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Power of Tests for Negative Binomial Regression Coefficients in Count Data

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ABSTRACT

In this study we focus on a negative binomial (NB) regression model to take account of regression coefficients in Poisson counts. An algorithm for estimating parameters was obtained and score test versus its alternative tests were presented for testing the significance of regression coefficients in Poisson regression model against NB regression model. The power of the score test is compared with their Likelihood ratio test (LRT) and Wald test via Monte Carlo simulation. The simulation result indicated that the Wald test is superior over the LRT and Score test in terms of its power. An Ethiopian under five children death rate data is used to illustrate the tests.

Keywords: *Count data, Negative binomial regression, Score test, regression parameter.*

1. INTRODUCTION

Count data is very common in various fields such as in biomedical science, public health and marketing. Poisson models are widely used in the regression analysis of count data and as a basis for count data analysis (Frome[6]; Lawless[9]). It is also appropriate for analyzing rate data. Poisson regression is a part of class of models in generalized linear models. It uses natural log as the link function and models the expected value of response variable. The natural log in the model ensures that the predicted values of response variable will never be negative. The response variable in Poisson regression is assumed to follow Poisson distribution. One requirement of the Poisson distribution is that the mean equals the variance. In real-life application, however, count data often exhibits overdispersion. Overdispersion occurs when the variance is significantly larger than the mean. When this happens, the data is said to be overdispersed. Overdispersion can cause underestimation of standard errors which consequently leads to wrong inference. Numerous methods have been suggested for dealing with this (Breslow[1]; Cameron and Trivedi[2]; Lawless[7]; Mc- Cullagh[11]; Hilbe[7]), and there have also been studies of the effect of overdispersion on Poisson- based methods (Cox[3]).

Analysts who choose the NB model over the Poisson model should justify the rejection of equi-dispersion and estimation of parameters in the Poisson model. A LRT or Wald test can be used, but the score test has the advantage that we only need to fit under the null hypothesis. The score test statistic for overdispersion parameter developed by Cameron and Trivedi[2] specifically for comparing the Poisson model against the negative binomial model, is a special case of the general score statistics later developed by Dean[4]. Furthermore, Dejen and Munisway[5] are provided a Monte Carlo simulation study to identify the power of score test statistic with existing tests for a negative binomial regression models in overdispersion parameter. In this paper, we propose a score test for testing regression parameters in a negative binomial regression model and compare the power of this test with the existing tests via Monte Carlo simulation study.

The outline of the paper is as follows: In Section 1.1 we introduce the negative binomial regression model and its estimation method. Score test and alternative tests for regression parameters in the model are discussed in Section 2 and 3, respectively. A simulation study for powers of two score tests and alternative tests will be presented in Section 4. Section 5 presents an example to illustrate our methodology and some conclusions are given in the last section.

1.1 Negative binomial regression model

Consider the negative binomial distribution with probability density function:

$$f(y_i;\mu_i,\alpha)=\frac{\Gamma(y_i+{}^1/_\alpha)}{y_i!\,\Gamma({}^1/_\alpha)}(1+\alpha\mu_i)^{\frac{-1}{\alpha}}\left(1+\frac{1}{\alpha\mu_i}\right)^{-y_i},y_i\geq 0 \qquad (1)$$

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with mean $E(Y_i) = \mu_i = exp(x_i^T\beta)$, and variance $Var(Y_i) = \mu_i(1 + \alpha\mu_i)$. In equation (1), the term α plays the role of a dispersion factor and it is a constant. Clearly, when $\alpha \to 0$, the NB distribution reduces to the usual standard Poisson distribution with parameter μ_i. For more details the reader is referred (Mc- Cullagh and Nelder[10]; Jansakul and Hinde[8]; Hilbe[7]).

Assume that each observation $y_i, i = 1, 2, \dots, n$ submits to negative binomial distribution, i.e. $y_i \sim NB(\alpha, \mu_i)$. Following the generalized linear model approach, we relate parameters μ_i to covariates $x_i \epsilon R^p$ through the log-link function so that

$$\log(\mu_i) = x_i^T\beta \tag{2}$$

Then we call equation (1) and (2) the negative binomial regression model, where β is a p- dimension regression coefficients, and $x_i^T = (x_{i1}, x_{i2}, \dots, x_{ip}), i = 1, 2, \dots, n$. The log-likelihood function of the NB regression model based on a sample of n independent observations is expressed as

$$\ell(\mu, \alpha;\ y_i) = \sum_{i=1}^{n}\{-\log(y_i) + \sum_{k=1}^{y_i}\log(\alpha y_i - \alpha k + 1) - (y_i + {}^{1}/_{\alpha})\log(1 + \alpha\mu_i) + y_i\log(\mu_i)\} \tag{3}$$

where, $\frac{\Gamma(y_i + {}^{1}/_{\alpha})}{y_i!\Gamma({}^{1}/_{\alpha})} = \prod_{k=1}^{y_i}(y_i + {}^{1}/_{\alpha} - k) = \alpha^{-y_i}\prod_{k=1}^{y_i}(\alpha y_i - \alpha k + 1)$.

We can obtain the maximum likelihood estimators by using the Newton-Raphson iterative method. By differentiating the log-likelihood function (3) with respect to α and β, we have

$$\frac{\partial\ell(\mu, \alpha)}{\partial\beta} = \frac{\partial\ell(\mu, \alpha)}{\partial\mu}\frac{\partial\mu}{\partial\beta} = \sum_{i=1}^{n}\frac{(y_i - \mu_i)}{1 + \alpha\mu_i}x_i;$$

$$\frac{\partial\ell(\mu, \alpha)}{\partial\alpha} = \sum_{i=1}^{n}\left\{\sum_{k=1}^{y_i}\frac{y_i - k}{\alpha y_i - \alpha k + 1} + \frac{\log(1 + \alpha\mu_i)}{\alpha^2} - \frac{(y_i + {}^{1}/_{\alpha})\mu_i}{1 + \alpha\mu_i}\right\};$$

so the score function is $U = \left(\frac{\partial\ell(\mu,\alpha)}{\partial\beta}, \frac{\partial\ell(\mu,\alpha)}{\partial\alpha}\right)$, and differentiating the log-likelihood function with respect to the parameters α and β

$$\frac{\partial^2\ell(\mu, \alpha)}{\partial\beta\partial\beta^T} = -\sum_{i=1}^{n}\left\{\frac{[1 + \alpha y_i]\mu_i}{(1 + \alpha\mu_i)^2}\right\}x_i{x_i}^T;$$

$$\frac{\partial^2\ell(\mu, \alpha)}{\partial\alpha^2} = \sum_{i=1}^{n}\left\{\left(\sum_{k=1}^{y_i}\frac{-(y_i - k)^2}{(\alpha y_i - \alpha k + 1)^2}\right) - \frac{2\log(1 + \alpha\mu_i)}{\alpha^3} + \frac{2\mu_i}{\alpha^2(1 + \alpha\mu_i)} + \frac{(y_i + {}^{1}/_{\alpha})\mu_i^2}{(1 + \alpha\mu_i)^2}\right\};$$

$$\frac{\partial^2\ell(\mu, \alpha)}{\partial\alpha\partial\beta} = \frac{\partial^2\ell(\mu, \alpha)}{\partial\beta\partial\alpha} = -\sum_{i=1}^{n}\left\{\frac{(y_i - \mu_i)\mu_i}{(1 + \alpha\mu_i)^2}\right\}x_i.$$

The observed Fisher information matrix:

$$I(\beta, \alpha) = \begin{bmatrix} I_{\beta\beta}(\beta, \alpha) & I_{\beta\alpha}(\beta, \alpha) \\ I_{\alpha\beta}(\beta, \alpha) & I_{\alpha\alpha}(\beta, \alpha) \end{bmatrix}, \tag{4}$$

where, the elements $I_{\beta\beta} = -E\left(\frac{\partial^2\ell(\mu,\alpha)}{\partial\beta\,\partial\beta^T}\right)$ is the $p \times p$ symmetric matrix, $I_{\beta\alpha} = I_{\alpha\beta}^T = -E\left(\frac{\partial^2\ell(\mu,\alpha)}{\partial\beta\partial\,\alpha}\right)$ is the $p \times 1$ matrix and $I_{\alpha\alpha} = -E\left(\frac{\partial^2\ell(\mu,\alpha)}{\partial\alpha^2}\right)$ is a scalar.

The Newton-Raphson iterative algorithm used the specification of initial values. Our suggestion is setting $\alpha \to 0$ and β using ML estimation obtained from the Poisson regression model. Let $\xi = (\alpha, \beta^T)^T$, under the usual regularity conditions for maximum likelihood estimation, when the sample size is large, $\hat{\xi} \sim N_p(\xi, I^{-1}(\alpha, \beta))$ approximately.

2. SCORE TESTS FOR REGRESSION PARAMETERS IN NB MODEL

In many applications, it is important to assess whether the assumed model is indeed appropriate. The score test developed by Cameron and Trivedi[2] specifically for comparing the Poisson model against the negative binomial model, is a special case of the general score statistics later developed by Dean[4]. In this study, we derive the method to test significance of regression coefficients in NB model and compare the power of this proposed test with existing tests using simulation study.

To test the effect of covariates on the parameter μ of NB regression model we should consider the following hypothesis

$$H_{02}: \beta^* = 0 \quad versus \quad H_{A2}: \beta^* \neq 0, \tag{5}$$

where, β^* is a subset of β without the intercept β_0. Based on the log-likelihood function ℓ (3) and partitioning the x_i as $(1, (x_i^*)^T)^T$, we can get the score function of β^* and β_0 as

$$\frac{\partial \ell(\mu,\alpha)}{\partial \beta^*} = \sum_{i=1}^{n} \frac{(y_i-\mu_i)}{1+\alpha\mu_i} x_i^*; \tag{6}$$

$$\frac{\partial \ell(\mu,\alpha)}{\partial \beta_0} = \sum_{i=1}^{n} \frac{(y_i-\mu_i)}{1+\alpha\mu_i}; \tag{7}$$

and the second derivatives of the log-likelihood are given by:

$$\frac{\partial^2 \ell(\mu,\alpha)}{\partial \beta^* \partial \beta^{*T}} = -\sum_{i=1}^{n} \left\{\frac{[1+\alpha y_i]\mu_i}{(1+\alpha\mu_i)^2}\right\} x_i^* x_i^{*T}; \tag{8}$$

$$\frac{\partial^2 \ell(\mu,\alpha)}{\partial {\beta_0}^2} = -\sum_{i=1}^{n} \left\{\frac{[1+\alpha y_i]\mu_i}{(1+\alpha\mu_i)^2}\right\}; \tag{9}$$

$$\frac{\partial^2 \ell(\mu,\alpha)}{\partial \beta^* \partial \beta_0} = \frac{\partial^2 \ell(\mu,\alpha)}{\partial \beta_0 \partial \beta^*} = -\sum_{i=1}^{n} \left\{\frac{[1+\alpha y_i]\mu_i}{(1+\alpha\mu_i)^2}\right\} x_i^*. \tag{10}$$

Let $\hat{\xi}_1 = \left(\hat{\alpha}, \hat{\beta}_0, 0^T\right)^T$ be the REML estimates of parameter ξ under null hypothesis H_{02}. Based on the relationship for the moments in a Poisson distribution with parameter μ and equations (8), (9) and (10), then we can write the blocking matrix $I_{\beta\beta}(\beta,\alpha)$ as follows:

$$I_{\beta\beta}(\beta,\alpha) = \begin{bmatrix} I_{\beta_0\beta_0}(\beta,\alpha) & I_{\beta_0\beta^*}(\beta,\alpha) \\ I_{\beta^*\beta_0}(\beta,\alpha) & I_{\beta^*\beta^*}(\beta,\alpha) \end{bmatrix}, \tag{11}$$

where,

$$I_{\beta^*\beta^*}(\beta,\alpha) = -E\left(\frac{\partial^2 \ell(\mu,\alpha)}{\partial \beta^* \partial \beta^{*T}}\right)\Bigg|_{\hat{\xi}_2} = \sum_{i=1}^{n} \frac{\hat{\mu}_i}{1+\hat{\alpha}\hat{\mu}_i} x_i^* x_i^{*T};$$

$$I_{\beta_0\beta_0}(\beta,\alpha) = -E\left(\frac{\partial^2 \ell(\mu,\alpha)}{\partial {\beta_0}^2}\right)\Bigg|_{\hat{\xi}_2} = \sum_{i=1}^{n} \frac{\hat{\mu}_i}{1+\hat{\alpha}\hat{\mu}_i};$$

$$I_{\beta_0\beta^*}(\beta,\alpha) = -E\left(\frac{\partial^2 \ell(\mu,\alpha)}{\partial \beta^* \partial \beta_0}\right)\Bigg|_{\hat{\xi}_2} = \sum_{i=1}^{n} \frac{\hat{\mu}_i}{1+\hat{\alpha}\hat{\mu}_i} x_i^*.$$

The general score test for testing $\beta^* = 0$ is

$$S_{\beta 2} = S_{\beta^*}^T(\beta,\alpha) {I_{22}^*}^{-1} S_{\beta^*}(\beta,\alpha)\Big|_{\hat{\xi}_2}$$

where, the score vector is given by the following:

$$S_{\beta^*} = S_{\beta^*}(\beta,\alpha)\big|_{\hat{\xi}_1} = \frac{\partial \ell(\mu,\alpha)}{\partial \beta^*}\bigg|_{\hat{\xi}_1} = \sum_{i=1}^{n} \left\{\frac{(y_i-\mu_i)}{1+\alpha\mu_i} x_i^*\right\} \tag{12}$$

and we denote the inverse of $I_{\beta\beta}(\beta,\alpha)\big|_{\hat{\xi}_2}$ as I^* which can be partitioned as $\begin{pmatrix} I_{11}^* & I_{12}^* \\ I_{21}^* & I_{22}^* \end{pmatrix}$. Due to the structure of $S_{\beta^*}(\beta,\alpha)\big|_{\hat{\xi}_1}$ only I_{22}^* is needed and is given as follows:

$$I_{22}^{*} = I_{(\beta^*\beta^*)}(\beta,\alpha) - I_{(\alpha\beta^*)}^{T}(\beta,\alpha) I_{(\alpha\alpha)}^{-1}(\beta,\alpha) I_{\alpha\beta^*}(\beta,\alpha)\Big|_{\hat{\xi}_2}$$

$$= X^{*T}\,\mathrm{diag}(v_i)\,X^{*}$$

where, $v_i = \frac{\hat{\mu}_i}{1+\hat{\alpha}\hat{\mu}_i}$.

Therefore, the score test statistic for testing the significant of coefficient of regression parameters value is given by:

$$S_{\beta 2} = \left(\frac{\partial \ell(\mu,\alpha)}{\partial \beta^*}\right)^{T} I_{22}^{*\,-1} \frac{\partial \ell(\mu,\alpha)}{\partial \beta^*}\Bigg|_{\hat{\xi}_1} \tag{13}$$

The standard asymptotic theory implies $S_{\beta 2}$ has a chi-square distribution with degrees of freedom equals the number of restricted regression parameters in the Poisson model.

3. ALTERNATIVE TESTS FOR OVERDISPERSION AND REGRESSION PARAMETERS

We test the significance of coefficient of explanatory variables x_i^* the hypothesis denoted as $H_{02}: \beta_i^* = 0 \;\; vs.\;\; H_{A2}: \beta_i^* \neq 0$ and the LRT test statistic for testing the null hypothesis is given by $LRT = -2\left(\ell(\hat{\alpha},\hat{\beta}_0) - \ell(\hat{\alpha},\hat{\beta}_0,\hat{\beta}^*)\right)$, where $\ell(\hat{\alpha},\hat{\beta}_0)$ and $\ell(\hat{\alpha},\hat{\beta}_0,\hat{\beta}^*)$ are the log-likelihood estimate of the parameters for restricted and unrestricted parameter, respectively. The associated Wald test statistic is $W_{\beta^*} = \hat{\beta}_i^{*T}\left[Cov(\hat{\beta}^*)\right]\hat{\beta}^*$, where $\hat{\beta}_i^*$ is the maximum likelihood estimate of coefficient x_i^*, $Cov(\hat{\beta}_i^*)$ is the variance-covariance matrix of these estimation, determined from the estimation of the variance-covariance matrix, $I(\beta_0,\beta^*)$. For an adequate model, the Wald test also has an asymptotic chi-square distribution with degrees of freedom equals the number of restricted parameters under the null hypothesis.

4. MONTE CARLO SIMULATION STUDY

A simulation study was conducted to examine the empirical size and power of the proposed score test against the alternative tests. In order to compare the power of proposed score test with the existing test statistics for testing the regression parameters in a NB model a limited simulation study was carried out under different situations. The simulations are performed for testing of regression coefficients in mean μ portion. The model used for simulation study is

$Y_i \sim NB(\alpha,\mu_i),\;\; i = 1,2,\dots,n;$ where, $\log(\hat{\mu}_i) = \beta_0 + \beta_1 x_i$.

The estimated values of parameters under the null hypothesis are treated as true values in simulation studies. The true values are respectively chosen as $\alpha = 0.4$, $\beta_0 = 1.0$ for testing the regression parameter. We first generate a set of random numbers from a uniform distribution in the interval [0, 1] as the values of x_i. To get values of y_i, a random variate is drawn from a NB model with true values of parameters, the value of x_i, and a given β_1. Repeating these procedure n times, we get a set of simulated data $\{y_i, x_i, i = 1,2,\dots n\}$. The values of score test and alternative tests are computed by formulas shown in Section 2 and 3, respectively. We take $\beta_1 = 0, 0.2, 0.4, 0.6, 1.0$. For each given values of parameters, we do 1000 replications i.e., the values of x_i's are fixed for each replication. Then the proportion of times which rejected the null hypothesis is just the simulated value of power. Here, all the statistics are compared with the χ_α^2 critical value at $\alpha = 0.05$ level.

The simulations are performed for samples of size $n = 40, 60, 80, 100, 200$ and 300 to get the simulated powers of the proposed test statistic and its alternative tests. For each set of generated data, a NB model is fitted for calculating the proposed score test and the existing tests followed by the powers of the tests. Results from the simulation study are presented in Table 1.

Table 1: Empirical power of the Wald, LRT and Score tests for regression parameter based on 1,000 replications generated from the NB regression model with $\log(\hat{\mu}_i) = 1 + \beta_1 x_i$.

n	*Method*	*Power*					
		$\boldsymbol{\beta_1 = 0.0}$	*0.2*	*0.4*	*0.6*	*0.8*	*1.0*
40	**Wald**	0.069	0.077	0.159	0.265	0.448	0.632
	LRT	0.062	0.074	0.149	0.255	0.432	0.612
	Score	0.057	0.064	0.130	0.231	0.408	0.589

60	**Wald**	0.051	0.09	0.189	0.392	0.606	0.816
	LRT	0.048	0.087	0.184	0.378	0.600	0.804
	Score	0.042	0.08	0.172	0.362	0.574	0.784
80	**Wald**	0.061	0.096	0.244	0.496	0.704	0.894
	LRT	0.058	0.094	0.237	0.486	0.702	0.887
	Score	0.052	0.091	0.228	0.464	0.694	0.883
100	**Wald**	0.060	0.116	0.319	0.603	0.797	0.951
	LRT	0.057	0.113	0.309	0.596	0.794	0.948
	Score	0.055	0.111	0.297	0.587	0.779	0.948
200	**Wald**	0.056	0.159	0.508	0.850	0.976	0.997
	LRT	0.054	0.157	0.506	0.848	0.976	0.997
	Score	0.053	0.156	0.499	0.844	0.975	0.997
300	**Wald**	0.058	0.230	0.684	0.959	0.996	1
	LRT	0.058	0.229	0.68	0.958	0.996	1
	Score	0.054	0.224	0.650	0.954	0.996	1

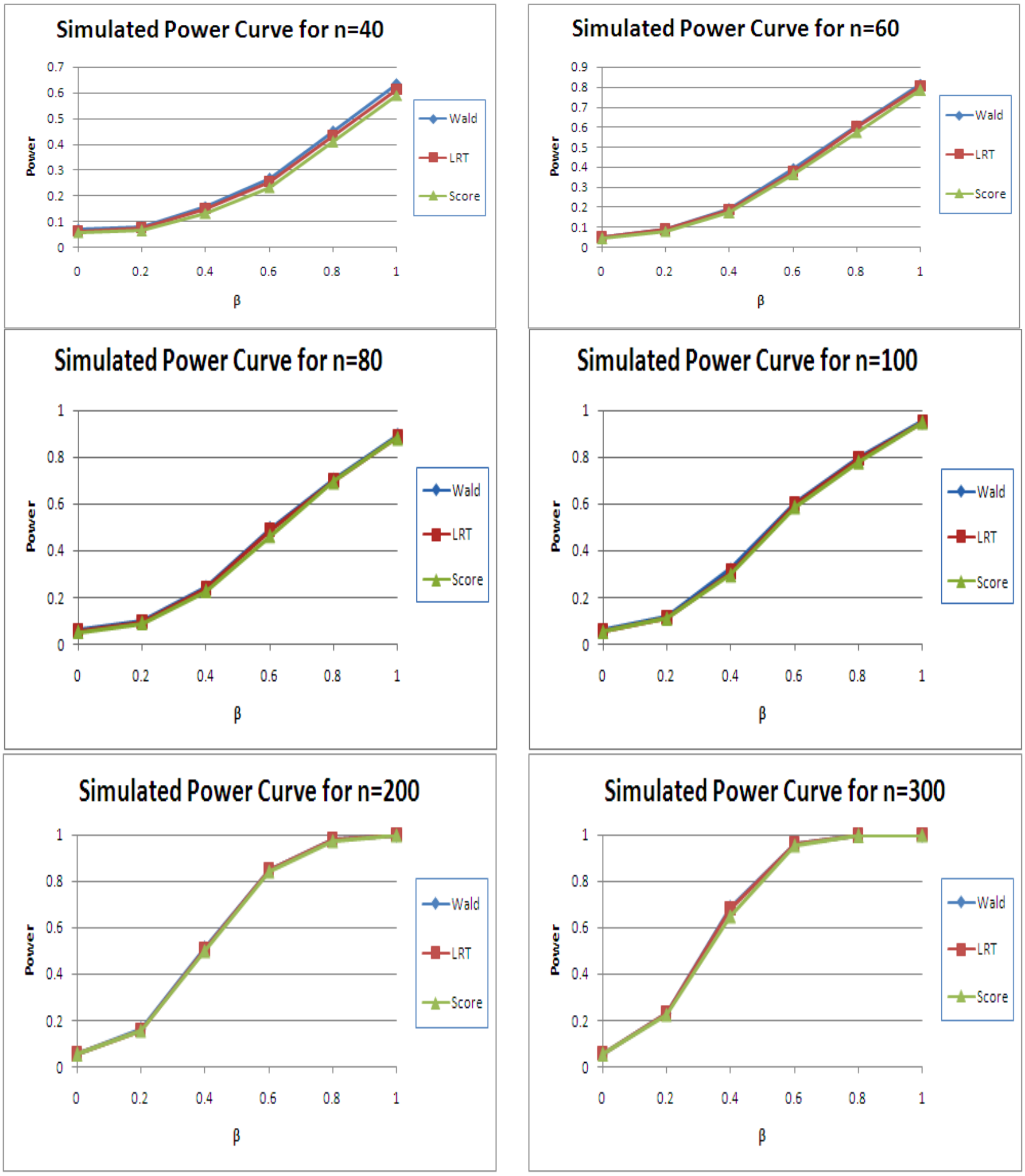

Fig. 1: The Simulated power curve of Wald, LRT and Score tests for regression parameter values based on negative binomial regression model.

The results shown in Table 1 displays that, in general, the statistic Score, Wald, and LRT maintains nominal level well for all values of α and β_1 considered. Only for small values n and β_1 there is some liberal behavior. Table 1 shows that the power of the tests for detecting the regression coefficient β_1 increase slowly for small n (n=40) and small β_1 (e.g., β_1 =0.0); but for larger values of n and β_1 the power increase and approach 1 quickly. It can be seen from Table 1 that as regression coefficient 1.0 or as n increase the power of the test increase very fast and approach 1 quickly. Some representative results are shown in Fig. 1 which displays that the plots of power functions of tests with respect to varying values of β_1. As shown in Fig. 1 as n increase, the power of tests also increases.

5. APPLICATION

To illustrate our methodology for fitting a NB model, we first consider the 2005 Demographic and Health Survey (DHS) which is obtained from the Central Statistical Agency, Ethiopia. Thus, this study analyzes responses from each of 9210 women (only those who have ever born a child), out of 14070 women of age 15-49 interviewed in 2005 DHS, on the counts of the number of deaths of children aged less than 5 years that the mother has experienced in her lifetime. The response variable of this study, Y_i, is a count, which gives the number of deaths of children aged less than 5 years that each mother has experienced in her lifetime.

The set of children death dataset in Ethiopia is then used to illustrate the score test with covariates in the NB model. The number of child death begins with a value of zero and grows from there. There are 9210 observations in the dataset, and the minimum count is 0 and the maximum count is 7, with mean 0.63 and median 0. The dispersion index (the ratio of variance to mean) is 1.7905. So the data exhibit over-dispersion. There are a number of variables in the dataset. Here we selected six important dummy explanatory variables, i.e., age of women at first birth (x_1), toilet facility (x_2), source of drinking water (x_3), education status of women (x_4), work status of women (x_5) and type of place of residence (x_6), from the variables and using the negative binomial regression model to fit the data for illustrating our results:

The estimated dispersion parameter in NB model is $\hat{\alpha} = 0.298942$ with standard error 0.028569; and the estimated regression parameters are $\hat{\beta}$ =(-1.9982, -0.1794, -0.0190, -0.01093, -0.4256, -0.1318, 0.2848). Based on the modeling information, the computed score statistics for regression parameters the computed score test is $\mathrm{S} = 1452.254$ with its corresponding p-value$= 0.0000$; The LRT statistics is 180.97 with p-value$= 0.0000$; and the Wald test statistics is 16968.43 with p-value$= 0.0000$. All the values of the tests indicate that the negative binomial model is suitable for this dataset.

The above analysis doesn't indicate which of the Poisson and negative binomial model will fit the data better. To cheek this, we fitted the maximum likelihood estimate of the parameters and the maximized log-likelihoods for them. The fitted statistics for Poisson and negative binomial models are shown in Table 2.

Table 2: The observed and predicted count percent of children who died before age five per mother by Poisson and NB models

Number of U5CD	Observed frequency	Observed Percent (%)	Predicted percent (%)	
			Poisson model	NB model
0	5897	64.0282	36.7879	41.6896
1	1881	20.4235	36.7879	28.7837
2	793	8.6102	18.3940	13.2487
3	379	4.1151	6.1313	5.0818
4	166	1.8024	1.5328	1.7543
5	60	0.6515	0.3066	0.5652
6	20	0.2172	0.0510	0.1735
7	14	0.1520	0.0073	0.0513

Clearly, from Fig. 2 and the value of AIC, BIC criteria in Table 2, there is a difference between Poisson and negative binomial model for this dataset. Then we can make a conclusion that the negative binomial model is essentially more appropriate than the Poisson model for the number of under 5 years children death in Ethiopia dataset.

Table 3: Model fitting of Poisson and NB models for number of under 5 years children death count dataset.

Model	Selection criteria		
	-2ℓ	AIC	BIC
Poisson Negative binomial	275.48 228.91	17203.81 17004.82	17253.71 17061.85

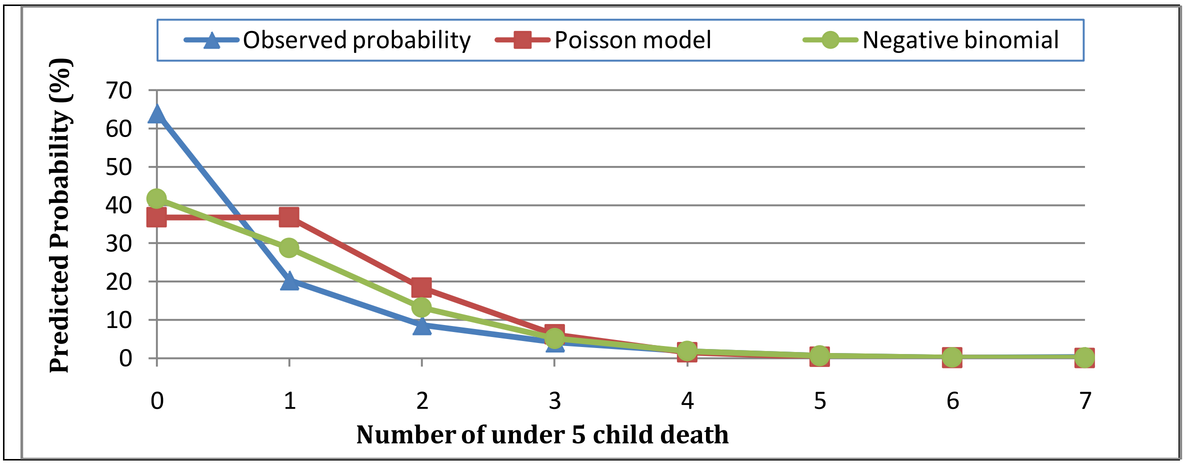

Fig. 2: Graphical comparison of Poisson and NB distribution for under 5 years children death

6. CONCLUSION

In this paper, we have presented the power of Poisson regression model versus negative binomial models based on the Score, Wald and LRT for regression coefficient parameter values. An algorithm for estimating parameters are obtained and a proposed score test and their alternative tests are presented for testing the significance of regression coefficients in Poisson regression model against the negative binomial regression model. A Monte Carlo simulation and application example are given to illustrate our method.

The simulation study and application example for regression parameter in NB model indicates the score test is highly misleading, and the Wald and LR tests should be used instead. The simulation result shows that for dataset that has small regression parameter values the Poisson regression is more appropriate while negative binomial regression is more appropriate for data that has high regression parameter values.

Our main work has been focused on the NB regression models without correlation between observations. However, it seems that it is reasonable to assume the correlation between observations. We will consider it in our future research.

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