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DOMINATOR COLORING OF SOME CLASSES OF GRAPHS

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ABSTRACT

In this paper, dominator coloring of fan graph, double fan graph, helm graph, gear graph, flower graph and sun flower graph are considered. We also obtain the corresponding chromatic number of the above graph families.

Key Words: *proper coloring and dominator coloring.*

AMS Subject Classification: *05C15, 05C69.*

1. PRELIMINARIES

In this section, we review the notions of fan graph, double fan graph, helm graph, gear graph, flower graph, sun flower graph and dominator coloring [1, 2, 3, 4, 5].

Definition 1.1. The *fan* graph denoted by F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex. F_n is a planar undirected graph with 2n+1 vertices and 3n edges.

Definition 1.2. *Double fan* $F_{2,\,n}$ is a graph isomorphic to $P_n + 2\,K_1$.

Definition 1.3. The *helm* H_n is the graph obtained from a wheel graph $W_{1,\,n}$ by attaching a pendant edge at each vertex of the n – cycle.

Definition 1.4. *Gear graph* G_r also known as a bipartite wheel graph, is a wheel graph $W_{1,\,n}$ with a vertex added between each pair of adjacent vertices of the outer cycle. Gear graph G has 2r+1 vertices and 3r edges.

Definition 1.5. A *flower* graph Fl_n is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Definition 1.6. The *sun flower graph* Sf_n is the resultant graph obtained from the flower graph of wheels $W_{1,\,n}$ by adding n pendant edges to the central vertex. The vertex set of sun flower graph is defined as follows: wheel graph is the graph on n+1 vertices constructed by connecting a single vertex to every vertex in an n cycle. By attaching a pendant edge at each vertex of the n – cycle in wheel graph, we get a helm graph with (2n+1) vertices and 3n edges. By joining each pendant vertex to the central vertex of the helm graph, we get a flower graph with (2n+1) vertices and 4n edges. By adding n pendant edges to the central vertex of the flower graph, we get a sunflower graph with
p = (3n+1) vertices and q = 5n edges. It is also defined as, V(n, p, q).

Definition 1.7. A *proper coloring* of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. The *chromatic number* $\chi(G)$ is the minimum number of colors required for a proper coloring of G. A *Color class* is the set of vertices, having the same color. The color class corresponding to the color i is denoted by C_i

Definition 1.8. A *dominator coloring* of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if {v} is a color class, then v dominates the color class {v}. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.

2. DOMINATOR COLORING ON SOME CLASSES OF GRAPHS

In this section, dominator coloring of fan graph, double fan graph, helm graph, gear graph, flower graph and sun flower graph are consider and also obtain the corresponding chromatic number of the above graph families.

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Theorem 2.1. For Fan graph F_n, $n \geq 2$, $\chi_d(F_n) = 3$.

Proof. By the definition of fan graph, F_n is obtained by joining n copies of the cycle graph C_3 with a common vertex. Let $V = \{v_1, v_2, \ldots, v_{2n+1}\}$ be the vertex set of F_n and let the vertex at the centre be labeled by v_1. F_n is a planar undirected graph with 2n+1 vertices and 3n edges.

A procedure to obtain a dominator coloring of F_n is as follows. Let the vertex v_1 be colored by color 1. The other two vertices of each copy of C_3 are colored by colors 2 and 3.

The above procedure guarantees a dominator coloring of F_n as v_1 dominates itself. Also vertices v_i, $2 \leq i \leq 2n+1$ dominate the color class 1, since they are adjacent to v_1.

Hence $\chi_d(F_n) = 3$.

The following example illustrates the procedure discussed in the above result.

Example 2.2. In figure 1, Fan graph F_4 is depicted with a dominator coloring.

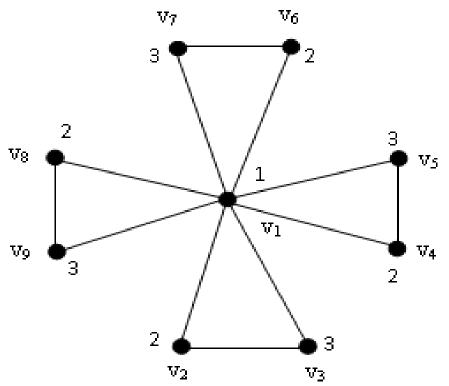

Figure: 1

The color classes of F_4 are $C_1 = \{v_1\}$, $C_2 = \{v_2, v_4, v_6, v_8\}$, $C_3 = \{v_3, v_5, v_7, v_9\}$. Therefore $\chi_d(F_4) = 3$.

Theorem 2.3. For Double fan graph $F_{2,n}$, $n \geq 2$, $\chi_d(F_{2,n}) = 3$.

Proof. Observe from the definition of double fan graph that $F_{2,n}$ is isomorphic to P_n+2K_1. Let $V_1 = \{v_i / 1 \leq i \leq n\}$ be the vertices of the path P_n and $V_2 = \{u_1, u_2\}$ be the vertices of $2K_1$. Then $V(F_{2,n}) = V_1 \cup V_2$. It has (n+2) vertices and (3n-1) edges.

A procedure to obtain a dominator coloring of $F_{2,n}$ is as follows. Since u_1 and u_2 are non-adjacent, they are colored by color 1. Since v_i, $1 \leq i \leq n$ is adjacent to u_1 and u_2, these vertices are alternatively colored by colors 2 and 3. So 3 colors are required to color these vertices.

The above procedure guarantees a dominator coloring of $F_{2,n}$ as v_i, $1 \leq i \leq n$ is adjacent to u_1 and u_2, these vertices dominate color class 1. Since u_1 and u_2 are adjacent to v_i, $1 \leq i \leq n$, they dominate color classes 2 and 3. So the given procedure gives a dominator coloring of $F_{2,n}$. Hence $\chi_d(F_{2,n}) = 3$.

Theorem 2.4. For Helm graph H_n, $n \geq 4$, $\chi_d(H_n) = n+1$.

Proof. By the definition of helm graph, H_n is obtained from a wheel by attaching a pendant edge at each vertex of the n – cycle. Let $V(H_n) = \{v_1\} \cup V_1 \cup V_2$, where v_1 is the central vertex, $V_2 = \{v_i / 2 \leq i \leq n+1\}$ be the vertices on the n – cycle and $V_3 = \{v_i / n+2 \leq i \leq 2n+1\}$ be the pendant vertices incident with n- cycle such that v_{n+i} is adjacent with v_i, $2 \leq i \leq n+1$. It has 2n+1 vertices and 4n edges.

The following procedure gives a dominator coloring of H_n. Since v_1 and v_{n+i}, $2 \leq i \leq 2n+1$ are non-adjacent, these vertices are colored by color 1. The remaining vertices v_i, $2 \leq i \leq n+1$ on the n – cycle are colored by i, since these vertices are adjacent to central vertex v_1 and pendant vertices v_i, $n+2 \leq i \leq 2n+1$.

In H_n, v_i, $2 \leq i \leq n+1$ dominate themselves. Since v_1 and v_{n+i}, $2 \leq i \leq 2n+1$ are adjacent to v_j, $2 \leq j \leq n+1$, these vertices dominate the color class j. So the given procedure gives a dominator coloring of H_n. Hence $\chi_d(H_n) = n+1$.

Theorem 2.5. For Gear graph G_r, $r \geq 3$, $\chi_d(G_r) = \lceil 2r / 3 \rceil + 2$.

Proof. It is clear from the definition of gear graph, G_r is obtained from a wheel graph $W_{1, n}$ with a vertex added between each pair of adjacent vertices of the outer cycle of wheel graph $W_{1, n}$. Let $V(G_r) = \{v_i / 1 \leq i \leq 2r+1\}$, where v_1 is a central vertex. It has 2r+1 vertices and 3r edges.

In G_r, v_1 is colored by color 1 and v_2 and v_4 are colored by color 2. When $r = 3k$, $k \geq 1$, the vertex v_{3+2i}, $0 \leq i \leq r - 1$ is colored by color 3, the vertex v_{6i}, $1 \leq i \leq (2r-1) / 6$ is colored by color (2i+2), the vertices v_{2i} and v_{2i+2}, i = 4, 7, 10, 13, …, r - 2 are colored by [2((i-3)/3)]. When $r = 3k+1$, $k \geq 1$, the vertex v_{3+2i}, $0 \leq i \leq r-1$ is colored by color 3, the vertex v_{6i}, $1 \leq i \leq (r - 1)/3$ is colored by color (2i+2), the vertices v_{2i} and v_{2i+2}, i = 4, 7, 10, 13, …, r-3 are colored by [2((i-3)/3)]. When $r = 3k+2$, $k \geq 1$, the vertex v_{3+2i}, $0 \leq i \leq r - 2$ is colored by color 3, the vertex v_{6i}, $1 \leq i \leq (r - 2) / 3$ is colored by color (2i+2), the vertices v_{2i} and v_{2i+2}, i = 4, 7, 10, 13, …, r-1 are colored by [2((i-3)/3)] and v_{2r+1} is colored by $\lceil 2r / 3 \rceil + 2$.

The above procedure guarantees a dominator coloring of G_r as v_1 dominates itself and even subscripted vertices dominate the color class 1. When $r = 3k$, $k \geq 1$, the vertex v_i, i = 3, 9, 15, …, 2r – 3 dominates color class of v_{i-1} and the vertices v_i and v_{i+2}, i = 5, 11, 17, …, 2r – 4 dominate the color class of v_{i+1}. When $r = 3k+1$, $k \geq 1$, the vertex v_i, i = 3, 9, 15, …, 2r+1 dominates color of v_{i-1} and the vertices v_i and v_{i+2}, i = 5, 11, 17, …, 2r – 3 dominate the color of v_{i+1}. When $r = 3k+2$, $k \geq 1$, the vertex v_i, i = 3, 9, 15, …, 2r-1 dominates color of v_{i-1}, the vertices v_i and v_{i+2}, i = 5, 11, 17, …, 2r – 3 dominate the color of v_{i+1} and the vertex v_{2r+1} dominates itself. So the given procedure gives a dominator coloring of G_r.

Hence $\chi_d(G_r) = \lceil 2r / 3 \rceil + 2$.

Theorem 2.6. For Flower graph Fl_n, $n \geq 3$, $\chi_d(Fl_n) = \begin{cases} 3 & \text{if } n \text{ is even} \\ 4 & \text{if } n \text{ is odd.} \end{cases}$

Proof. By the definition of flower graph, Fl_n is obtained from a helm graph by joining each pendant vertex to the central vertex. It has 2n+1 vertices and 4n edges. We refer the labelling given in the proof of theorem 2.5.

In Fl_n, v_1 is colored by color 1. When n is odd, the sequence of vertices v_i, $2 \leq i \leq n$ on the rim are colored by colors 2 and 3 alternatively and v_{n+1} is colored by 4. The remaining vertices v_i, $n+2 \leq i \leq 2n+1$ are colored by colors 3 and 2 alternatively. When n is even, the sequence of vertices v_i, $2 \leq i \leq n+1$ are colored by colors 2 and 3 alternatively and the remaining vertices v_i, $n+2 \leq i \leq 2n+1$ are colored by colors 3 and 2.

In Fl_n, the vertex v_1 dominates itself. The vertex v_i, $2 \leq i \leq 2n+1$ dominates the color class 1, since it is adjacent to v_1.

Hence $\chi_d(Fl_n) = \begin{cases} 3 & \text{if } n \text{ is even} \\ 4 & \text{if } n \text{ is odd.} \end{cases}$

Theorem 2.7. For Sun flower graph Sf_n, $n \geq 3$, $\chi_d(Sf_n) = \begin{cases} 3 & \text{if } n \text{ is even} \\ 4 & \text{if } n \text{ is odd.} \end{cases}$

Proof. By the definition of sun flower graph, Sf_n is obtained from a flower graph by joining each pendant edge to the central vertex. Let $V(Sf_n) = \{v_i / 1 \leq i \leq 3n+1\}$, where v_1 is the central vertex. It has 3n+1 vertices and 5n edges.

In Sf_n, v_1 is colored by color 1. When n is odd, the sequence of vertices v_i, $2 \leq i \leq n$ on n – cycle are colored by colors 2 and 3 alternatively and v_{n+1} is colored by 4. The vertex v_i, $n+2 \leq i \leq 2n+1$ is colored by colors 2 and 3 alternatively and the vertex v_i, $2n+2 \leq i \leq 3n+1$, is colored by any one of the colors 2, 3 or 4. When n is even, the sequence of vertices v_i, $2 \leq i \leq n+1$ are colored by colors 2 and 3 alternatively and the sequence of vertices v_i, $n+2 \leq i \leq 2n+1$ are colored by colors 3 and 2 and the vertex v_i, $2n+2 \leq i \leq 3n+1$ is colored by any one of the colors 2 or 3.

In Sf_n, the vertex v_1 dominates itself. The vertex v_i, $2 \leq i \leq 3n+1$ dominates the color class 1, since these vertices are adjacent to v_1. Hence $\chi_d(Sf_n) = \begin{cases} 3 & \text{if } n \text{ is even} \\ 4 & \text{if } n \text{ is odd.} \end{cases}$

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