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EFFECT OF HALL CURRENTS ON MHD UNSTEADY FLOW OF VISCO-ELASTIC OLDROYD FLUID THROUGH A RECTANGULAR CHANNEL

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ABSTRACT

The purpose of present study is to analyze the effect of Hall currents on MHD unsteady flow of visco-elastic Oldroyd fluid through a rectangular channel in the presence of magnetic field. Here we considered the first and second order Oldroyd fluid and investigate the same problem in this new visco-elastic model. The numerical expressions of the velocity profiles have been given for both first and second order fluids.

INTRODUCTION

The study of the Hall currents has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through the reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

In the classical theory of viscous flow tremendous development have been observed as evidenced by the informative monographs of Batchelor [1] and various other authors Maity et. al [9], Kumar [4], Kundu and Sengupta [5], Sharma [11], Kundu and Sengupta [6], Rahman and Alam Sarkar [10], Varshney and Singh [12], Kumar and Singh [3], Mahdy [7], Mahdy et al [8], Hayat et al [2] and Varshney and Singh [13].

In recent years the development of new areas of fluid mechanics are very remarkable. For this one may refer to the review of literature in the connection with non-Newtonian fluids, polymetric liquids and visco-elastic liquids of different types with Hall currents. Moreover some interesting problems in this area have been investigated by Kumar [3] and his research collaborators.

In the present study we have considered the problem of Kumar [3] with Hall currents.

Basic theory and equations of motion for first order Oldroyd fluid

For slow motion, the rheological equations for Oldroyd visco-elastic fluid are:

$$\tau_{ij} = -p'\delta_{ij} + \tau'_{ij}$$

$$\left(1+\lambda_1'\frac{\partial}{\partial t'}\right)\tau'_{ij} = 2\mu\left(1+\mu_1'\frac{\partial}{\partial t'}\right)e_{ij}$$

and $e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right)$

where τ_{ij} is the stress tensor, τ'_{ij} is the deviatoric stress tensor, e_{ij} is the rate of strain tensor, p' is the pressure, λ_1' is the stress relaxation time parameter, μ_1' is the strain rate retardation time parameter, δ_{ij} is the metric tensor in Cartesian co-ordinates, μ is the coefficient of viscosity and $v_{i,j}$ is the velocity components.

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Let us consider the walls of the rectangular channel to the planes $x' = \pm a$ and $y' = \pm b,$ where z′-axis is taken towards the direction of motion 0, 0, w′ (x′, y′, t′) are respectively the velocity components along x′, y′, z′ direction where w′ (x′, y′, t′) is the axial velocity of the fluid. A transient pressure gradient $-Pe^{-\omega' t'}$ varying with time is applied to the fluid.

Following the stress-strain relation the equation for unsteady motion is given by

$$\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)\frac{\partial w'}{\partial t'} = -\frac{1}{\rho}\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)\frac{\partial p'}{\partial z'} + \upsilon\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)\nabla^2 w' - \left(\frac{\sigma B_0^2}{\rho(1+m_1^2)}\right)\left(1+\lambda_1' \frac{\partial}{\partial t'}\right)w' \tag{1}$$

Introducing the non dimensional quantities

$$w = \frac{w'a}{\upsilon}, \qquad p = \frac{p'a^2}{\rho\upsilon^2}, \qquad t = \frac{t'\upsilon}{a^2}$$

$$\omega = \frac{\omega' a^2}{\upsilon}, \qquad (x, y, z) = \frac{1}{a}(x', y', z')$$

$$\lambda_1 = \lambda_1' \frac{\upsilon}{a^2}, \qquad \mu_1 = \mu_1' \frac{\upsilon}{a^2}$$

The equation (1) becomes

$$\left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t} = \left(1+\lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \left(1+\mu_1 \frac{\partial}{\partial t}\right)\nabla^2 w - \left(\frac{M^2}{(1+m_1^2)}\right)\left(1+\lambda_1 \frac{\partial}{\partial t}\right)w \tag{2}$$

where $M = aB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number).

The boundary conditions are

w = 0, when $x = \pm 1, \quad -\frac{b}{a} \le y \le \frac{b}{a}$ (3)

w = 0, when $y = \pm\frac{b}{a}, \quad -1 \le x \le 1$ (4)

From the nature of boundary conditions, we have chosen the solution of (2) as

w = W(x) cos my $e^{-\omega t}$ (5)

Condition (4) will be satisfied if $\cos m\frac{b}{a} = 0$

or $m = (2n+1)\frac{\pi a}{2b}$, n = 0, 1, 2, 3… (6)

Now the boundary conditions (3) become

W = 0 when x = ± 1 (7)

We construct the solution as the sum of all possible solutions for each value of n

$$w = \sum_{n=0}^{\infty} W(x)\cos my \; e^{-\omega t} \tag{8}$$

By putting $\frac{\partial p}{\partial z} = -Pe^{-\omega t}\,(\omega > 0)$ in (2) and using (7) we get

$$\sum_{n=0}^{\infty}\left\{\frac{d^2}{dx^2}W(x) - m^2W(x)\right\}\cos my + \sum_{n=0}^{\infty}\left\{\frac{(1-\lambda_1\omega)\left(\omega - \frac{M^2}{(1+m_1^2)}\right)}{(1-\mu_1\omega)}\right\}W(x)\cos my + \frac{(1-\lambda_1\omega)}{(1-\mu_1\omega)}P = 0 \tag{9}$$

Equating the coefficient of cos my equal to zero, we get

$$\frac{d^2}{dx^2}W - \frac{K_1^2}{a^2} + A_n = 0 \tag{10}$$

where $K_1^2 = \left\{m^2 - \frac{(1-\lambda_1\omega)\left(\omega - \frac{M^2}{(1+m_1^2)}\right)}{(1-\mu_1\omega)}\right\}a^2$

and $$A_n = \frac{(-1)^n\,4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)} \tag{11}$$

Solving equation (9) and using boundary condition (7) we get

$$W(x) = \frac{(-1)^{n+1}\,4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)K_1^2}\left\{1 - \frac{\cosh\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}$$

Thus the velocity of the fluid is given by

$$w(x,y,t) = \sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}\,4P(1-\lambda_1\omega)}{(2n+1)\pi(1-\mu_1\omega)K_1^2}\left\{1 - \frac{\cosh\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}\right]e^{-\omega t}\cos(2n+1)\frac{\pi a y}{2b} \tag{12}$$

Thus equation (12) represents the velocity of first order Oldroyd visco-elastic fluid.

BASIC THEORY AND EQUATION OR MOTION FOR SECOND ORDER OLDROYD FLUID

Or slow motion, the rheological equations for second order Oldroyd visco-elastic fluid are.

$$\tau_{ij} = -p'\delta_{ij} + \tau'_{ij}$$

$$\left(1 + \lambda_1'\frac{\partial}{\partial t'} + \lambda_2'\frac{\partial^2}{\partial t'^2}\right)\tau'_{ij} = 2\mu\left(1 + \mu_1'\frac{\partial}{\partial t'} + \mu_2'\frac{\partial^2}{\partial t'^2}\right)e_{ij}$$

and $e_{ij} = \frac{1}{2}\left(v_{i,j} + v_{j,i}\right)$

where τ_{ij} is the stress tensor, τ'_{ij} is the deviatoric stress tensor, e_{ij} is the rate of strain tensor, p' is the pressure, λ_1' is the stress relaxation time parameter, μ_1' is the strain rate retardation time parameter, λ_2' is the additional material constant, μ_2' is the additional material constant, δ_{ij} is the metric tensor in Cartesian co-ordinates, μ is the coefficient of viscosity and v_i is the velocity components, m_1 is Hall currents parameter.

Let us consider the walls of the rectangular channel to be the planes $x' = \pm a$ and $y' = \pm b$, where z′-axis is taken towards the direction of motion 0, 0, w′ (x′, y′, t′) are respectively the velocity components along x′, y′, z′ direction where w′ (x′, y′, t′) is the axial velocity of the fluid. A transient pressure gradient $-Pe^{-\omega' t'}$ varying with time is applied to the fluid.

Following the stress-strain relation the equation for unsteady motion is given by

$$\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)\frac{\partial w'}{\partial t'}=-\frac{1}{\rho}\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)\frac{\partial p'}{\partial z'}$$

$$+\upsilon\left(1+\mu'_1\frac{\partial}{\partial t'}+\mu'_2\frac{\partial^2}{\partial t'^2}\right)\nabla^2 w'-\left(\frac{\sigma B_0^{2}{}'}{\rho(1+m_1^2)}\right)\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)w' \tag{13}$$

Introducing the non dimensional quantities

$$w=\frac{w'a}{\upsilon},\quad \omega=\frac{\omega' a^2}{\upsilon}\quad p=\frac{p'a^2}{\rho\upsilon^2},\quad (x,y,z)=\frac{1}{a}(x',y',z')$$

$$t=\frac{t'\upsilon}{a^2},\quad \lambda_1=\lambda'_1\frac{\upsilon}{a^2},\quad \mu_1=\mu'_1\frac{\upsilon}{a^2}\quad \mu_2=\mu'\frac{\upsilon^2}{a^4},\quad \lambda_2=\lambda'_2\frac{\upsilon^2}{a^4}$$

in equation (13), we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial w}{\partial t}=-\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}+\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t}\right)\nabla^2 w$$

$$-\left(\frac{M^2}{(1+m_1^2)}\right)\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)w \qquad \text{.......(14)}$$

where $M=aB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number).

Solution of the problem

Let the possible solution (14) as

w = W (x) cos my $e^{-\omega t}$

Here we used the same boundary conditions of fluid given by (7) after using (4) and (6). We construct the solution as the sum of all possible solutions for each value of n.

$$w=\sum_{n=0}^{\infty}W(x)\cos my\ e^{-\omega t} \tag{15}$$

and $\frac{\partial p}{\partial z}=-Pe^{-\omega t},(\omega>0)$

Using (15) and putting the value of $\frac{\partial p}{\partial z}$ in (14), we get

$$\sum_{n=0}^{\infty}\left[\frac{d^2}{dx^2}W(x)-m^2W(x)\right]\cos my+\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega-\frac{M^2}{(1+m_1^2)}\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\times\sum_{n=0}^{\infty}W(x)\cos my+\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}P=0 \tag{16}$$

If we express $\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}P$ as a Fourier series in the interval of $-\frac{b}{a}\le y\le\frac{b}{a}$ and equate the coefficient of cos my to zero, we get

$$\frac{d^2}{dx^2}W(x)-\frac{K_1^2}{a^2}W+A_n=0 \tag{17}$$

where $A_n=\dfrac{(-1)^n 4P(1-\lambda_1\omega+\lambda_2\omega^2)}{(2n+1)\pi(1-\lambda_1\omega+\mu_2\omega^2)}$

$$K_1^2=\left\{m^2-\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega-\frac{M^2}{(1+m_1^2)}\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right\}a^2 \tag{18}$$

Now we obtain the solution by using the boundary condition (7) as follows

$$W(x)=\frac{(-1)^{n+1}}{(2n+1)}\frac{4P}{\pi}\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)K_1^2}\left\{1\frac{\cos\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}$$

So the velocity of the fluid is given by

$$W(x,y,t)=\frac{(-1)^{n+1}}{(2n+1)}\frac{4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)K_1^2}\left\{1-\frac{\cos\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}\times e^{-\omega t}\cos(2n+1)\frac{\pi ay}{2b} \tag{19}$$

Thus the velocity of second order Oldroyd visco-elastic fluid is given by equation (19).

DEDUCTION

Case I: If we put $\mu_1=0$ in equation (12) we shall obtain the Maxwell fluid which is given below

$$w(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{(2n+1)}\frac{4P(1-\lambda_1\omega)}{\pi K_1^2}\left\{1-\frac{\cosh\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}\right]e^{-\omega t}\cos(2n+1)\frac{\pi ay}{2b}$$

where $K_1^2=\left[m^2-(1-\lambda_1\omega)\left(\omega-\dfrac{M^2}{(1+m_1^2)}\right)\right]a^2$

Case II: Putting $\mu_1=0$ and $\lambda_1=0$ in equation (12) we shall obtain the purely viscous fluid which is given below

$$w(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{(2n+1)}\frac{4P}{\pi K_1^2}\left\{1-\frac{\cosh\frac{K_1}{a}x}{\cosh\frac{K_1}{a}}\right\}\right]e^{-\omega t}(2n+1)\frac{\pi ay}{2b}$$

where $K_1^2=\left\{m^2-\left(\omega-\dfrac{M^2}{(1+m_1^2)}\right)\right\}a^2$

Case III: Putting $m_1=0$ in equation (12) we shall obtain the same result as Kumar and Singh [3].

DISCUSSION

The expressions for velocities (in the presence of Hall currents) of first order Oldroyd visco-elastic fluid by (12) and for second order visco-elastic fluid is given by equations (19) respectively.

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