ON R*-CLOSED SETS IN TOPOLOGICAL SPACES

C. Janaki* & Renu Thomas**

*Asst. Professor, Dept of Mathematics, L. R. G Govt. Arts College for Women, Tirupur (TN), India
**Asst. Professor, Dept. of Mathematics, Sree Narayana Guru College, Coimbatore (TN), India

(Received on: 28-07-12; Revised & Accepted on: 22-08-12)

ABSTRACT

In this paper, a new class of sets called R*-closed sets in topological spaces is introduced and their properties are discussed.

Mathematical subject classification 2000: 54AO5.

Keywords: R*-closed sets, R*-open sets, R*-continuous, R*-irresoluteness.

1. INTRODUCTION


Throughout this paper, we consider spaces on which no separation axioms are assumed unless explicitly stated. The topology of a given space X is denoted by \( \tau \) and \((X, \tau)\) is replaced by X if there is no confusion. For \( A \subseteq X \), the closure and the interior of A in X are denoted by \( \text{cl}(A) \) and \( \text{int}(A) \) respectively.

2. PRELIMINARIES

Definition 2.1. A subset A of a topological space \((X, \tau)\) is called

(1) a regular open [19] if \( A = \text{int}(\text{cl}(A)) \) and regular closed [19] if \( A = \text{cl}(\text{int}(A)) \)
(2) a pre open [13] if \( A \subseteq \text{int}(\text{cl}(A)) \) and pre closed [13] if \( \text{cl}(\text{int}(A)) \subseteq A \)
(3) a semi open [10] if \( A \subseteq \text{cl}(\text{int}(A)) \) and semi closed [10] if \( \text{int}(\text{cl}(A)) \subseteq A \)
(4) a semi-preopen [1] if \( A \subseteq \text{cl}(\text{int}(\text{cl}(A))) \) and semi-pre closed [1] if \( \text{int}(\text{cl}(\text{int}(A))) \subseteq A \)

The intersection of all regular closed(semi-closed, pre-closed, semi-pre-closed) subset of \((X, \tau)\) containing A is called the regular closure (semi-closure, pre-closure, semi-pre-closure resp.) of A and is denoted by \( rcl(A) \) (\( scl(A), pcl(A), spcl(A) \) resp.)

Definition 2.2. [6] A subset A of a space \((X, \tau)\) is called regular semi open set, if there is a regular open set \( U \) such that \( U \subseteq A \subseteq \text{cl}(U) \). The family of all regular semi open sets of \( X \) is denoted by \( \text{RSO}(X) \).

Lemma 2.3. [5] Every regular semi open set in \((X, \tau)\) is semi open but not conversely.

Lemma 2.4. [8] If \( A \) is regular semi open in \((X, \tau)\), then \( X \setminus A \) is also an regular semi open.

Lemma 2.5. [8] In a space \((X, \tau)\), the regular closed sets, regular open sets and clopen sets are regular semi open.
Definition 2.6. [8] A subset $A$ of a space $(X, \tau)$ is said to be semi regular open if it is both semi open and semi closed.

Definition 2.7. A subset of a topological space $(X, \tau)$ is called

1. a generalized closed (briefly g-closed) [11] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
2. a semi generalized closed (briefly sg-closed) [3] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $X$.
3. a generalized semi closed (briefly gs-closed) [2] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
4. a generalized semi preclosed (briefly gsp-closed) [6] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $X$.
5. a regular generalized (briefly rg-closed) [15] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
6. a generalized pre-closed (briefly gp-closed) [12] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
7. a generalized pre regular closed (briefly gpr-closed) [9] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
8. a weakly generalized closed (briefly wg-closed) [14] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
9. a weakly closed (briefly w-closed) [18] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open in $X$.
10. a semi weakly generalized closed (briefly swg-closed) [14] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open.
11. a regular weakly generalized closed (briefly rwg-closed) [14] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
12. a regular w-closed (briefly rw-closed)[4] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular semi open in $X$.
13. a regular generalized weak closed set (briefly rgw-closed) [16] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular semi open.

The complements of the above mentioned closed sets are their respective open sets.

3. **R*-CLOSED SETS AND R*-OPEN SETS**

Definition 3.1. A subset $A$ of a space $(X, \tau)$ is called R*-closed if $\text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular semi open in $(X, \tau)$.

We denote the set of all R*-closed sets in $(X, \tau)$ by $R^*C(X)$.

Example 3.2. Let $X= \{a, b, c\}$ $\tau= \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$

R*-closed sets are $\{X, \phi, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Remark 3.3. Closed sets and R*-closed sets are independent of each other.

Example 3.4. Let $X= \{a, b, c, d\}$ $\tau= \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

R*-closed sets are $\{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Theorem 3.5.

(1) Every R*-closed set is rg-closed.
(2) Every R*-closed set is gpr-closed.
(3) Every R*-closed set is rwg-closed.
(4) Every R*-closed set is rw-closed.
(5) Every R*-closed set is pr-closed.
(6) Every R*-closed set is rgw-closed.

Proof: Straight forward.

Converse of the theorem need not be true as seen in the following example.

Example 3.6

a) Let $X= \{a, b, c, d\}$ and $\tau= \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$

Let $A= \{c\}$. $A$ is rg-closed, gpr closed, rwg closed, but not R*-closed set.

b) Let $X= \{a, b, c, d\}$ and $\tau= \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$
Let $A = \{d\}$ is rw-closed set but not $R^*$-closed set.

c) Let $X = \{a, b, c, d,\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$

$R^*C(X) = \{\{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X, \emptyset\}$

Let $A = \{c\}$. $A$ is pr-closed set but not $R^*$-closed set.

d) Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{a, b\}\}$

$R^*C(X) = \{X, \emptyset, \{c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$

$A = \{b\}$ is rgw closed set but not $R^*$-closed set.

**Remark 3.7.** $g$-closed, $gs$-closed, $gp$-closed, $gsp$-closed sets are independent with $R^*$-closed sets.

**Example 3.8.** In example 3.4. $A = \{a, b\}$ is $R^*$-closed set but it is not $g$-closed, $gs$-closed, $gp$-closed and $gsp$-closed.

$B = \{d\}$ is $g$-closed, $gs$-closed, $gp$-closed and $gsp$-closed but not $R^*$-closed set.

**Remark 3.9.** The following example shows that $R^*$-closed sets are independent of $wg$-closed, $w$-closed, $sg$-closed, $swg$-closed.

**Example 3.10.** Let $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$.

1. Closed sets are $\{X, \emptyset, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{d\}\}$

2. $R^*$-closed set are $\{X, \emptyset, \{a, b\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}.$

3. $wg$-closed sets are $\{\{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X, \emptyset\}.$

4. $w$-closed sets are $\{\{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \emptyset\}.$

5. $sg$-closed sets are $\{\{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, X, \emptyset\}.$

6. $swg$-closed sets are $\{\{c\}, \{d\}, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X, \emptyset\}.$

**Remark 3.11.** From the above discussion we have the following diagram.

![Diagram of Relationships between Set Types]

**Theorem 3.12.** The union of the two $R^*$-closed sets is an $R^*$-closed subset of $X$.

**Proof:** Assume that $A$ and $B$ are $R^*$-closed sets in $X$. Let $U$ the regular semi open in $X$ such that $(A \cup B) \subset U$. Then $A \subset U$ and $B \subset U$. Since $A$ and $B$ are $R^*$-closed, $rcl(A) \subset U$ and $rcl(B) \subset U$ respectively. Hence $rcl(A \cup B) \subset U$. Therefore $A \cup B$ is $R^*$-closed.

**Remark 3.13** The intersection of two $R^*$-closed sets in $X$ need not be $R^*$-closed in $X$.

**Example 3.14.** Let $X = \{a, b, c, d\}, \tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}\}$. $R^*$-closed sets are $\{X, \emptyset, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}.$

© 2012, IJMA. All Rights Reserved
Let \( A = \{a, d\} \) and \( B = \{a, b\} \)

Therefore \( A \cap B = \{a\} \notin R^*\)-closed set.

**Theorem 3.15.** If a subset \( A \) of \( X \) is \( R^*\)-closed set in \( X \), then \( rcl(A) \setminus A \) does not contain any non-empty regular semi open set in \( X \).

**Proof:** Suppose that \( A \) is \( R^*\)-closed set in \( X \). Let \( U \) be a regular semi open set such that \( rcl(A) \setminus A \supset U \) and \( U \neq \emptyset \). Now \( U \subseteq X \setminus A \) implies \( A \subseteq X \setminus U \). Since \( U \) is regular semi open, by Lemma 2.4 \( X \setminus U \) is also regular semi open in \( X \). Since \( A \) is \( R^*\)-closed in \( X \), by definition \( rcl(A) \subseteq X \setminus U \). So \( U \subseteq X \setminus rcl(A) \), hence \( U \subseteq rcl(A) \cap X \setminus rcl(A) = \emptyset \). This shows that \( U = \emptyset \), which is a contradiction.

Hence \( rcl(A) \setminus A \) does not contain any non-empty regular semi open set in \( X \).

**Remark 3.16.** If \( rcl(A) \setminus A \) contain no non-empty regular semi open subset of \( X \), then \( A \) need not to be \( R^*\)-closed

**Example 3.17.** In example 3.1 \( X = \{a, b, c, d\} \) and \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b, c\}\} \)

Let \( A = \{a\} \). \( rcl(A) \setminus A = \{c, d\} \) does not contain a non-empty regular semi open set, but \( A = \{a\} \) is not \( R^*\)-closed.

**Corollary 3.18.** If a subset \( A \) of \( X \) is \( R^*\)-closed in \( X \), then \( rcl(A) \setminus A \) does not contain any non-empty regular open set in \( X \).

**Proof:** Follows from theorem 3.13, since every regular open set is regular semi open.

**Corollary 3.19.** If a subset \( A \) of \( X \) is \( R^*\)-closed set in \( X \) then \( rcl(A) \setminus A \) does not contain any non-empty regular closed set in \( X \).

**Proof:** Follows the theorem 3.13 and the fact that every regular closed set is regular semi open.

**Theorem 3.20.** For any element \( x \in X \). The set \( X \setminus \{x\} \) is \( R^*\)-closed or regular semi open.

**Proof:** Suppose \( X \setminus \{x\} \) is not regular semi open, then \( X \) is the only regular semi open set containing \( X \setminus \{x\} \). This implies \( rcl(X \setminus \{x\}) \subseteq X \). Hence \( X \setminus \{x\} \) is \( R^*\)-closed or regular semi open set in \( X \).

**Theorem 3.21.** If \( A \) is regular open and \( R^*\)-closed. Then \( A \) is regular closed and hence \( r\)-clopen.

**Proof:** Suppose \( A \) is regular open and \( R^*\)-closed. \( A \subseteq A \) and by hypothesis \( rcl(A) \subseteq A \). Also \( A \subseteq rcl(A) \), so \( rcl(A) \cap A \). Therefore \( A \) is regular closed and hence \( r\)-clopen.

**Theorem 3.22.** If \( A \) is an \( R^*\)-closed subset of \( X \) such that \( A \subseteq B \subseteq rcl(A) \), then \( B \) is an \( R^*\)-closed set in \( X \).

**Proof:** Let \( A \) be an \( R^*\)-closed set of \( X \) such that \( A \subseteq B \subseteq rcl(A) \). Let \( U \) be a regular semi open set of \( X \) such that \( B \subseteq U \), then \( A \subseteq U \). Since \( A \) is \( R^*\)-closed, we have \( rcl(A) \subseteq U \). Now \( rcl(B) \subseteq rcl(rcl(A)) = rcl(A) \subseteq U \), therefore \( B \) is an \( R^*\)-closed set in \( X \).

**Remark 3.23.** The converse of the theorem 3.22 need not be true.

**Example 3.24.** Consider the topological space \( (X, \tau) \), where \( X = \{a, b, c, d\} \),

\[ \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \]

Let \( A = \{c\}, B = \{c, d\}, rcl\{c\} = \{c, d\} \)

Then \( A \) and \( B \) are such that \( A \subseteq B \subseteq rcl(A) \) where \( B \) is \( R^*\)-closed set in \( (X, \tau) \) but \( A \) is not \( R^*\)-closed set in \( (X, \tau) \).

**Theorem 3.25.** Let \( A \) be the \( R^*\)-closed in \( (X, \tau) \). Then \( A \) is regular closed if and only if \( rcl(A) \setminus A \) is regular semi open.

**Proof:** Suppose \( A \) is regular closed in \( X \). Then \( rcl(A) = A \) and so \( rcl(A) \setminus A = \emptyset \), which is regular semi open in \( X \).

Conversely, suppose \( rcl(A) \setminus A \) is regular semi open in \( X \). Since \( A \) is \( R^*\)-closed by theorem 3.13 \( rcl(A) \setminus A \) does not contain any non-empty regular semi open set in \( X \). Then \( rcl(A) \setminus A = \emptyset \). Hence \( A \) is regular closed in \( X \).
Theorem 3.26. If a subset A of a topological space X is both regular semi open and $R^*$-closed, then it is regular closed.

**Proof:** By hypothesis, we have $rcl(A) \subseteq A$. Hence A is regular closed.

Theorem 3.27. In a topological space X, if $RSO(X) = \{X, \emptyset\}$, then every subset of X is an $R^*$-closed set.

**Proof:** Let X be a topological space and $RSO(X) = \{X, \emptyset\}$. Let A be any subset of X. Suppose $A = \emptyset$, then $\emptyset$ is an $R^*$-closed set in X. Suppose $A \neq \emptyset$, then X is the only regular semi open set containing A and so $rcl(A) \subseteq X$. Hence A is $R^*$-closed set in X.

Remark 3.28. The converse of theorem 3.27 need not to be true in general as seen from the following example.

Example 3.29. Let $X = \{a, b, c, d\}$ be with the topology $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}\}$. Then every subset of X is $R^*$-closed set in X. But $RSO(X) = \{X, \emptyset, \{a, b\}, \{c, d\}\}$.

Definition 3.30. A subset A in X is called $R^*$-open if $A^c$ is $R^*$-closed in X.

Theorem 3.31. A subset A of X is said to be $R^*$-open if $F \subseteq rint(A)$ whenever F is regular semi open and $F \subseteq A$.

**Proof:** Necessity. Let F be regular semi open such that $F \subseteq A$. $X - A \subseteq X - F$. Since $X - A$ is $R^*$-closed, $rcl(X - A) \subseteq X - F$. Thus $F \subseteq rint(A)$.

Sufficiency. Let U be any regular semi open set such that $X - A \subseteq U$. We have $X - U \subseteq A$ and by hypothesis $X - U \subseteq rint(A)$. That is $rcl(X - A) = X - rint A \subseteq U$. Therefore $(X - A)$ is $R^*$-closed and hence A is $R^*$-open.

Theorem 3.32. Finite intersection of $R^*$-open sets is $R^*$-open.

**Proof:** Let A and B be $R^*$-open sets in X. Then $A^c \cup B^c$ is $R^*$-closed set. This implies $(A \cap B)^c$ is $R^*$-closed set. Therefore $A \cap B$ is $R^*$-open.

Theorem 3.33. If A is $R^*$-closed subset of $(X, \tau)$ and F be a regular closed set in $rcl(A) \setminus A$, then $R^*$-open set.

**Proof:** Let A be an $R^*$-closed subset of $(X, \tau)$ and F be a regular closed set such that $F \subseteq rcl(A) - A$. By corollary 3.19, $F = \emptyset$ and thus $F \subseteq rint(rcl(A) - A)$.

By Theorem 3.31, $rcl(A) - A$ is $R^*$-open.

Lemma 3.34. If the regular open and regular closed sets of X coincide, then all subset of X are $R^*$-closed (and hence all are $R^*$-open).

**Proof:** Let A be a subset of X which is regular open such that $A \subseteq U$ and U is regular open, then $rcl(A) \subseteq rcl(U) \subseteq U$.

Therefore A is $R^*$-closed.

4. $R^*$-CONTINUOUS AND $R^*$-IRRESOLUTE FUNCTIONS

Definition 4.1. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $R^*$-continuous function if every $f^{-1}(V)$ is $R^*$-closed in $(X, \tau)$ for every closed set $V$ in $(Y, \sigma)$.

Definition 4.2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $R^*$-irresolute if $f^{-1}(V)$ is $R^*$-closed in $(X, \tau)$ for every $R^*$-closed set $V$ in $(Y, \sigma)$.

Example 4.3. Let $X = \{a, b, c, d\}$ $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$ and $Y = X$ $\sigma = \{Y, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity mapping, then $f$ is $R^*$-continuous.
Example 4.4. \( X = \{a, b, c, d\} \) \( \tau = \{X, \phi, \{a\}, \{a, b, c, d\}, \{a, b\}, \{a, b, c\}\} \) \( Y = X \) and \( \sigma = \{Y, \phi, \{a\}, \{a, c\}, \{a, b\}\} \)

Define the function \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = d, f(b) = a, f(c) = b, f(d) = c \).

The inverse image of every \( R^* \)-closed sets is \( R^* \)-closed under \( f \). Hence \( f \) is \( R^* \)-irresolute.

Remark 4.5. The composition of two \( R^* \)-continuous function need not be \( R^* \)-continuous.

Example 4.6. Let \( X = \{a, b, c, d\} = Y = Z \) \( \tau = \{X, \phi, \{a\}, \{a, c\}, \{a, b\}\} \) \( \sigma = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}\} \).

Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = d, f(c) = b, f(d) = c \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) by \( g(a) = a, g(b) = d, g(c) = b, g(d) = c \). Here both \( f \) and \( g \) are \( R^* \)-continuous but \( g \circ f \) is not \( R^* \)-continuous.

Remark 4.7. \( R^* \)-continuity and continuity are independent concepts.

Example 4.8. Let \( X = \{a, b, c, d\} \) \( \tau = \{X, \phi, \{a\}, \{a, c\}, \{a, b\}\} \) and \( Y = \{a, b, c\} \) \( \sigma = \{Y, \phi, \{a\}, \{a, c\}\} \).

Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b, f(c) = d, f(d) = c \).

The function \( f \) is \( R^* \)-continuous but not \( R^* \)-irresolute.

Example 4.9. Let \( X = \{a, b, c, d\} \) \( \tau = \{X, \phi, \{a\}, \{a, b\}\} \) \( \sigma = \{Y, \phi, \{a\}, \{b\}\} \).

Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b, f(c) = d, f(d) = c \).

The function \( f \) is \( R^* \)-continuous but not \( R^* \)-irresolute.

Theorem 4.12.

(a) Every \( R^* \)-continuous mapping is \( rw \)-continuous

(b) Every \( R^* \)-continuous mapping is \( rg \)-continuous.

(c) Every \( R^* \)-continuous mapping is \( pr \)-continuous

(d) Every \( R^* \)-continuous mapping is \( rwg \)-continuous

(e) Every \( R^* \)-continuous mapping is \( rgw \)-continuous

(f) Every \( R^* \)-continuous mapping is \( gpr \)-continuous.

Proof: Obvious

The converse of the above need not be true as seen in the following examples.

Example 4.13. Consider \( X = \{a, b, c, d\} \) \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b, c\}\} \) and \( Y = \{a, b, c, d\} \) \( \sigma = \{Y, \phi, \{a, b, c\}\} \).

Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) the identity mapping.

The mapping \( f \) is both \( rw \)-continuous and \( rg \)-continuous but not \( R^* \)-continuous.

Example 4.14. \( X = \{a, b, c, d\} \) \( \tau = \{X, \phi, \{a\}, \{b, c\}\} \) and \( Y = \{a, b, c, d\} \) \( \sigma = \{Y, \phi, \{a, b, c\}\} \).

Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b, f(c) = d, f(d) = c \).

The function \( f \) is \( pr \)-continuous, \( rwg \)-continuous, \( rgw \)-continuous and \( gpr \)-continuous but not \( R^* \)-continuous.
Remark 4.15. From the above theorem the following diagram is implicated.

Theorem: 4.16.
Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions, then

1. \( g \circ f \) is \( R^* \)-continuous if \( g \) is \( \alpha \)-continuous and \( f \) is \( R^* \)-continuous.
2. \( g \circ f \) is \( R^* \)-irresolute if \( g \) is \( R^* \)-irresolute and \( f \) is \( R^* \)-irresolute.
3. \( g \circ f \) is \( R^* \)-continuous if \( g \) is \( R^* \)-continuous and \( f \) is \( R^* \)-irresolute.

Proof:
(1). Let \( V \) be closed set in \( (Z, \eta) \). Then \( g^{-1}(V) \) is closed set in \( (Y, \sigma) \), since \( g \) is continuous and \( R^* \)-continuity of \( f \) implies \( f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \) is \( R^* \)-closed in \( (X, \tau) \). That is \( (g \circ f)^{-1}(V) \) is \( R^* \)-closed in \( (X, \tau) \). Hence \( g \circ f \) is \( R^* \)-continuous.

(2). Let \( V \) be \( R^* \)-closed set in \( (Z, \eta) \). Since \( g \) is irresolute, \( g^{-1}(V) \) is \( R^* \)-closed set in \( (Y, \sigma) \). As \( f \) is \( R^* \)-irresolute \( f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \) is \( R^* \)-closed in \( (X, \tau) \). Hence \( g \circ f \) is \( R^* \)-irresolute.

(3). Let \( V \) be closed in \( (Z, \eta) \). Since \( g \) is \( R^* \)-continuous, \( g^{-1}(V) \) is \( R^* \)-closed in \( (Y, \sigma) \). As \( f \) is \( R^* \)-irresolute \( f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \) is \( R^* \)-closed in \( (X, \tau) \). Therefore \( g \circ f \) is \( R^* \)-continuous.

REFERENCES

© 2012, IUMA. All Rights Reserved


Source of support: Nil, Conflict of interest: None Declared