International Journal of Mathematical Archive-3(11), 2012, 3941-3946

On $\pi g\theta$ -Closed sets in Topological spaces

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(Received on: 24-10-12; Revised & Accepted on: 25-11-12)

ABSTRACT

In this paper a new class of sets called $\pi g \theta$ -closed is introduced and its properties are studied. Further the notion of $\pi g \theta$ - $T_{1/2}$ space and $\pi g \theta$ -continuity are introduced.

Mathematics Subject Classification: 54A05.

Key words: $\pi g \theta$ -closed set, $\pi g \theta$ -open set, $\pi g \theta$ -continuity and $\pi g \theta$ - $T_{\frac{1}{2}}$ space.

1. INTRODUCTION

Velicko[23] introduced the notions of θ -open subsets, θ -closed subsets and θ -closure, for the sake of studying the important class of H-closed spaces in terms of arbitrary filterbases. Dontchev and Maki [2] alone have explored the concept of θ -generalized closed sets. The finite union of regular open set is π -open and subsequently the complement of π -open set is said to be π -closed, which has been highlighted by Zaitsev [23]. Dontchev and Noiri have explored the concept of quasi normal spaces and π g-closed sets. The studies of π g α -closed set [11], π gp-closed set [19], π gb-closed set[20], π gs-closed set[4] were introduced later.

The prime objective of this study is to explore the idea of $\pi g \theta$ -closed sets, its properties, characterization and its functions.

2. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) will be replaced by X if there is no chance of confusion.

Let us recall the following definitions which we shall require later.

Definition 2.1: A subset A of space (X, τ) is called

- (1) a pre open set[16] if $A \subset int(cl(A))$ and a pre closed set if $cl(int(A)) \subset A$;
- (2) a semi-open set[3] if $A \subset cl(int(A))$ and semi-closed if $int(cl(A)) \subset A$;
- (3) a α -open set [17] if A \subset int(cl(int(A))) and α -closed set if cl(int(cl(A))) \subset A;
- (4) a semi- pre open set[2] if $A \subset cl(int(cl(A)))$ and semi-pre closed set if $int(cl(int(A))) \subset A$;
- (5) a regular open set [21] if A = int (cl(A)) and a regular closed set if A = cl(int(A)).

Definition 2.2: A subset A of a space(X, τ) is called

(1) a generalized closed (briefly g-closed) set [13] if $cl(A) \subset U$ whenever $A \subset U$ and U is open.

- (2) a semi-generalized closed (briefly sg-closed)set [5] if $scl(A) \subset U$ whenever $A \subset U$ and U is semi-open.
- (3) a generalized semi-closed (briefly gs-closed)set [3] if $scl(A) \subset U$ whenever $A \subset U$ and U is open.
- (4) a α -generalized closed (briefly α g-closed) set [15] if α cl(A) \subset U whenever A \subset U and U is open in (X, τ).

(5) a generalized α -closed (briefly g α -closed) set[14] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is α -open in (X, τ) .

(6) a θ -generalized closed (briefly, θ g-closed) set [7] if $cl_{\theta}(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ) .

- (7) π g -closed set [8] if cl(A) \subset U, whenever A \subset U and U is π -open.
- (8) π ga-closed set [11] if α cl(A) \subset U, whenever A \subset U and U is π -open.
- (9 π gs-closed set [4] if scl(A) \subset U, whenever A \subset U and U is π -open.
- (10) π gb -closed set [20] if bcl(A) \subset U, whenever A \subset U and U is π -open.
- (11) π gp -closed set [19] if pcl(A) \subset U, whenever A \subset U and U is π -open.

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International Journal of Mathematical Archive- 3 (11), Nov. - 2012

The compliment of g-closed (resp. sg-closed, gs-closed, ag-closed, ga-closed, π g-closed, π ga-closed, π gs-closed, π g θ -closed, π gp-closed) is g-open (resp. sg- open, gs- open, ag- open, θ g- open, π ga- open, π g θ - open, π g- open,

Definition 2.3: Let (X, τ) be a topological space. A point $x \in X$ is said to be in the θ -closure of a subset $A \subseteq X$ if for each open neighbourhood U of x we have $cl(U) \cap A \neq \Phi$. We shall denote the θ -closure of A by $cl_{\theta}(A)$. A subset $A \subseteq X$ is called θ - closed if $A = cl_{\theta}(A)$. The compliment of θ - closed is θ - open.

Definition 2.4: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

(1) π -open map [23] if f(F) is π -open in Y for every π -open in X.

(2) π -irresolute [23] if $f^{-1}(V)$ is π -closed in (X, τ) for every π -closed of (Y, σ) ;

(3) θ -irresolute [18] if for each θ -open set V in Y, $f^{1}(V)$ is θ -open in X;

(4) θ -continuous [18] if for each open set V in Y, $f^{1}(V)$ is θ -open in X.

3. πgθ-CLOSED SETS

We introduce the following definition.

Definition 3.1: A subset A of (X, τ) is called $\pi g \theta$ -Closed set (briefly $\pi g \theta$ -closed) if $cl_{\theta}(A) \subset U$ whenever $A \subset U$ and U is π -open. By $\pi G \theta C$ (τ) we mean the family of all $\pi g \theta$ -closed subsets of the space (X, τ). The compliment of $\pi g \theta$ -closed is $\pi g \theta$ -open.

Theorem 3.2:

- **1.** Every θ closed set is $\pi g \theta$ -closed.
- **2.** Every θ g-closed set is π g θ -closed.
- **3.** Every $\pi g \theta$ -closed set is πg -closed.
- **4.** Every $\pi g \theta$ -closed set is $\pi g \alpha$ -closed.
- **5.** Every $\pi g \theta$ -closed set is $\pi g s$ -closed.
- **6.** Every $\pi g \theta$ -closed set is $\pi g b$ -closed.
- **7.** Every $\pi g \theta$ -closed set is $\pi g p$ -closed.

Proof: Straight forward.

Converse of the above need not be true as seen in the following examples.

Example 3.3: Let $X = \{a, b, c\}, \tau = \{\Phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$. Let $A = \{b\}$. Then A is $\pi g \theta$ -closed but not θ -closed

Example 3.4: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{b\}, \{d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Let $A = \{a, d\}$. Then A is $\pi g \theta$ - closed but not θg -closed.

Example 3.5: Let $X = \{a, b, c, d, e\}$, $\tau = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $A = \{a, e\}$. Then A is π g-closed but not π g θ -closed.

Example 3.6: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$, Let $A = \{c\}$. Then A is $\pi g \alpha$ -closed but not $\pi g \theta$ -closed.

Example 3.7: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$, Let $A = \{a\}$. Then A is π gs-closed but not π g θ -closed.

Example 3.8: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{a, c\}$. Then A is π gb-closed but not π g θ - closed.

Example 3.9: Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let $A = \{c\}$. Then A is π gp-closed but not π g θ -closed. π g θ -closed is independent of closedness, α - closedness, semi –closedness, sg-closedness, gs-closedness, as seen in the following examples.

Example 3.10: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$.(i) Let $A = \{d\}$. Then A is $\pi g \theta$ -closed but not g-closed.

Example 3.11: Let $X = \{a, b, c, d, e\}, \tau = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let $A = \{e\}$. Then A is g-closed but not $\pi g \theta$ -closed.

Example 3.12: Let $X = \{a, b, c, \}$, $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let $A = \{b\}$. Then A is $\pi g \theta$ -closed but not closed, α - closed, semi –closed.

Example 3.13: Let $X = \{a, b, c\}, \tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let $A = \{a\}$. Then A is closed, α - closed, semi-closed but not $\pi g \theta$ -closed.

Example 3.14: Let $X = \{a, b, c, d\}, \tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}.$ (i) Let $A = \{a, b, c\}$. Then A is $\pi g\theta$ -closed but neither sg-closed nor gs-closed. (ii) Let $A = \{a\}$. Then A is sg-closed, gs-closed but not $\pi g\theta$ -closed.

Example 3.15: Let $X = \{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. (i) Let $A = \{b, d\}$. Then A is $\pi g \theta$ -closed but neither αg - closed nor $g \alpha$ - closed. (ii) Let $A = \{c\}$. Then A is αg - closed, $g \alpha$ - closed but not $\pi g \theta$ -closed.

Remark 3.16: The above discussion are summarized in the following diagram.



 $1 = \pi g \theta$ -closed, $2 = \theta$ -closed, $3 = \theta g$ -closed, $4 = \pi g$ -closed, $5 = \pi g \alpha$ -closed, $6 = \pi g s$ -closed, $7 = \pi g b$ -closed, $8 = \pi g p$ -closed.

Theorem 3.17: If A is regular open and $\pi g \theta$ -closed, then A is θ -closed.

Proof: Let A be regular open and $\pi g\theta$ -closed. Since every regular open is π -open and since A is $\pi g\theta$ -closed, $cl_{\theta}(A) \subset A$. Then $A = cl_{\theta}(A)$. Hence A is θ -closed.

Theorem 3.18: Let A be $\pi g \theta$ -closed in (X, τ) . Then $cl_{\theta}(A)$ - A does not contain any non-empty π -closed set.

Proof: Let F be a non-empty π -closed set such that $F \subset cl_{\theta}(A) - A$. Since A is $\pi g\theta$ -closed, $A \subset X - F$ where X - F is π -open implies $cl_{\theta}(A) \subset X$ - F. Hence $F \subset X$ - $cl_{\theta}(A)$. Now, $F \subset cl_{\theta}(A) \cap X$ - $cl_{\theta}(A)$ implies $F = \phi$ which is a contradiction. Therefore $cl_{\theta}(A)$ - A does contain any non-empty π -closed set.

Corollary 3.19: Let A be $\pi g \theta$ -closed in (X, τ). Then A is θ -closed if and only if $cl_{\theta}(A)$ -A is π -closed.

Proof: Let A be θ -closed. Then $cl_{\theta}(A) = A$ implies $cl_{\theta}(A) - A = \phi$ which is π -closed. Assume $cl_{\theta}(A) - A = \phi$ is π -closed. Then $cl_{\theta}(A) - A = \phi$. Hence $cl_{\theta}(A) = A$.

Theorem 3.20: If A is $\pi g\theta$ -closed set and B is any set such that $A \subset B \subset cl_{\theta}(A)$, then B is $\pi g\theta$ -closed set.

Proof: Let $B \subset U$ and U be π -open. Given $A \subset B$. Then $A \subset U$. Since A is $\pi g\theta$ -closed, $A \subset U$ implies $cl_{\theta}(A) \subset U$. By assumption it follows that $cl_{\theta}(B) \subset cl_{\theta}(A) \subset U$. Then B is $\pi g\theta$ - closed.

Theorem 3.21: A finite union of $\pi g\theta$ -closed sets is always a $\pi g\theta$ -closed.

Proof: Let A, $B \in \pi G\theta C(X)$. Let U be any π -open set such that $(A \cup B) \subseteq U$. Since $cl_{\theta}(A \cup B) = cl_{\theta}(A) \cup cl_{\theta}(B) \subseteq U \cup U=U$. This implies $cl_{\theta}(A \cup B) \subseteq U$. Hence $A \cup B$ is also a $\pi g\theta$ -closed set.

Remark 3.22: Finite intersection of $\pi g \theta$ -closed sets need not be $\pi g \theta$ -closed.

Example 3.23: Let $X = \{a, b, c, d\}$ and $\tau = \{\Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, d\}, X\}$. Let $A = \{a, b, c\}$ and $B = \{a, b, d\}$. Clearly A and B are $\pi g \theta$ -closed sets. But $A \cap B = \{a, b\}$ is not a $\pi g \theta$ -closed set.

4. π gθ -OPEN SETS

Definition 4.1: A set $A \subset X$ is called $\pi g \theta$ -open if and only if its complement is $\pi g \theta$ -closed.

Remark 4.2: $cl_{\theta}(X-A) = X - int_{\theta}(A)$. By $\pi G\theta O(\tau)$ we mean the family of all $\pi g\theta$ -open subsets of the space (X, τ) .

Theorem 4.3: If $A \subset X$ is $\pi g \theta$ -open if and only if $F \subset int_{\theta}(A)$ whenever F is π -closed and $F \subset A$.

Proof: Necessity: Let A be $\pi g\theta$ -open Let F be π -closed and F \subset A. Then X-A \subset X-F where X-F is π -open. By assumption, $cl_{\theta}(X-A) \subset X$ -F. By remark 4.2, X- $int_{\theta}(A) \subset (X$ -F). Thus F $\subset int_{\theta}(A)$.

Sufficiency: Suppose F is π -closed and F \subset A such that F \subset int_{θ}(A). Let X-A \subset U where U is π -open. Then X-U \subset A where X-U is π -closed. By hypothesis, X-U \subset int_{θ}(A) which implies $cl_{\theta}(X-A) \subset X$ - int_{θ}(A) \subset U. Thus X-A is $\pi g\theta$ - closed and A is $\pi g\theta$ -open.

Theorem 4.4: If $int_{\theta}(A) \subset B \subset A$ and A is $\pi g\theta$ -open, then B is also $\pi g\theta$ -open.

Proof: Let $int\theta(A) \subset B \subset A$. Thus X-A \subset X-B $\subset cl_{\theta}(X-A)$. Since X – A is $\pi g\theta$ - closed, by theorem 3.20, X-A \subset X-B $\subset cl_{\theta}(X-A)$ implies X-B is $\pi g\theta$ - closed.

Remark 4.5: For any $A \subset X$, $int_{\theta}(cl_{\theta}(A)-A) = \Phi$.

Theorem 4.6: If $A \subset X$ is $\pi g \theta$ -closed, then $cl_{\theta}(A)$ -A is $\pi g \theta$ - open.

Proof: Let A be $\pi g\theta$ -closed. Let F be π -closed. F \subset cl_{θ}(A) –A.By theorem3.18, F = Φ . By remark 4.5, int_{θ}(cl_{θ}(A)-A) = Φ . Thus F \subset int_{θ}(cl_{θ}(A)-A) = Φ . Thus cl_{θ}(A) –A is $\pi g\theta$ – open.

Definition 4.7: A space (X, τ) is called a $\pi g\theta$ -T_{1/2} space if every $\pi g\theta$ –closed set is θ -closed.

Theorem 4.8:

(i) $\theta O(\tau) \subset \pi G \theta O(\tau)$.(ii) A space (X, τ) is $\pi g \theta$ -T_{1/2} space iff $\theta O(\tau) = \pi G \theta O(\tau)$.

Proof: Let A be θ -open, then X-A is θ -closed. So X-A is of $\pi g \theta$ -closed. Then A is of $\pi g \theta$ -open. Hence $\theta O(\tau) \subset \pi G \theta O(\tau)$.

(ii)Necessity: Let (X,τ) be $\pi g\theta$ -T_{1/2} space. Let $A \in \pi G\theta O(\tau)$. Then X-A is $\pi g\theta$ -closed. By hypothesis, X - A is θ -closed, thus $A \in \Theta O(\tau)$. Thus $\pi G\theta O(\tau) = \theta O(\tau)$.

Sufficiency: Let $\theta O(\tau) = \pi G \theta O(\tau)$. Let A be of $\pi g \theta$ -closed. Then X-A is $\pi g \theta$ -open. X-A $\in \pi G \theta O(\tau)$. This implies X-A $\in \theta O(\tau)$. Hence A is θ -closed. This implies(X, τ) is a $\pi g \theta$ -T_{1/2} space.

Theorem 4.9: For a topological space(X, τ) the following are equivalent . (i)X is a $\pi g\theta$ -T_{1/2} spa ce. (ii)Every singleton set is π -closed or θ -open.

Proof: (i) \Rightarrow (ii): Let X be a $\pi g \theta$ -T_{1/2} space. Let $x \in X$ and assuming that $\{x\}$ is not π -closed. Then clearly X – $\{x\}$ is trivially $\pi g \theta$ -closed in X. Since X is $\pi g \theta$ -T_{1/2} space, X- $\{x\}$ is θ -closed. Therefore $\{x\}$ is θ -open.

(ii) \Rightarrow (i) Assume every singleton of X is either π -closed or θ -open. Let A be $\pi g \theta$ -closed set. Let $\{x\} \in cl_{\theta}(A)$.

Case (i): Let $\{x\}$ be π -closed. Suppose $\{x\} \notin cl_{\theta}(A)$. Then $\{x\} \in cl_{\theta}(A)$. A. By theorem 3.18, $\{x\} \in A$. Hence $cl_{\theta}(A) \subset A$.

Case (ii) : Let $\{x\}$ be θ -open. Since $\{x\} \in cl_{\theta}(A)$, we have $\{x\} \cap A \neq \Phi$ implies $\{x\} \in A$. Therefore $cl_{\theta}(A) \subset A$. Hence A is θ -closed.

5. $\pi g \theta$ -continuous and $\pi g \theta$ -irresolute functions

Definition 5.1.: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called $\pi g\theta$ -continuous if every $f^{1}(V)$ is $\pi g\theta$ -closed in (X, τ) for every closed set V of (Y, σ) .

Definition 5.2: A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called $\pi g \theta$ -irresolute if $f^{1}(V)$ is $\pi g \theta$ -closed in (X, τ) for every $\pi g \theta$ -closed set V in (Y, σ) .

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Remark 5.3: 1. $\pi g\theta$ -irresolute function is independent of θ -irresoluteness, as seen in the following examples.

Example 5.4:(a).Let $X=Y=\{a, b, c, d\}$, $\tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, X\}$, $\sigma = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Then f is θ -irresolute but not $\pi g \theta$ -irresolute, since f¹{b, c, d} = {b, c, d} is not $\pi g \theta$ -closed in (X, τ) .

Example 5.4: (b). Let $X=Y=\{a, b, c, d\}, \tau = \{\Phi, \{a\}, \{d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, X\}, \sigma = \{\Phi, \{a\}, \{c\}, \{d\}, \{a, d\}, \{a, c\}, \{c, d\}, \{a, c, d\}, \{a, b, d\}, X\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Then f is $\pi g \theta$ -irresolute but not θ -irresolute, since f¹{a, b, d}={a, b, d} is not θ -closed in (X, τ) .

Remark 5.3:2. Every θ -continuous is $\pi g\theta$ -continuous The converse of the above need not be true as seen in the following examples.

Example 5.5: Let $X=Y=\{a, b, c, d, e\}, \tau = \{\Phi, \{a, b\}, \{c\}, \{a, b, c\}, X\}, \sigma = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Let f: (X, τ) \rightarrow (Y, σ) be an identity function. Then f is $\pi g \theta$ -continuous but not θ -continuous, since $f^1\{c, d, e\}=\{c, d, e\}$ is not θ -closed in (X, τ).

Remark 5.6: Composition of two $\pi g\theta$ -continuous function need not be $\pi g\theta$ -continuous.

Example 5.7: Let $X=Y=Z = \{a, b, c, d, e\}, \tau = \{\Phi, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}, \sigma = \{\Phi, \{a, b\}, \{c\}, \{a, b, c\}, X\}, \eta = \{\Phi, \{e\}, \{a, b, c, d\}, X\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an identity function. Let g: $(Y,\sigma) \rightarrow (Z,\eta)$ be an identity function. Both f and g are $\pi g \theta$ -continuous but gof is not $\pi g \theta$ -continuous, since $(gof)^{-1}\{a, b, c, d\} = \{a, b, c, d\}$ is not $\pi g \theta$ -closed in (X, τ) .

Definition 5.8: A function f: $X \rightarrow Y$ is said to be pre- θ -closed if f(U) is θ -closed in Y for each θ -closed set in X.

Preposition 5.9: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be π - irresolute and pre- θ - closed map. Then f(A) is $\pi g \theta$ –closed in Y for every $\pi g \theta$ –closed set A of X.

Proof: Let A be any $\pi g\theta$ -closed set of X and U be any π - open set of Y containing f(A). Since f is π -irresolute, f¹(U) is π -open in X and A \subseteq f¹(U). Therefore we have cl_{θ}(A) \subseteq f¹(U) and hence f (cl_{θ}(A)) \subseteq U.

Since f is pre- θ -closed, $cl_{\theta}(f(A)) \subseteq cl_{\theta}(f(cl_{\theta}(A))) = f(cl_{\theta}(A)) \subseteq U$. Hence f(A) is $\pi g\theta$ -closed in Y.

Theorem 5.10: Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be a function.

- (i) If f is $\pi g \theta$ -irresolute and X is $\pi g \theta$ -T_{1/2} space, then f is θ -irresolute.
- (*ii*) If f is $\pi g\theta$ -continuous and X is $\pi g\theta$ -T_{1/2} space then f is θ -continuous.

Proof:

(i) Let V be θ - closed in Y. Since f is $\pi g \theta$ -irresolute, f¹(V) is $\pi g \theta$ -closed in X.

Since X is $\pi g\theta$ -T_{1/2} space, f¹(V) is θ -closed in X. Hence f is θ -irresolute.

(ii)Let V be closed in Y. Since f is $\pi g \theta$ –continuous, $f^{1}(V)$ is $\pi g \theta$ –closed in X.

By assumption, it is θ –closed. Therefore f is θ –continuous.

Theorem 5.11: If the bijective f: $(X, \tau) \rightarrow (Y, \sigma)$ is θ –irresolute and π -open map then f is $\pi g \theta$ –irresolute.

Proof: Let V be $\pi g\theta$ -closed in Y. Let $f^{1}(V) \subset U$ where U is π -open in X. Then $V \subset f(U)$ and f(U) is π -open implies $cl_{\theta}(V) \subset f(U)$. This implies $f^{1}(cl_{\theta}(V)) \subset U$. Since f is θ -irresolute, $f^{1}(cl_{\theta}(V))$ is θ -closed. Hence $cl_{\theta}(f^{1}(V)) \subset cl_{\theta}(f^{1}(Cl_{\theta}(V))) = f^{1}(cl_{\theta}(V)) \subset U$. Therefore f is $\pi g\theta$ -irresolute.

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Source of support: Nil, Conflict of interest: None Declared