

Strongly g-closed sets and strongly g-closed sets**

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ABSTRACT

*In this paper we introduce and investigate the concept of strongly g-closed set and strongly g**-closed set.*

Keywords: *Strongly g-closed set, strongly g**-closed set.*

1. INTRODUCTION

N. Levine [2] introduced generalized closed sets in 1970, M.K.R.S. Veera Kumar [7] introduced the generalised g*-closed sets in 2000. R. Parimelazhagan and V. Subramania Pillai[5] introduced the strongly g*-closed set in 2012. In this paper we introduce and study the concept of strongly g-closed sets and strongly g**-closed sets.

2. PRELIMINARIES

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $\text{cl}(A)$, $\text{int}(A)$ and $C((X, \tau))$ denote the closure of A , interior of A and the closed sets of (X, τ) respectively.

Let us recall the following definitions, which are useful in this sequel.

Definition 2.1: A subset A of a space (X, τ) is called a

1. semi-open set [1] if $A \subseteq \text{cl}(\text{int}(A))$ and a semi-closed set if $\text{int}(\text{cl}(A)) \subseteq A$.
2. regular-open set [1] if $A = \text{int}(\text{cl}(A))$ and a regular-closed set if $\text{int}(\text{cl}(A)) = A$
3. pre-open set [3] if $A \subseteq \text{int}(\text{cl}(A))$ and pre-closed set if $\text{cl}(\text{int}(A)) \subseteq A$.

Definition 2.2: A subset A of a space (X, τ) is called a

1. regular generalized closed (briefly rg-closed) set [4] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ) .
2. generalized closed (briefly g-closed) set [2] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
3. generalized g star (briefly g*-closed) set [7] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ) .
4. generalized g star star (briefly g**-closed) set [6] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in (X, τ) .
5. strongly g star closed (briefly strongly g*-closed) set [5] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X, τ) .

3. BASIC PROPERTIES OF STRONGLY g-closed sets and strongly g-closed sets**

We introduce the following definitions.

Definition 3.1: Let (X, τ) be a topological space and A be its subset. Then A is said to be a strongly g-closed set if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 3.2: Let (X, τ) be a topological space and A be its subset. Then A is said to be a strongly g^{**} -closed set if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in X .

Proposition 3.3: Every closed set is strongly g -closed.

Proof follows from the definitions.

Proposition 3.4: Every closed set is strongly g^{**} -closed.

Proof follows from the definitions.

The converse of the above propositions need not true in general as seen in the following examples.

Example 3.5: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{a, c\}, X\}$. Then $A = \{a, b\}$ is a strongly g -closed set but not a closed set of (X, τ) .

Example 3.6: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$. Then $A = \{b\}$ is a strongly g^{**} -closed set but not a closed set of (X, τ) .

Proposition 3.7: Every strongly g^* -closed set is strongly g -closed.

Proof follows from the definitions.

The converse of the above propositions need not true in general as seen in the following examples.

Example 3.8: Let $X = \{a, b, c, d\}$ and $\tau = \{\varphi, \{a\}, X\}$. Then $A = \{a, b\}$ is strongly g -closed but not strongly g^* -closed in (X, τ) .

Proposition 3.9: Every strongly g^{**} -closed set is strongly g -closed.

Proof follows from the definitions.

Proposition 3.10: Every strongly g^* -closed set is strongly g^{**} -closed but not conversely.

Proof follows from the definitions.

Example 3.11: In example (3.8), $A = \{a, b\}$ is strongly g^{**} -closed but not strongly g^* -closed in (X, τ) .

Proposition 3.12: Every g -closed set is strongly g -closed but not conversely.

Proof follows from the definitions.

Example 3.13: Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$. Then $A = \{b\}$ is strongly g -closed but not g -closed in (X, τ) .

Proposition 3.14: Every g^{**} -closed set is strongly g^{**} -closed but not conversely.

Proof follows from the definitions.

Example 3.15: Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$. Then $A = \{b\}$ is strongly g -closed but not g -closed in (X, τ) .

Proposition 3.16: Every g^* -closed set is strongly g -closed but not conversely.

Proof follows from the definitions.

Example 3.17: In example (3.6) $A = \{b\}$ is strongly g -closed but not g^* -closed in (X, τ) .

Proposition 3.18: Every g^* -closed set is strongly g^{**} -closed but not conversely.

Proof follows from the definitions.

Example 3.19: In example (3.6) $A = \{b\}$ is strongly g^{**} -closed but not g^* -closed in (X, τ) .

Remark 3.20: g^{**} -closedness and strongly g^* -closedness are independent as seen in the following examples.

Example 3.21: In example (3.5) $A = \{c\}$ is strongly g^* -closed but not g^{**} -closed in (X, τ) and in example (3.8) $B = \{a, b\}$ is g^{**} -closed but not strongly g^* -closed in (X, τ)

Remark 3.22: g -closedness and strongly g^* -closedness are independent as seen in the following examples.

Example 3.23: In example (3.13) $A = \{b\}$ is strongly g^* -closed but not g -closed in (X, τ)

Example 3.24: In example (3.8) $A = \{a, b\}$ is g -closed but not strongly g^* -closed in (X, τ)

Remark 3.25: g -closedness and strongly g^{**} -closedness are independent as seen in the following examples.

Example 3.26: In example (3.13) $A = \{b\}$ is strongly g^{**} -closed but not g -closed in (X, τ)

Example 3.27: In example (3.10) $A = \{a, b\}$ is g -closed but not strongly g^{**} -closed in (X, τ)

Proposition 3.28: Every g^{**} -closed set is strongly g -closed but not conversely.

Proof follows from the definitions.

Example 3.29: In example (3.5) $A = \{c\}$ is strongly g -closed but not g^{**} -closed in (X, τ) .

Theorem 3.30: If A subset of a topological space (X, τ) is both open and strongly g -closed then it is closed.

Proof: Suppose A is both open and strongly g -closed. Since A is strongly g -closed $\text{cl}(\text{int}(A)) \subseteq A$. That is $\text{cl}(A) = \text{cl}(\text{int}(A)) \subseteq A$. $\therefore A$ is closed.

Corollary 3.31: If A subset of a topological space (X, τ) is both open and strongly g -closed then it is both regular open and regular closed in X .

Corollary 3.32: If A subset of a topological space (X, τ) is both open and strongly g -closed then it is rg -closed in X .

Theorem 3.33: If A subset of a topological space (X, τ) is both open and strongly g^{**} -closed then it is closed.

Proof is similar to theorem (3.30).

Corollary 3.34: If A subset of a topological space (X, τ) is both open and strongly g^{**} -closed then it is both regular open and regular closed in X .

Corollary 3.35: If A subset of a topological space (X, τ) is both open and strongly g^{**} -closed then it is rg -closed in X .

Theorem 3.36: If A subset of a topological space (X, τ) is both strongly g -closed and semi-open then it is g -closed.

Proof: Since A is strongly g -closed $\text{cl}(\text{int}(A)) \subseteq A$ whenever $A \subseteq U$ and U is open in X .

$\text{cl}(\text{int}(A)) \supseteq A$ since A is semi-open. Then $\text{cl}(A) \subseteq \text{cl}(\text{int}(A)) \subseteq U$. Hence A is g -closed.

Corollary 3.37: If A subset of a topological space (X, τ) is both strongly g -closed then it is g -closed in X .

Proof: Since every open set is semi-open the result follows from the above theorem.

Theorem 3.38: If A subset of a topological space (X, τ) is both strongly g**-closed and semi-open then it is g**-closed.

Corollary 3.39: If A subset of a topological space (X, τ) is both strongly g**-closed then it is g**-closed in X.

Theorem 3.40: If A subset of a topological space (X, τ) is both strongly g-closed then $\text{cl}(\text{int}(A)) - A$ contains no non empty closed set.

Proof: Suppose F is a closed set such that $F \subseteq \text{cl}(\text{int}(A)) - A$. Then $F \subseteq \text{cl}(\text{int}(A)) \cap A^c$, which implies $F \subseteq \text{cl}(\text{int}(A))$ and $F \subseteq A^c$. $F \subseteq A^c$ implies $A \subseteq F^c$ where F^c is open. Therefore $\text{cl}(\text{int}(A)) \subseteq F^c$ since A is strongly g-closed.

Then $F \subseteq (\text{cl}(\text{int}(A)))^c$. Hence $F \subseteq (\text{cl}(\text{int}(A))) \cap (\text{cl}(\text{int}(A)))^c = \emptyset$.

Therefore $\text{cl}(\text{int}(A)) - A$ contains no non empty closed set.

Theorem 3.41: If A subset of a topological space (X, τ) is both strongly g**-closed then $\text{cl}(\text{int}(A)) - A$ contains no non empty closed set.

Proof: Since every open set is g*-open the result follows from the previous theorem.

Theorem 3.42: If A is strongly g-closed and $A \subseteq B \subseteq \text{cl}(\text{int}(A))$, then B is strongly g-closed.

Proof: Let $B \subseteq U$ where U is open which implies $A \subseteq U$ where U is open.

$\therefore \text{cl}(\text{int}(A)) \subseteq U$, since A is strongly g-closed. $\text{cl}(\text{int}(B)) \subseteq \text{cl}(B) \subseteq \text{cl}(\text{int}(A)) \subseteq U$. $\therefore B$ is strongly g-closed.

Theorem 3.43: If A is strongly g**-closed and $A \subseteq B \subseteq \text{cl}(\text{int}(A))$, then B is strongly g**-closed.

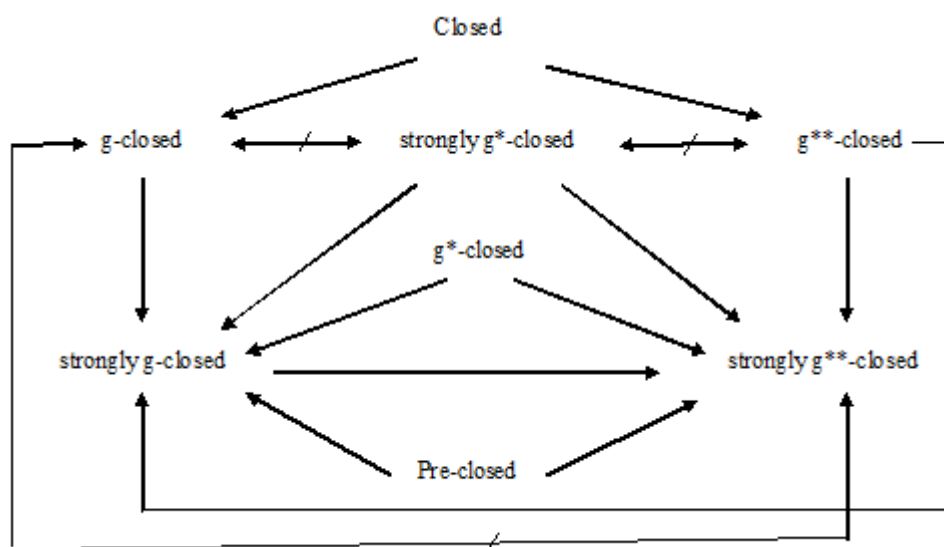
Proof is similar to the above theorem.

Proposition 3.44: Every pre-closed set is (i) strongly g-closed (ii) strongly g**-closed but not conversely.

Proof follows from the definitions.

Example 3.45: In example (3.5) $A = \{a, b\}$ is strongly g-closed and strongly g**-closed but it is not pre-closed since $\text{cl}(\text{int}(A)) = X \not\subseteq A$

The above results can be represented in the following figure.



Where $A \longrightarrow B$ (resp. $A \longleftrightarrow B$) represents A implies B (resp. A and B are independent).

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