $R^*$ -closed, we have  $\text{rcl}(A) \subset U$ . Now  $\text{rcl}(B) \subset \text{rcl}(\text{rcl}(A)) = \text{rcl}(A) \subset U$ , therefore  $B$  is an  $R^*$ -closed set in  $X$ .

**Remark 3.23.** The converse of the theorem 3.22 need not be true.

**Example 3.24.** Consider the topological space  $(X, \tau)$ , where  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ . Let  $A = \{c\}$ ,  $B = \{c, d\}$ ,  $\text{rcl}\{c\} = \{c, d\}$

Then  $A$  and  $B$  are such that  $A \subset B \subset \text{rcl}(A)$  where  $B$  is  $R^*$ -closed set in  $(X, \tau)$  but  $A$  is not  $R^*$ -closed set in  $(X, \tau)$ .

**Theorem 3.25.** Let  $A$  be the  $R^*$ -closed in  $(X, \tau)$ . Then  $A$  is regular closed if and only if  $\text{rcl}(A) \setminus A$  is regular semi open.

**Proof:** Suppose  $A$  is regular closed in  $X$ . Then  $\text{rcl}(A) = A$  and so  $\text{rcl}(A) \setminus A = \emptyset$ , which is regular semi open in  $X$ .

Conversely, suppose  $\text{rcl}(A) \setminus A$  is regular semi open in  $X$ . Since  $A$  is  $R^*$ -closed by theorem 3.13  $\text{rcl}(A) \setminus A$  does not contain any non-empty regular semi open set in  $X$ . Then  $\text{rcl}(A) \setminus A = \emptyset$ . Hence  $A$  is regular closed in  $X$ .

**Theorem 3.26.** If a subset  $A$  of a topological space  $X$  is both regular semi open and  $R^*$ - closed, then it is regular closed.

**Proof:** By hypothesis, we have  $\text{rcl}(A) \subset A$ . Hence  $A$  is regular closed.

**Theorem 3.27.** In a topological space  $X$ , if  $\text{RSO}(X) = \{X, \emptyset\}$ , then every subset of  $X$  is an  $R^*$ -closed set.

**Proof:** Let  $X$  be a topological space and  $\text{RSO}(X) = \{X, \emptyset\}$ . Let  $A$  be any subset of  $X$ . Suppose  $A = \emptyset$ , then  $\emptyset$  is an  $R^*$ -closed set in  $X$ . Suppose  $A \neq \emptyset$ , then  $X$  is the only regular semi open set containing  $A$  and so  $\text{rcl}(A) \subset X$ . Hence  $A$  is  $R^*$ - closed set in  $X$ .

**Remark 3.28.** The converse of theorem 3.27 need not to be true in general as seen from the following example.

**Example 3.29.** Let  $X = \{a, b, c, d\}$  be with the topology  $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}\}$ . Then every subset of  $X$  is  $R^*$ -closed set in  $X$ . But  $\text{RSO}(X) = \{X, \emptyset, \{a, b\}, \{c, d\}\}$ .

**Definition 3.30.** A subset  $A$  in  $X$  is called  $R^*$ -open if  $A^c$  is  $R^*$ -closed in  $X$ .

**Theorem 3.31.** A subset  $A$  of  $X$  is said to be  $R^*$ -open if  $F \subseteq \text{rint}(A)$  whenever  $F$  is regular semi open and  $F \subseteq A$ .

**Proof: Necessity** .Let  $F$  be regular semi open such that  $F \subseteq A$ .  $X - A \subseteq X - F$ . Since  $X - A$  is  $R^*$ -closed,  $\text{rcl}(X - A) \subseteq X - F$ . Thus  $F \subseteq \text{rint}(A)$ .

**Sufficiency.** Let  $U$  be any regular semi open set such that  $X - A \subseteq U$ . We have  $X - U \subseteq A$  and by hypothesis  $X - U \subseteq \text{rint}(A)$ . That is  $\text{rcl}(X - A) = X - \text{rint}(A) \subseteq U$ . Therefore  $(X - A)$  is  $R^*$ -closed and hence  $A$  is  $R^*$ -open.

**Theorem 3.32.** Finite intersection of  $R^*$ -open sets is  $R^*$ -open.

**Proof:** Let  $A$  and  $B$  be  $R^*$ -open sets in  $X$ . Then  $A^c \cup B^c$  is  $R^*$ -closed set. This implies  $(A \cap B)^c$  is  $R^*$ -closed set. Therefore  $A \cap B$  is  $R^*$ -open.

**Theorem 3.33.** If  $A$  is  $R^*$ -closed subset of  $(X, \tau)$  and  $F$  be a regular closed set in  $\text{rcl}(A) \setminus A$ , then  $R^*$ -open set.

**Proof:** Let  $A$  be an  $R^*$ -closed subset of  $(X, \tau)$  and  $F$  be a regular closed subset such that  $F \subseteq \text{rcl}(A) - A$ . By corollary 3.19,  $F = \emptyset$  and thus  $F \subseteq \text{rint}(\text{rcl}(A) - A)$ .

By Theorem 3.31,  $\text{rcl}(A) - A$  is  $R^*$ - open.

**Lemma 3.34.** If the regular open and regular closed sets of  $X$  coincide, then all subset of  $X$  are  $R^*$ -closed (and hence all are  $R^*$ -open).

**Proof:** Let  $A$  be a subset of  $X$  which is regular open such that  $A \subseteq U$  and  $U$  is regular open, then  $\text{rcl}(A) \subseteq \text{rcl}(U) \subseteq U$ .

Therefore  $A$  is  $R^*$ -closed.

#### 4. $R^*$ -CONTINUOUS AND $R^*$ -IRRESOLUTE FUNCTIONS

**Definition 4.1.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called  $R^*$ -continuous function if every  $f^{-1}(V)$  is  $R^*$ closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .

**Definition 4.2.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called  $R^*$ -irresolute if  $f^{-1}(V)$  is  $R^*$ -closed in  $(X, \tau)$  for every  $R^*$ -closed set  $V$  in  $(Y, \sigma)$ .

**Example 4.3.** Let  $X = \{a, b, c, d\}$   $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$  and  $Y = X$

$\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ .  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the identity mapping, then  $f$  is  $R^*$ -continuous.

**Example 4.4.**  $X = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$   $Y = X$  and  $\sigma = \{Y, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$   
Define the function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = d, f(b) = a, f(c) = b, f(d) = c$ .

The inverse image of every  $R^*$ -closed sets is  $R^*$ -closed under  $f$ . Hence  $f$  is  $R^*$ -irresolute.

**Remark 4.5.** The composition of two  $R^*$ -continuous function need not be  $R^*$ -continuous.

**Example 4.6.** Let  $X = \{a, b, c, d\} = Y = Z$   $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$   
 $\sigma = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ ,  $\eta = \{X, \phi, \{a\}, \{d\}, \{a, d\}, \{a, b\}, \{a, b, d\}\}$ .

Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = d, f(c) = b, f(d) = c$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  by  $g(a) = a, g(b) = d, g(c) = b, g(d) = c$ . Here both  $f$  and  $g$  are  $R^*$ -continuous but  $g \circ f$  is not  $R^*$ -continuous.

**Remark 4.7.**  $R^*$ -continuity and continuity are independent concepts.

**Example 4.8.** Let  $X = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$  and  $Y = \{a, b, c, d\}$   $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$  defined by the function  $f: (X, \tau) \rightarrow (Y, \sigma)$ , the identity mapping.  $f$  is  $R^*$ -continuous function but not continuous.

**Example 4.9.** Let  $X = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$   $Y = \{a, b, c, d\}$   $\sigma = \{Y, \phi, \{a, b, c\}\}$   $f: X \rightarrow Y$ , the identity mapping.  $f$  is continuous but not  $R^*$ -continuous.

**Remark 4.10.** Every  $R^*$ -irresolute function is  $R^*$ -continuous but not conversely.

**Example 4.11.** Let  $X = \{a, b, c, d\}$   $\tau = \{\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}, X, \phi\}$   $Y = \{a, b, c, d\}$   $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ . Define  $f: X \rightarrow Y$ , the identity mapping.  $f$  is  $R^*$ -continuous but not  $R^*$ -irresolute.

**Theorem 4.12.**

- (a) Every  $R^*$ -continuous mapping is rw-continuous
- (b) Every  $R^*$ -continuous mapping is rg-continuous.
- (c) Every  $R^*$ -continuous mapping is pr-continuous
- (d) Every  $R^*$ -continuous mapping is rwg-continuous
- (e) Every  $R^*$ -continuous mapping is rgw-continuous
- (f) Every  $R^*$ -continuous mapping is gpr-continuous.

**Proof:** Obvious

The converse of the above need not be true as seen in the following examples.

**Example 4.13.** Consider  $X = \{a, b, c, d\}$   $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$  and  $Y = \{a, b, c, d\}$ ,  $\sigma = \{Y, \phi, \{a, b, c\}\}$ .  
Define  $f: (X, \tau) \rightarrow (Y, \sigma)$ , the identity mapping.

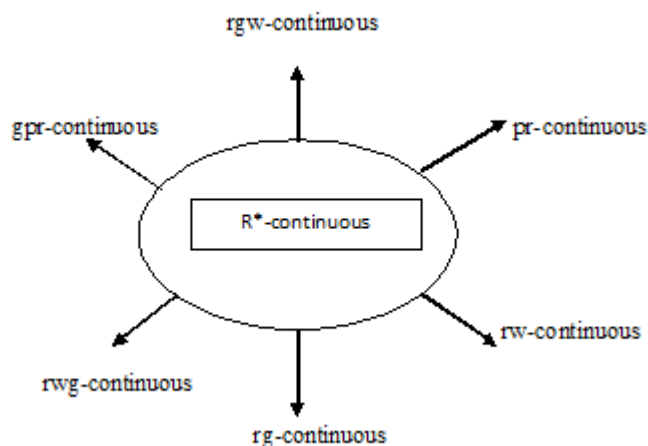
The mapping  $f$  is both rw-continuous and rg-continuous but not  $R^*$ -continuous.

**Example 4.14.**  $X = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}\}$  and  $Y = \{a, b, c, d\}$ ,  $\sigma = \{Y, \phi, \{a, b, c\}\}$

Define  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = b, f(c) = d, f(d) = c$

The function  $f$  is pr-continuous, rwg-continuous, rgw-continuous and gpr-continuous but not  $R^*$ -continuous.

**Remark 4.15.** From the above theorem the following diagram is implicated.



**Theorem:4.16.**

Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be any two function, then

- (1).  $g \circ f$  is  $R^*$ -continuous if  $g$  is continuous and  $f$  is  $R^*$ -continuous.
- (2).  $g \circ f$  is  $R^*$ -irresolute if  $g$  is  $R^*$ -irresolute and  $f$  is  $R^*$ -irresolute.
- (3).  $g \circ f$  is  $R^*$ -continuous if  $g$  is  $R^*$ -continuous and  $f$  is  $R^*$ -irresolute.

**Proof:**

- (1). Let  $V$  be closed set in  $(Z, \eta)$ . Then  $g^{-1}(V)$  is closed set in  $(Y, \sigma)$ , since  $g$  is continuous and  $R^*$ -continuity of  $f$  implies  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $R^*$ -closed in  $(X, \tau)$ . That is  $(g \circ f)^{-1}(V)$  is  $R^*$ -closed in  $(X, \tau)$ . Hence  $g \circ f$  is  $R^*$ -continuous.
- (2). Let  $V$  be  $R^*$ -closed set in  $(Z, \eta)$ . Since  $g$  is irresolute,  $g^{-1}(V)$  is  $R^*$ -closed set in  $(Y, \sigma)$ . As  $f$  is  $R^*$ -irresolute  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $R^*$ -closed in  $(X, \tau)$ . Hence  $g \circ f$  is  $R^*$ -irresolute.
- (3). Let  $V$  be closed in  $(Z, \eta)$ . Since  $g$  is  $R^*$ -continuous,  $g^{-1}(V)$  is  $R^*$ -closed in  $(Y, \sigma)$ . As  $f$  is  $R^*$ -irresolute  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $R^*$ -closed in  $(X, \tau)$ . Therefore  $(g \circ f)$  is  $R^*$ -continuous.

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