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MHD Flow Over a Nonlinear Stretching Sheet: An Approximate Analytical Solution

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ABSTRACT

The magnetohydrodynamic (MHD) flow of a viscous incompressible and electrically conducting fluid over a continuously stretching surface is considered. The governing equation is derived as a function of two parameters: the magnetic parameter M and stretching parameter β and is solved via the method of stretching of variables which is one of the best available approximate methods. The results are compared and are in good agreement with that of direct numerical solution and previous studies. The effects of the above parameters are shown on velocity profile and skin friction coefficient.

Keywords: *Boundary layer flow; Stretching sheet; MHD; Approximate method; Least squares; Defect f*unction.

1. INTRODUCTION

A theoretical study of the boundary layer flow of an incompressible viscous fluid over a continuously stretching sheet is a subject of interest since it has many engineering applications that include cooling of a metallic plate in a cooling bath, aerodynamic extrusion of plastic sheets, hot rolling, wire drawing, metal extrusion and spinning, drawing of copper wires, etc. These flows are also encountered in the glass and polymer industry. In one of the earliest studies, Sakiadis [1] theoretically investigated the boundary layer flow over a continuously stretching sheet which moves with constant speed and solved the boundary value problem numerically. Results of Sakiadis [1] were confirmed experimentally by Tsou et al. [2]. Crane [3] obtained an exact solution for the flow of an incompressible fluid over a linearly stretching sheet. Afzal [4] studied heat transfer from a arbitrarily stretching surface and gave closed form solutions for some parameters and solved numerically for other parameters. Batallar [5] gives the numerical study of a viscous flow over a nonlinear stretching sheet with heat transfer. This study includes the sheet with prescribed surface temperature and heat flux. Numerous other stretching sheet problems have been discussed, for example, the problem of suction/injection on the sheet surface has been studied by Erickson [6], Gupta and Gupta [7] have analyzed this problem at a constant surface temperature, Siddappa and Khapate [8] and Rajagopal et al. [9] have studied by including viscoelastic fluids and showed that as viscoelastic parameter increases the skin friction decreases.

All these studies did not include the influence of magnetohydrodynamic (MHD) viscous flow. MHD flows are often encountered in applications such as power generators, electrostatic filters, cooling reactors, etc. MHD flow controls forces and stabilizes the boundary layer flow. MHD flow also appears in industrial applications such as the cooling of continuous strips or filaments, annealing and thinning of copper wires, etc. The properties of final product depends on the rate of cooling. In an uniform magnetic field, rate of cooling of sheets can be controlled and desired characteristics of product are obtained. Application of MHD flows in the purification of molten metals from non-metallic ones is done by applying uniform magnetic field. Pavlov [10] studied the MHD flow over a stretching sheet which has practical applications in polymer industry and gave the exact solution of boundary layer equation. Many aspects of MHD flow on a stretching surface were reported in the literature, for example, Kumaran et al. [11] have obtained an exact solution of MHD flow past a quadratically stretching sheet with suction/injection, Ishak et al. [12] studied the two-dimensional MHD stagnation point flow towards a stretching sheet with variable surface temperature and solve the problem both analytically, in circumstances, in which an exact solution to the problem is obtained to some parameters and otherwise numerically using Keller-box method, homotopy analysis method is used by Ghotbi [13] to study MHD viscous flow over a non-linear stretching sheet and obtained a uniformly valid Taylor series based analytical solution, Hayat et al. [14] have investigated the above problem and solved the boundary layer equations using series solution based on modified decomposition method and Pad´e approximants, etc.

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Each problem involving boundary layer equations contains many involved parameters and needs to solve an entire set of partial differential equations which is a major computation task. Indeed an amount of work involved is so high that it is rarely attempted. Because of this complexity, it is necessary to devise other methods for boundary layer analysis, most of which do not require them to be solved exactly. All these methods are approximate by nature, but most of them give better accuracy in lesser time and can be used for wide range of physical variables. One of such approximate methods is the method of stretching of variables which is applied for the solution of MHD viscous flow over a non-linear stretching sheet problem. This method is quite easy to use especially for nonlinear ordinary differential equations and requires less computer time compared to numerical method (for example, shooting method, Keller-box method etc.) and easy to solve compared to other approximate methods (for example, modified decomposition, homotopy analysis method etc.). Exploring this method for the solution of the Falkner-Skan equation, Kudenatti and Awati [15] compared the solution with their new exact analytical solution for various parameters involved and showed that there is a nice agreement between this method and direct numerical solution. Kudenatti et al. [16] have successfully used this approximate analytical method to solve a class of boundary value problems for non-linear stretching sheet. So, this approximate analytical method is rather general and can be applied to other class of non-linear problems in science and engineering.

Presentation of the paper proceeds as follows. In the next section mathematical formulation of the problem in question is derived along with the boundary conditions. Section 3 gives the solution to the problem by means of method of stretching of variables. The results thus obtained are plotted in Figures and Tables. These results are discussed in detail in Section 4. In the concluding section, summary of the paper is given.

2. MATHEMATICAL FORMULATION

Consider the steady two-dimensional MHD flow of an incompressible viscous fluid over a stretching sheet moving with constant velocity. The *y*-axis is extending along the upward direction and normal to the surface of the sheet and *x* is extending along the parallel to the sheet. Under the absence of induced magnetic field, the magnetic field *B* is applied normal to the stretching sheet. The set of boundary layer equations is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u \tag{2.2}$$

where *u* and *v* are the velocity components in the *x* and *y* directions, υ is the kinematic viscosity, σ is the electrical conductivity, ρ is the fluid density of the fluid and $B(x)=B_0x^{(n-1)/2}$. The corresponding boundary conditions are

at $\quad y=0\ :\ u=U,\quad v=0,$ (2.3a)

and as $y\to\infty\ :\ u=0.$ (2.3b)

Introducing the stream functions $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$ which satisfy the equation (2.1) where

$$\psi=\sqrt{\frac{2\nu xU(x)}{1+n}}f(\eta),\quad \eta=y\sqrt{\frac{(1+n)U(x)}{2\nu x}},\ \text{and } U(x)=U_\infty x^n \tag{2.4}$$

Using the above similarity transformations, the system (2.2)-(2.3) can be converted into a nonlinear ordinary differential equation as

$$f'''+ff''-\beta f'^2-Mf'=0 \quad '=\frac{d}{d\eta}, \tag{2.5}$$

and the boundary conditions take the form as

$$f(0)=0,\qquad f'(0)=1,\qquad f'(\infty)=0, \tag{2.6}$$

where *f* is the dimensionless stream function, $\beta=2n/n+1$ is the nonlinear stretching parameter and $M=2\sigma B_0^2/\rho U_\infty(n+1)$ is the Hartman (magnetic) parameter. Note that *U=U(x)* is used to define dimensionless stream function f and n is a positive constant. The system (2.5)-(2.6) readily admits an exact analytical solution for linearly stretching boundary $\beta = 1\ (n = 1)$

$$f = \frac{1 - \exp\left(-\sqrt{1+M}\eta\right)}{\sqrt{1+M}} \tag{2.7}$$

and for other value of β, we present an approximate analytical method in the next section.

3. METHOD OF STRETCHING OF VARIABLES

Many attempts have been tried to develop methods for the solution of boundary value problems over an infinite domain, which are necessarily approximate methods because these are not universally valid and the accurate solutions. For example, Pad´e approximants have been used in modified decomposition method to accelerate the convergence of the series solution. Also, some numerical techniques require special treatment of the boundary condition at infinity. But method of stretching of variables gives the best approximate solution. It provides us a simple way to find out the required derived quantities. In the method of stretching of variables, we have to choose suitable velocity profile f' such that the derivative boundary conditions satisfied automatically and integration of f' will satisfy the remaining boundary conditions. Substitution of this resulting function into the given equation gives the residual of the form $R(\xi,\alpha)$ and it is called defect function. Using least squares method, the residual of the defect function can be minimized. The solution to the third order nonlinear boundary value problem over an infinite domain characterizing the flow of a viscous fluid impinging normally to a wall from which the fluid is extracted at a uniform rate has been given by Ariel [17] using method of stretching of variables. Chakraborty and Mazumdar [18] have given an approximate solution to the problem of steady laminar flow of MHD fluid over a stretching sheet. More details about the method of stretching of variables are given in [17, 18]. We introduce two variables ξ and F as

$$\xi = \alpha\eta,\ F(\xi) = \alpha f(\eta) \tag{3.1}$$

where $\alpha \succ 0$ is an amplification factor. In view of equation (3.1),equation (2.5) transformed to

$$\alpha^2 F''' + FF'' - \beta F'^2 - MF' = 0, \qquad ' = \frac{d}{d\xi} \tag{3.2}$$

and the boundary conditions in (2.6) become

$$F(0) = 0, \quad F'(0) = 1, \quad F'(\infty) = 0. \tag{3.3}$$

We choose a trail velocity profile

$$F' = \exp(-\xi) \tag{3.4}$$

which automatically satisfies the derivatives boundary conditions in (3.3). Integrating this velocity profile from 0 to ξ using first boundary condition in (3.3) and then substituting this result into equation (3.2), we get

$$R(\xi,\alpha) = (1-\beta)\exp(-2\xi) + \left(\alpha^2 - M - 1\right)\exp(-\xi) \tag{3.5}$$

$R(\xi,\alpha)$ in the above expression is called the residual or defect function of the equation (3.2). With the help of least squares method as discussed by Ariel [18], for minimizing the defect function (3.5), we take

$$\frac{\partial}{\partial\alpha}\int_0^\infty R^2(\xi,\alpha)d\xi = 0. \tag{3.6}$$

Substituting (3.5) into equation (3.6) and solving cubic equation in α for positive root, we get

$$\alpha = \frac{1}{\sqrt{3}}(1 + 3M + 2\beta)^{\frac{1}{2}}. \tag{3.7}$$

Once the amplification factor is calculated, then using equation (3.1), original function f can be written as

$$f = \frac{1}{\alpha}(1 - \exp(-\alpha\eta)) \tag{3.8}$$

with α defined in (3.7). Thus, equation (3.8) gives the solution of the system (2.5)-(2.6) for all values of stretching parameter β and magnetic parameter M. It is interesting to note that the exact solution given in (2.7) is well established from equation (3.8) for $\beta = 1$.

4. RESULTS AND DISCUSSION

Application of method of stretching of variables to investigate the effects of various physical parameters on viscous flow over a non-linearly stretching sheet has revealed the important features of boundary layer flow when subjected to a uniform magnetic field. The method provides us with a simple way to compute the accurate solution and ensures less computer time. All solutions have been examined as a function of stretching parameter β and magnetic parameter M. Solution (3.8) to the system (2.5)-(2.6) for all values of β and M embeds the known exact solution (2.7) for $\beta = 1$ as a special case.

In order to check the accuracy of the results from equation (3.8), Table 1 compares the values of skin friction $f''(0)$ with that of direct numerical solution of the system (2.5)-(2.6) for different parameters of β and M. It is found that there is a nice agreement between two solutions. Every effort was made to obtain a high accuracy solution for the values of large M. The possibility of slightly inaccurate solution for small M is clearly associated with the slower convergence approach to mainstream conditions. Also from the Table 1, it is interesting to note that for higher values of M (i.e. $3M \geq 2\beta$ for given β), the skin friction changes marginally. As the application of magnetic field is increased, the magnitude of the skin friction $f''(0)$, as visible in these solutions, increases for all values of the parameter β. This also has been shown in the Figure 1 for different values of M. Solutions that are obtained by Hayat et al.[14] can also be recovered. They have computed the skin friction using modified decomposition method with 15th order Pad´e approximants and results are comparable to the present method

Some of the computed velocity profiles $f'(\eta)$ are presented in Figures 2(a-d) for different values of parameters M and β. These figures demonstrate that the velocity of the fluid decreases with increasing M. It is well-known that, the application of uniform magnetic field normal to the flow direction leads to the so-called Lorentz force (LF). Thus, the variation of M leads to change in the LF which has tendency to resist the velocity of the fluid. Hence, effect of magnetic field is to reduce the velocity component parallel to the stretching surface. A common feature of these profiles, irrespective of β, is that f' decreases for increasing M.

5 CONCLUSIONS

In this paper, the method of stretching of variables is applied to theoretically study the effect of magnetic field on boundary layer flow of viscous fluid over a nonlinear stretching sheet. Results thus obtained are compared with direct numerical solution and also with previous studies. It is found that the method gives an explicit analytical expression in terms of the parameters involved and turns out to be the best approximate method for nonlinear problems. It is shown that magnitude of skin friction increases with increasing M and β.

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Table 1: Comparison of values of skin friction $f''(0)$ obtained from the expression (3.8) with that of direct numerical solution for the stretching and magnetic parameters.

M	$\beta=0.0$		$\beta=1.0$	
	Present Method	Numerical	Present Method	Numerical
1	-1.15470	-1.16329	-1.41421	-1.41422
2	-1.52752	-1.53131	-1.73205	-1.73205
5	-2.30940	-2.31048	-2.44948	-2.44949
10	-3.21455	-3.21494	3.31662	-3.31662
20	-4.50924	-4.50933	-4.58257	-4.58258
50	-7.09459	-7.09462	-7.14142	-7.14143
100	-10.01665	-10.01666	-10.04987	-10.04988
500	-22.36813	-22.36813	-22.38302	-22.38303
1000	-31.62804	-31.62804	-31.63858	-31.63858

M	$\beta=1.5$		$\beta=5.0$	
	Present Method	Numerical	Present Method	Numerical
1	-1.52752	-1.52528	-2.16024	-2.15290
2	-1.82574	-1.82450	-2.38047	-2.37538
5	-2.51661	-2.51618	-2.94392	-2.94152

10	-3.36650	-3.36633	-3.69684	-3.69582
20	-4.61880	-4.61877	-4.86483	-4.86446
50	-7.16472	-7.16472	-7.32575	-7.32569
100	-10.06644	-10.06644	-10.18168	-10.18167
500	-22.39047	-22.39048	-22.44251	-22.44254
1000	-31.64385	-31.64385	-31.68069	-31.68071

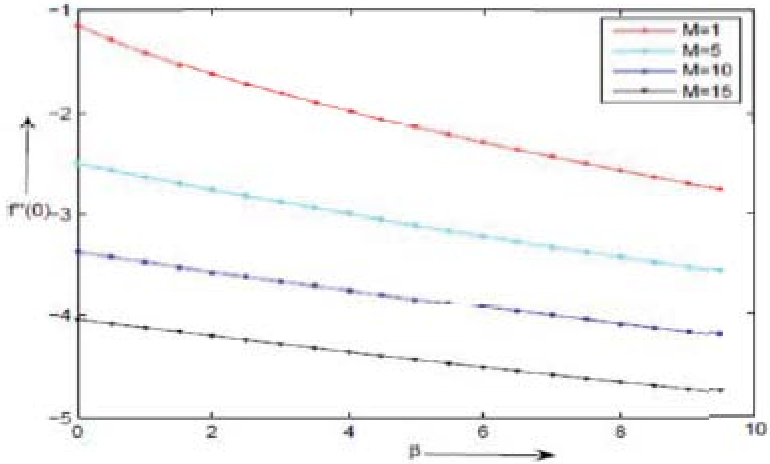


Figure 1: Variation of Skin friction with stretching parameter for different values of Magnetic parameter.

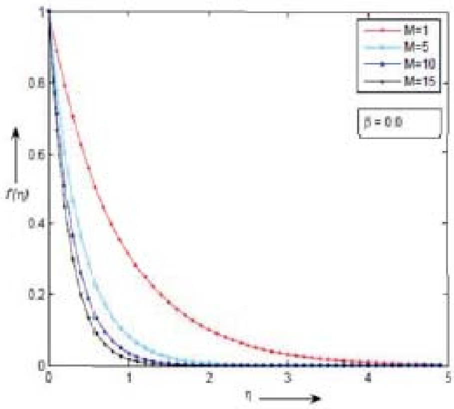
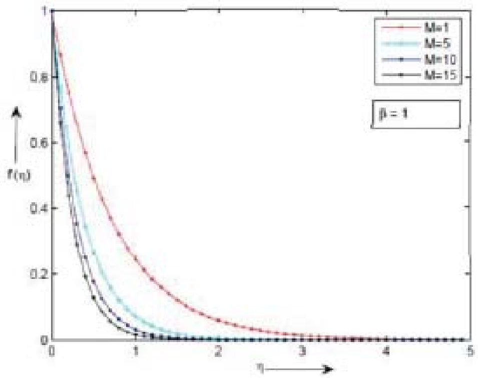


Fig. 2(a):



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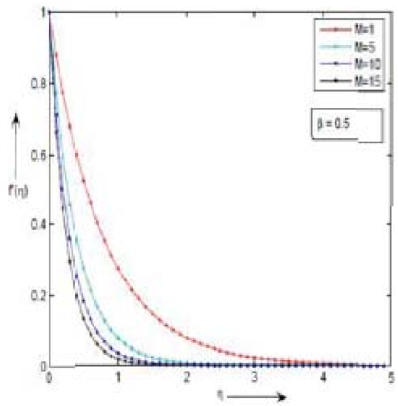

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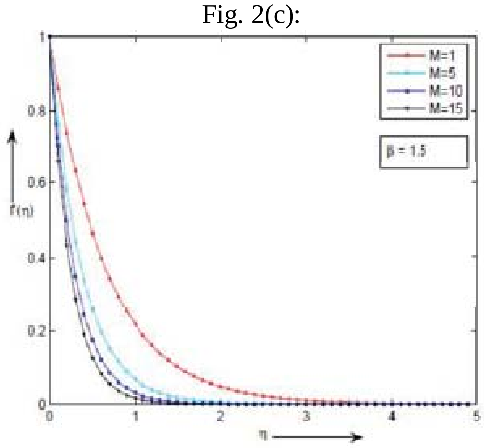

Fig. 2(d):

Figure 2: Variation of velocity profile for different values of stretching & magnetic parameters.

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