International Journal of Mathematical Archive-3(9), 2012, 3253-3260
IJMA Available online through www.ijma.info ISSN 2229 – 5046

RADIATION EFFECT ON HYDROMAGNETIC CONVECTIVE FLOW OF VISCOUS LIQUID PAST AN INFINITE VERTICAL PLATE IN THE PRESENCE OF A HEAT SOURCE

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(Received on: 26-07-12; Revised & Accepted on: 03-09-12)

ABSTRACT

A study of radiation effect on free convection flow of an incompressible, electrically conducting, viscous liquid through time dependent porous medium past an infinite, isothermal, vertical, porous plate is presented. A constant heat source is considered in the flow region in the presence of transversely applied uniform magnetic field and a time dependent suction velocity. To obtain the analytical solutions of velocity field and temperature field, perturbation technique is used. The expressions for skin friction and rate of heat transfer are also obtained. The effect of Magnetic parameter, Permeability parameter, Grashof number, Radiation Parameter on velocity and temperature is discussed graphically. In this study velocity of liquid increases with the increase in G_r *(Grashof number),* k_o *(Permeability parameter) and N (Radiation parameter), but it decreases with the increase in M (Magnetic parameter).*

Key words: *Radiation, Viscous liquid, Porous medium, Magnetic field, onvective Flow, Time dependent suction velocity, Heat source/sink.*

INTRODUCTION

Many scientists have studied problems on free convection under the influence of magnetic field due to its application in Astrophysics, Geophysics, Engineering, aerodynamics etc. Extensive reviews of the free convection flows have been done by Ede [2], Gebhart [4] and Nield and Bejan [8].

Laminar free convection flow of a viscous fluid past an infinite porous plate was discussed by Soundalgekar [15]. Purushothaman et.al. [12] have studied free convection in an infinite porous medium due to a pulsating point heat source. Jha [6] studied a unsteady mixed convection flow past an infinite vertical porous plate with constant heat source. This study was further extended by Jha [7] to investigate the effects of Hall current in wall temperature oscillation on free convective and mass transfer flow in a rotating porous medium with constant heat source. Prathiban and Patil [10] studied the effects of non-uniform boundary temperature on thermal instability in a porous medium with internal heat source. Pop et al [9] presented an analysis on mixed convection in a porous medium produced by a line heat source.

In this series, Singh and Kumar [13] presented an analysis of free convection in oscillating MHD flow of a visoelastic Rivlin-Ericksen in porous medium with constant heat sources. Hsu *et. al.* [5] have analyzed natural convection of micro polar fluid in an enclosure with heat sources. Dass and Lahiri [1] studied three-dimensional coupled thermoplastic interactions due to instantaneous point heat source. Singh et. al. [14] have discussed free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate considering the presence of a heat source. Gireesha and Bagewadi [3] have discussed a study of two-dimensional dusty fluids flow under transverse magnetic field. Recently, Pundhir et al [11] have studied hydro magnetic convective flow past an infinite vertical plate in the presence of a heat source.

In this study we consider the work of Pundhir et. al. [11] with radiation. The aim of present investigation is to study radiation effect on hydro magnetic convective flow of viscous liquid past an infinite vertical plate in the presence of a heat source.

FORMULATION OF THE PROBLEM

In Cartesian system, we consider the two dimensional flow of an electrically conducting incompressible viscous liquid through a porous medium past an infinite isothermal vertical porous plate with constant heat source and radiation at

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y = 0 in the presence of a uniform magnetic field B_o applied normal to the flow. The porous medium is a non-homogeneous. Therefore, it can be taken as a function of t say k(t). The analysis to the present problem is based on the following assumptions:

1. The x-axis is along the plate y-axis normal to it.
2. A uniform magnetic filed $\vec{B}_0 = (\mu_e H_0)$ is applied and acts in the y-direction.
3. The permeability of the medium is an exponential decreasing function of time.
4. The plate temperature varies is the presence of a heat source.
5. The magnetic Reynold's number is small so the induced magnetic field is negligible.
6. The plate is infinite in extent; therefore, all the physical variables except pressure depend on y and t only.
7. The suction velocity is an exponential decreasing function of time.

The governing equations for continuity, momentum of energy for the present problem under the present configuration are:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k(t)}\right)u \tag{2}$$

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial y} = \frac{K_T}{\rho C_P}\frac{\partial^2 T}{\partial y^2} - \frac{S}{\rho C_P}(T - T_\infty) - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y} \tag{3}$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, ν is the Kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_o is the magnetic induction, S is the source/sink coefficient, q_r is the radiative heat flux.

By using Rosseland approximation for the radiation we take

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{4}$$

where σ^* the Stefan - Boltzmann constant and k^* the mean absorption coefficient.

We assume that the temperature differences within the flow are such that T^4 may be expressed as a linear function of temperature.

This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \simeq 4T_\infty^3 T - 3T_\infty^4 \tag{5}$$

The continuity equation (1) shows that v is a function of time only. Therefore, we assume:

$$v = -v_0(1 + \varepsilon e^{-nt}) \tag{6}$$

where $v_0 > 0$ is a real constant, n is a positive constant, ε is a small quantity (<<1) and the negative sign indicates that the suction is towards the plate.

Using assumption (6) we assume:

$$k(t) = k_0(1 + \varepsilon e^{-nt}) \tag{7}$$

where k_0 is the constant permeability of the medium.

The boundary conditions for the present problem are:

$$u = U_o\,(1 + \varepsilon e^{-nt}) \quad T = T_\infty(1 + \varepsilon e^{-nt}) \text{ at } y = 0$$
$$u \to 0\,,\ T \to T_\infty \text{ as } y \to \infty \tag{8}$$

Substituting (6) and (7) in (2) and (3), we get :

$$\frac{\partial u}{\partial t} - v_o\left(1+\varepsilon e^{-nt}\right)\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2 u}{\rho} - \frac{\nu u}{k_o(1+\varepsilon e^{-nt})} \tag{9}$$

$$\frac{\partial T}{\partial t} - v_o\left(1+\varepsilon e^{-nt}\right)\frac{\partial T}{\partial y} = \frac{K_T}{\rho C_P}\frac{\partial^2 T}{\partial y^2} - \frac{S}{\rho C_P}(T - T_\infty) - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y} \tag{10}$$

On introducing the following non dimensional quantities

$$u^* = \frac{u}{U_o},\ t^* = \frac{t v_o^2}{\nu},\ y^* = v_o\frac{y}{\nu},\ T^* = \frac{T - T_\infty}{\nu},\ n^* = \frac{n\nu}{v_o^2},$$

Equations (9) and (10) after dropping the asterisks (*) can be written as

$$\frac{\partial u}{\partial t} - (1+\varepsilon e^{-nt})\frac{\partial u}{\partial y} = G_r T + \frac{\partial^2 u}{\partial y^2} - M^2 u - \frac{u}{k_o(1+\varepsilon e^{-nt})} \tag{11}$$

$$\frac{\partial T}{\partial t} - \left(1+\varepsilon e^{-nt}\right)\frac{\partial T}{\partial y} = \frac{1}{P_r}\left(1+\frac{4N}{3}\right)\frac{\partial^2 T}{\partial y^2} - ST \tag{12}$$

with the boundary conditions :

$$u = (1 + \varepsilon e^{-nt})\ T = (1 + \varepsilon e^{-nt}) \text{ at } y = 0$$
$$u \to 0\,,\ T \to 0 \quad \text{as } y \to \infty \tag{13}$$

where

$$k_0^* = \frac{k_0 v_0^2}{\nu^2} \quad \text{(Permeability Parameter)}$$

$$S^* = \frac{\nu S}{\rho C_p v_0^2} \quad \text{(Heat source parameter)}$$

$$M = \sqrt{\frac{\sigma B_0^2 \nu}{\rho v_0^2}} \quad \text{(Magnetic parameter)}$$

$$G_r = \frac{g B \nu^2}{U_0 v_0^2} \quad \text{(Grashof number)}$$

$$P_r = \frac{\mu C_P}{K_T} \quad \text{(Prandtl number)}$$

$$N = \left(\frac{4\sigma^* T_\infty^3}{k^* K_T}\right) \quad \text{(Radiation parameter)}$$

SOLUTION OF THE PROBLEM

The solutions of equations (11) and (12) are:

$$u = u_o + \varepsilon u_1 e^{-nt}$$
$$T = T_o + \varepsilon T_1 e^{-nt} \quad .... \quad (14)$$

Substituting (14) in the equations (11) and (12) and comparing the harmonic and non harmonic terms, we get

for u:
$$u_0'' + u_0' - M_1 u_0 = -G_r T_0 \quad (15)$$
$$u_1'' + u_1' - M_2 u_1 = -G_r T_1 \quad (16)$$

for T:
$$(1+4N/3)\, T_o'' + P_r T_o' - S P_r T_o = 0 \quad (17)$$
$$(1+4N/3)\, T_1'' + P_r T_1' - P_r S_1 T_1 = -P_r T_0' \quad (18)$$

where

$$M_1 = M^2 + \frac{1}{k_0},\ M_2 = M^2 + \frac{1}{k_0} - n,\ S_1 = S - n$$

with the boundary conditions

$$u_o = 1,\ u_1 = 1,\ T_o = 1,\ T_1 = 1 \text{ at } y = 0$$
$$u_o \to 0,\ u_1 \to 0,\ T_o \to 0,\ T_1 \to 0 \text{ as } y \to \infty \quad .... \quad (19)$$

The solution of the above coupled equation (15) to (18) under the boundary condition (19), we get:

$$u_0(y) = (1 + a_2)e^{-m_3 y} - a_2 e^{-m_1 y} \quad .... \quad (20)$$

$$u_1 = (1 - a_6 - a_7 - a_8)e^{-m_4 y} + a_6 e^{-m_3 y} + a_7 e^{-m_2 y} + a_8 e^{-m_1 y} \quad (21)$$

$$T_0(y) = e^{-m_1 y} \quad (22)$$

$$T_1(y) = (1 + a_1)e^{-m_2 y} - a_1 e^{-m_1 y} \quad .... \quad (23)$$

where

$$m_1 = \frac{P_r + \sqrt{P_r^2 + 4P_r S(1+4N/3)}}{2(1+4N/3)}$$
$$m_2 = \frac{P_r + \sqrt{P_r^2 + 4P_r S_1(1+4N/3)}}{2(1+4N/3)}$$
$$m_3 = \frac{1 + \sqrt{1 + 4M_1}}{2}$$
$$m_4 = \frac{1 + \sqrt{1 + 4M_2}}{2}$$
$$a_1 = \frac{-m_1 P_r}{(1+4N/3)m_1^2 - m_1 P_r + S_1 P_r}$$
$$a_2 = \frac{G_r}{m_1^2 - m_1 - M_1}$$

$$a_3 = (M_2 - \frac{1}{k_0})(1 + a_2)$$

$$a_4 = -(1 + a_1)G_r$$

$$a_5 = a_1 G_r + \frac{a_2}{k_0} - a_2 m_1$$

$$a_6 = \frac{a_3}{m_3^2 - m_3 - M_2}$$

$$a_7 = \frac{a_4}{m_2^2 - m_2 - M_2}$$

$$a_8 = \frac{a_5}{m_1^2 - m_1 - M_2}$$

Substituting the above values of u_0, u_1, T_0 and T_1 in (14), we obtain

$$u = (1 + a_2)e^{-m_3 y} - a_2 e^{-m_1 y} + \varepsilon[(1 - a_6 - a_7 - a_8)e^{-m_4 y} + a_6 e^{-m_3 y} + a_7 e^{-m_2 y} + a_8 e^{-m_1 y}]e^{-nt} \quad (24)$$

$$T = e^{-m_1 y} + \varepsilon[(1 + a_1)e^{-m_2 y} - a_1 e^{-m_1 y}]e^{-nt} \quad (25)$$

The skin friction coefficient (τ) at the plate at y = 0 is:

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = m_1 a_2 - (1 + a_2)m_3 + \varepsilon[(a_6 + a_7 + a_8 - 1)m_4 - a_6 m_3 - a_7 m_2 - a_8 m_1]e^{-nt} \quad (26)$$

The rate of heat transfer in terms of Nusselt number (N_u) at the plate at y = 0 is:

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = m_1 + \varepsilon[m_1 a_1 - (1 + a_1)m_2]e^{-nt} \quad (27)$$

RESULTS AND DISCUSSION

The velocity Profile of liquid is tabulated in Table-I and plotted in Fig.-I having five Graphs for n = 2, t = 1, ε = 0.003, S = 3, P_r = 0.71 and following different values of G_r, M, k_o and N.

	G_r	M	k_o	N
For Graph-I	12	2	5	1
For Graph-II	18	2	5	1
For Graph-III	12	3	5	1
For Graph-IV	12	2	40	1
For Graph-V	12	2	5	4

It is observed from Fig.-I that all velocity Graphs of liquid is decreasing sharply up to y = 3, then after velocity in each Graphs begins to decrease and tends to zero with the increase in y. It is also observed from Fig.-I that velocity of liquid increases with the increase in G_r, k_o and N, but it decreases with the increase in M.

The temperature Profile is tabulated in Table-II and plotted in Fig.-II having three Graphs. It is observed that temperature increases with the increase in N.

Skin friction Profile is tabulated in Table-III and plotted in Fig.-III having five Graphs. It is observed that skin friction increases with the increase in G_r, k_o and N, but it decreases with the increase in M and t.

PARTICULAR CASE

When N is equal to zero, this problem reduces to the problem of Pundhir *et al* [11].

CONCLUSION

1. Velocity of liquid increases with the increase in N (Radiation parameter).
2. Temperature of liquid increases with the increase in N (Radiation parameter).
3. Skin friction of liquid increases with the increase in N (Radiation parameter).

ACKNOWLEDGEMENT

The authors are very thankful to the Principal and the Head, Department of Mathematics, S.V. College, Aligarh for providing necessary facilities.

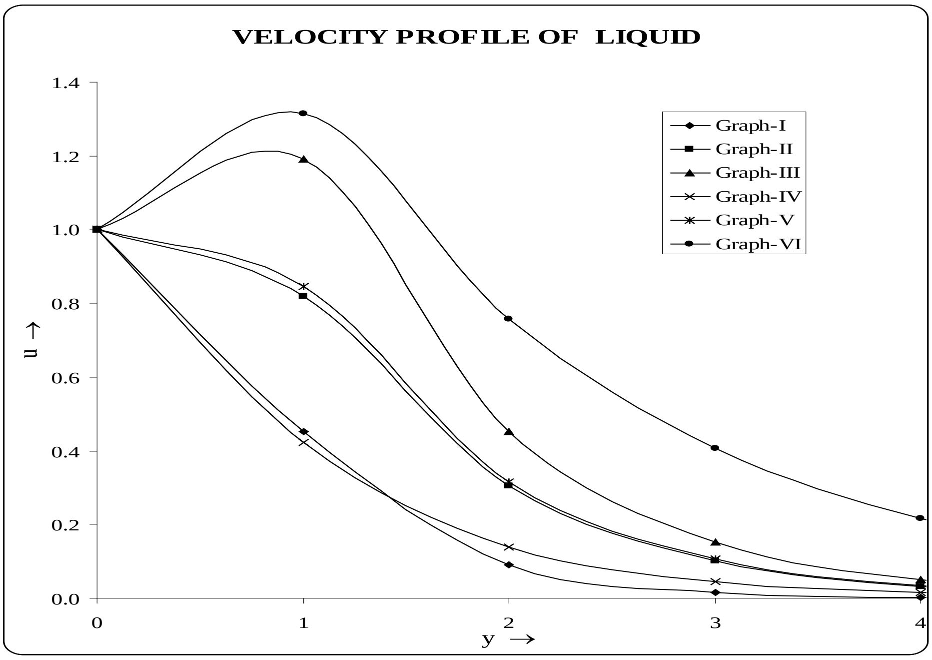

Fig.-I

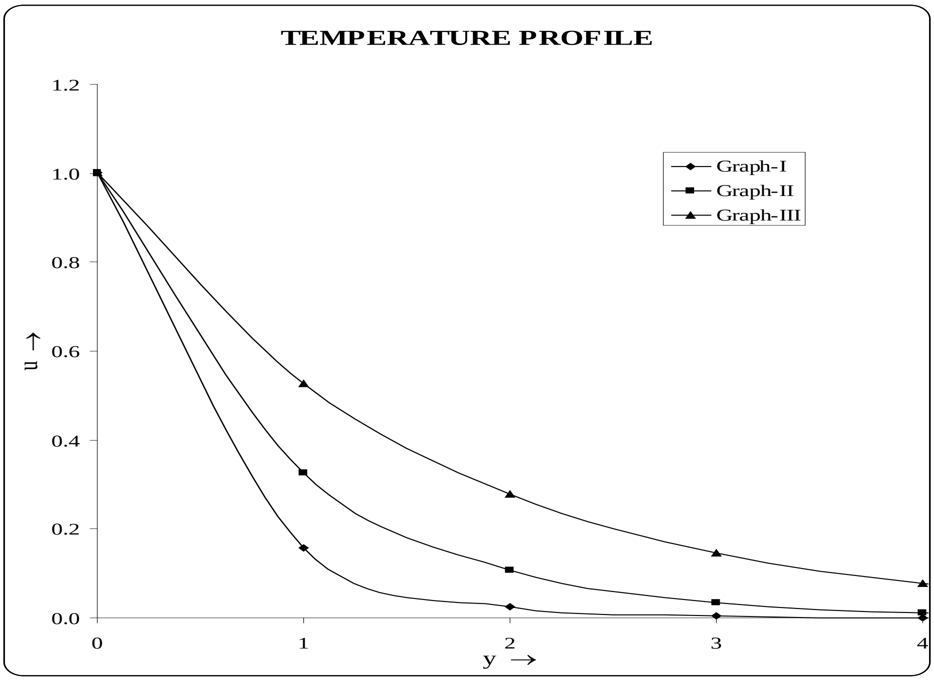

Fig.-II

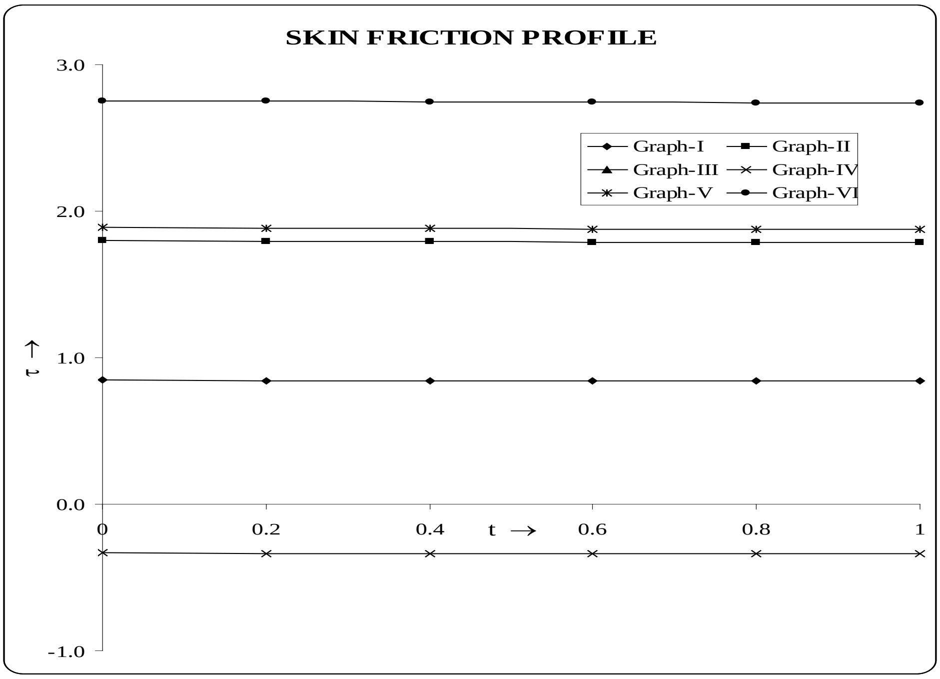

Fig.-III

Table-I: Values of velocity of liquid at n = 2, t = 1, S = 3, P_r = 0.71, ε = 0.003 and different values of G_r, M, k_o and N.

y	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	1.00041	1.00041	1.00041	1.00041	1.00041	1.00041
1	0.45167	0.81978	1.19162	0.42195	0.84481	1.31404
2	0.09211	0.30386	0.45258	0.14040	0.31582	0.75745
3	0.01595	0.10187	0.15243	0.04590	0.10620	0.40774
4	0.00260	0.03346	0.05012	0.01499	0.03492	0.21754

Table-II: Values of temperature at n = 2, t = 1, S = 3, P_r = 0.71, ε = 0.003 and different values of N.

y	Graph-I	Graph-II	Graph-III
0	1.00041	1.00041	1.00041
1	0.15733	0.32646	0.52756
2	0.02475	0.10654	0.27821
3	0.00389	0.03477	0.14672
4	0.00061	0.01135	0.07738

Table-III: Values of skin friction at n = 2, S = 3, P_r = 0.71, ε = 0.003 and different values of G_r, M, k_o and N.

t	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0.84576	1.79790	3.99833	-0.33381	1.88763	2.75451

0.2	0.84386	1.79381	3.99091	-0.33453	1.88354	2.74841
0.4	0.84259	1.79107	3.98593	-0.33502	1.88079	2.74432
0.6	0.84174	1.78923	3.98259	-0.33534	1.87895	2.74158
0.8	0.84116	1.78800	3.98035	-0.33556	1.87771	2.73974
1	0.84078	1.78718	3.97885	-0.33570	1.87688	2.73851

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Source of support: Principal and the Head, Department of Mathematics, S.V. College, Aligarh, Conflict of interest: None Declared